Arithmetic Made Easy

APPROVED AS TEXT BOOK IN

1. The Punjab } *vide* Director's Circular No. 1750-G II., dated 31-1-33.

2 Burma } *vide* Director's Circular No 192 dated 38-6-34

3. Rajputana
4. Ajmer
5 Marwara } *vide* Prospectus of Rajputana Board 1935
6 Central India
7. Gwalior

8. Central Provinces
9 Berar } *vide* Secretary, High School Education Board's Order No 1027, dated 2-2-34.

10. Bombay Presidency } *vide* Bombay Govt *Gazette*, June 13, 1935 Notice No 86 (*a*)-1-c.

11 N W F. Province } *vide* Director's Circular No. 2447-200, dated 4-2-35

12 Sind Province } *vide* D P I's letter No. S—150 (*a*)-15940 E, dated 24-2 1938

ALSO APPROVED BY

The Directors of Education of Bikaner, Mysore, Jaipur, Jodhpur, Indore & Kotah States.

REVIEW.

This is a neatly got-up book for pupils preparing for the Matriculation and School Leaving certificate Examinations The authors have taken particular care in arranging the subject according to the capacity of the students and explaining the principles and suggesting the methods of solution. The Exercises are given with a view to stimulate interest. A cursory glance through the book shows that every effort has been made to help the students to grasp the main ideas in the mathematical problems. We can confidently recommend the book for use in schools. Its usefulness has been increased by the inclusion of a large number of examples from public examination papers. The pupil will get a practical knowledge of Arithmetic by using this book.

OPINION.

SHRI DURBAR HIGH SCHOOL, BUNDI.

July 21, 1933

You will be glad to know that your " Arithmetic Made Easy " has met our appreciation The book has been introduced as a Text-Book from the seventh class right up to the Tenth Class of the above institution. The book covers all the course prescribed by the Board of High School and Intermediate Education, Rajputana, Central India, and Gwalior for the High School examination Chapters have been arranged in a systematic order Easy and simple examples lead gradually to those of increasing difficulty. The book contains all types of questions Frequent use is made of diagrams for the purpose of illustration Algebraic methods have been used to help clearness and simplicity *I think there is no other more suitable book in the field in these days.* Thanking you as well as congratulating you on bringing out a so nicely got up and extremely useful book.

(Sd.)   C. H. D. SINGH.

Mathematical Teacher

Arithmetic Made Easy

FOR INDIAN SCHOOLS.

BY

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AND

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AUTHOR OF ALGEBRA MADE SIMPLE AND ATALIQ-I-HISAB ETC ,

The Mathematical House,

Mohan Lal Road, LAHORE.

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PREFACE.

The readiness with which our "Algebra Made Simple" was received by the teachers in our schools and the success that we achieved in that book prompted us to write this volume too.

We do not claim anything new or original in the book but *the system of grading almost the whole lot of examples and the method of solving them will be found by the teachers to be interesting and unique* The examples in each Chapter have been very carefully selected and then graded and we have freely used simple algebraical artifices to solve some of them which would have been otherwise difficult to do

Throughout the book will be found a very large number of examples , most of them, with the exception of a few at the end of some of the exercises and the three miscellaneous exercises, are rather of an easy type. We have made each Chapter of the book complete as far as it goes and hence some of the elementary portion of the book will be found to be rather difficult to students reading in the lower classess.

Many examples, especially those given in the Miscellaneous sets, have been taken from various Indian and foreign University papers, some of which have been appended, intact, at the end of the book only to show the students the nature of the questions set at University examinations.

We can boldly assert that there is no other book which contains all that is required from students preparing for the Matriculation Examination. We have explained all theorems as simply and methodically as possible.

In spite of our desire and our best efforts in that respect, we could not make the volume smaller but the teachers using the book in the class, might here and there omit the detailed descriptions of certain portions to save time

We spared no pains to make the printing and the general get-up of the book as best as we could but we are painfully conscious that there is yet much room for improvement and we intend to do the same in the next edition of the book, if the necessity for it ever arises

It is not impossible that there might have crept in some typographical errors here and there and we shall be greatly obliged if those, who use the book would kindly communicate to us any such mistake they might come across.

In conclusion, we shall feel our labours amply rewarded, if those for whom the book has been written, derive any the least benefit from it.

LAHORE   K M G.

10th February, 1930.   L R D

Preface to the Eighth Edition

In this edition, the book has been thoroughly revised. All the examples have been worked anew to ensure accuracy in the examples and the answers. We hope that no error now remains in this edition.

We render our hearty thanks to the teachers of Mathematics in the Province and outside who pointed out errors and communicated suggestions for the improvement of this book ˈ

LAHORE.   AUTHORS

July 1, 1936

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Arithmetic Made Easy

CHAPTER I.

NUMERATION AND NOTATION

§1   **Arithmetic** may be regarded as a science of num bers but it has a theoretical and also a practical side   In its theoretical part, it deals with numbers and enunciates gene ral principles obtained from an enquiry into their properties. In its practical part, it concerns itself with the application of these principles to specific numbers for practical purposes While a knowledge of the theory is essential for a clear understanding of the subject, an intelligent grasp of the principles underlying the solution of a problem or involved in the *modus operandi* of it or in deducing a formula to economise time and save labour is also important   Accuracy and rapidity are the principal objects to be borne in mind in actual practice.

§2   The digits used in Arithmetic are the following —

0,	1,	2,	3,	4,	5,	6,	7,	8,	9
zero,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine

Out of these, the figures 1 to 9 are called **significant** digits as distinguished from 0 which is called **insignificant** digit.   The fundamental digit is 1 which is often called a unit or unity, meaning a single object, the other digits 2, 3, 4, etc. therefore mean, 2, 3, 4, etc , units respectively Various numbers may be formed with these digits by repeating one or more of them any number of times or by using different digits and their repetitions.

These digits have also another value.   Thus in 27, the value of the digit 2 is not two but it means 2 tens, *i. e*, twenty   Hence every digit has a **local value** and an **intrinsic value**   In 33707 the local values of the digits 3, 3, 7, 0, 7 are 30000 (*viz* 3 ten thousands), 3000 (*viz* 3 thousands), 700 (*viz* 7 hundreds), 0 tens and 7 units respec-

tively , while the intrinsic values of 3, 7, 0 are 3 units, 7 units and nothing   0 is a digit having no digit value but it increases the local value of each digit to its left and does not affect the value of any digit succeeding it   The others are significant digits   In any number, the first significant digit is the one which has the highest local value.

§3   The above is the Arabic system of Notation The other system, *viz* , the Roman system of Notation is as follows —

I=1,V=5,X=10,L=50, C=100, D=500, M=1,000

I= 1	XI= 11	XXX= 30	CCC= 300
II= 2	XII= 12	XL= 40	CD= 400
III= 3	XIII= 13	L= 50	D= 500
IV= 4	XIV= 14	LX= 60	DC= 600
V= 5	XV= 15	LXX= 70	DCC= 700
VI= 6	.	LXXX= 80	DCCC= 800
VII= 7	and so on un-	XC= 90	CM= 900
VIII= 8	til we come to	C=100	LM= 950
IX= 9	XX= 20	CC=200	M=1000
X=10			ML=1050

It is clear from the above that I or X or L or C when placed to the right of a bigger number indicates addition and when placed to the left denotes subtraction   A bar placed over a number multiplies its value by one thousand, Thus $\overline{DC}$=600,000

§4.   The Indian system is like the Arabic system up to ten thousand [see § 7].   But in this system hundred thousand is called a **lac** and hundred lacs a **crore.**

§5   **Abstract** and **Concrete** numbers.   'Two books' or 'Three rupees' is said to be a concrete quantity because the number tells us not only *how many* things are taken but also of *what kind*   If there is no mention of the kind, the number is said to be abstract, *e g.,* 2, 3

The numbers 1, 2, 3,   are called **natural** numbers. The alternate numbers 1, 3, 5,   are called **odd** numbers and the alternate numbers 2, 4, 6,   are called **even** numbers Two or more numbers taken successively in any series are said to be **consecutive,** as 57, 58, 59 or 103, 104, 105, 106. etc

§6   The method of expressing numbers by means of the symbols or figures is called **Notation** and the art of reading in words a number expressed by figures is called **Numeration**, which is, therefore, the *reverse* of Notation

§7   The following scheme, known as the **Numeration Table** will show how to read a number in words ,—

16 Hundreds of thousands of billions	17 Tens of thousands of billions	16 Thousands of billions	15 Hundreds of billions	14 Tens of billions	13 Billions	12 Hundreds of thousands of millions	11 Tens of thousands of millions	10 Thousands of millions	9 Hundreds of millions	8 Tens of millions	7 Millions	6 Hundreds of thousands	5 Tens of thousands	4 Thousands	3 Hundreds	2 Tens	1 Unit

Note   A million billion is called **trillion**, a million trillion is called **quadrillion**   Similarly we have **quintillion**, **sextillion**, septillion, octillion, nonillion.

Example   In accordance with the above Table, write the numbers of (1) 35000478, (2) 760829145, (3) 31200780020 in words

Rule   *Put a comma after the first 3 figures from right, another comma after the next three figures, then a comma after every 6 figures respectively   Now the first figure in the first period stands for the unit's place, the second figure for ten's place, the third figure for the hundred's place   The 3 digits in the next period will give so many thousands and the next period of six will give so many millions, next billions and so on.   Hence*

(1) Thirty-five millions, four hundred and seventy-eight,

(2) Seven hundred and sixty millions, eight hundred and twenty nine thousand, one hundred and forty-five

(3) Thirty-one thousand and two hundred million seven hundred and eighty thousand and twenty.

EXERCISE A

Express each of the following in words —

1	3460	**2**	15070	**3**	200903
4	10005008	**5**	506720891	**6**	407080293

Express in figures .—

7.   Two hundred thousand and three hundred

8   Three million, five thousand and six

9.   Four hundred and six million, seven thousand and five

10   Four hundred thousand million and twenty-one

11   Six trillion, two hundred billion, twenty-seven thousand million, five hundred thousand and nine

Write down the local value of each digit in the following numbers —

12.	47012	**13**	1213456
14	4446607	**15**	37960012456

Express in Roman notation the following numbers —

16	37, 67, 79, 84, 99	**17**	107, 289, 702, 868
18	1920, 2070, 4659, 7200	**19**	200018, 10010001

Express in Arabic notation the following numbers —

20	DCLXMXXXII	**21**	MDCCCLXIII
22	MDCCCLXXX	**23**	MDCCLXVI
24	MMMMDCX	**25**	DLXCCXXVII.

26   Write the largest and the smallest numbers of 5 digits

27   Write the largest and the smallest numbers with the symbols 2, 5, 9, 0, 8

28   Write down all the numbers that can be formed by the digits 5, 4, 8 taken altogether.

29   What change takes place in the value of 5 in 23405 if it changes place with 3 ?

30.   What is the greatest number of five figures which begins with 7 and ends with 3 ?

31   What is the greatest number of six figures which begins with 8 and ends with 2 ?

32   Form the biggest number and the smallest number by the digits of the number 2304951

CHAPTER II

THE FOUR FUNDAMENTAL OPERATIONS.

ADDITION

§1   To add is to find a number equal to two or more numbers taken together   The number thus obtained is called their **sum** or **amount** and the numbers to be added are called **summands**.

§2   **The sign of addition** The sign + is read **plus.** It is used to indicate that the numbers between which it is placed are to be added.   Thus 8 + 9 means that 9 is to be added to 8   It is read 'eight *plus* nine.'

§3   **The sign of equality** The sign = is called the sign of equality   It is read **equal** or **is equal to**   Thus 8 + 9 = 17 indicates that the sum of 8 and 9 is equal to 17. It will be read " eight plus nine is equal to seventeen ".

§4   Accuracy and rapidity in the process of addition depends upon learning thoroughly the **addition table** (which we subjoin here)

1 and 1 are 2	2 and 1 are 3	3 and 1 are 4	4 and 1 are 5	5 and 1 are 6	6 and 1 are 7	7 and 1 are 8	8 and 1 are 9	9 and 1 are 10
2 3 2	4 2	5 2	6 2	7 2	8 2	9 2	10 2	11
3 4 3	5 3	6 3	7 3	8 3	9 3	10 3	11 3	12
4 5 4	6 4	7 4	8 4	9 4	10 4	11 4	12 4	13
5 6 5	7 5	8 5	9 5	10 5	11 5	12 5	13 5	14
6 7 6	8 6	9 6	10 6	11 6	12 6	13 6	14 6	15
7 8 7	9 7	10 7	11 7	12 7	13 7	14 7	15 7	16
8 9 8	10 8	11 8	12 8	13 8	14 8	15 8	16 8	17
9 10 9	11 9	12 9	13 9	14 9	15 9	16 9	17 9	18

This table can be extended for numbers larger than ten   We give here some questions for practice

1   Add 2 to   2,   4,   6 and so on up to 100.
2.   „   3 „   3,   6,   9   „   „   99
3   „   4 „   4,   8, 12   „   „   100
4   „   5 „   5, 10, 15   „   „   100.

5   Add 6 to   6, 12, 18 and so on up to 96.
6   „   7 „   7, 14, 21   „   „   98
7   „   8 „   8, 16, 24   „   „   96
8   „   9 „   9, 18, 27   „   „   99.
9. „ 10 „ 10, 20, 30   „   „   100

Such exercises are very useful   The teacher can make the boy practice some more exercises beginning   each exer cise with any other number

The following is the rule for addition —

Rule   *Write down the numbers to be added one under another so that units may come under units, tens under tens, hundreds under hundreds, and so on, then draw a straight line under the last number. Now add up the figures in the first vertical column on the right hand   Put down the last figure of this sum beneath the line and carry the remaining figure or figures to the preceding column. Proceed in the same manner till the last column is added   Put down the complete sum of the last column   The result thus obtained will be the sum or amount of the summands*

Example 1.   Add together 8935,   8205,   5576, 89375 and 27.

Sol   Arrange the numbers according to   the rule given above and proceed thus —

8935 1st col 5 + 5 + 6 + 5 + 7 = 28 , put down 8 & carry 2
8205 2nd „   2 + 3 + 0 + 7 + 7 + 2 = 21, „   1   „   2
5576 3rd „   2 + 9 + 2 + 5 + 3   = 21, „   1   „   2
89375 4th „   2 + 8 + 8 + 5 + 9   = 32, „   2   „   3
27   5th „   3 + 8   = 11, „   11

112118   The sum is 112118 **Ans.**

Test of Correctness   We can test the correctness of the addition by *casting out nines.*   Thus —

Cast out nines from the sum of the digits of each summand   and note down the remainders , then cast out nines from the sum of the remainders and note down the remainder   This remainder will correspond with the remainder if we cast out nines from the digits of the sum

Obesrve the test of the abova examples

$$8935 \begin{vmatrix} 8+9+3+5 \end{vmatrix} = 25 = 18 \div 7, \textbf{7} \text{ is rem of 1st smd.}$$
$$8205 \begin{vmatrix} 8+2+0+5 \end{vmatrix} = 15 = 9+6, \textbf{6} \quad \text{,,} \qquad 2\text{nd} \quad \text{,,}$$
$$5576 \begin{vmatrix} 5+5+7+6 \end{vmatrix} = 23 = 18+5, \textbf{5} \quad \text{,,} \qquad 3\text{rd} \quad \text{,,}$$
$$89375 \begin{vmatrix} 8+9+3+7+5 \end{vmatrix} = 32 = 27+5, \textbf{5} \quad \text{,,} \qquad 4\text{th} \quad \text{,,}$$
$$\underline{27} \begin{vmatrix} 2+7 \end{vmatrix} = 9 = 9+0, \text{no rem} \qquad 5\text{th} \quad \text{,,}$$
$$112118 \begin{vmatrix} \textbf{7+6+5+5} \end{vmatrix} = 23 = 18+5, \text{ 5 is the final rem}$$
$$1+1+2+1+1+8 = 14 = 9+5, \text{ 5 is rem of the sum}$$

Since the final remainder corresponds with the remainder of the sum, the operation is correct

EXERCISE 1.

Add together —

1		2		3.		4.	
	21		28		38		58
	25		56		48		59
	35		46		59		75
	55		57		81		96
5	872	6	721	7	981	8	321
	537		826		302		526
	935		156		856		905
	821		322		706		738
9.	8937	10	5304	11.	89356	12	79356
	531		728		3562		215302
	7431		15631		302		18019
	295		439		160025		514
	302		8050		56829		918202

Find the value of —

13   $8502 + 502 + 90321 + 83562 + 1613204.$
14   $5230 + 1519 + 21203 + 25625 + 1635209.$
15   $1802 + 27 + 124 + 335 + 95639 + 1835234.$

Find the sum of the following and test the correctness of the sum —

16   15935, 8102, 9356304, 1650235.
17   89, 514, 893521, 89356201, 21935.
18.   165, 9356, 85312, 5010101, 89356

Add together (19 and 20) .—

19   Nine hundred and twenty-seven , fifty thousand and eleven , five thousand and nine , eighty-five thousand six hundred and twenty-six , two thousand six hundred and nine

20   Three hundred and   forty nine   thousand and
eleven , ten million, five hundred and six , nine thousand
and sixteen , sixty-five thousand and two , five hundred and
sixty thousand and seven

21   There are 3 trees in my garden , one of them has
574 apples on it, the second has 200 and the third has 426
on it   How many apples are there in my garden ?

22   I was born in the year 1875 A D , in which year
shall I be 60 years old ?

23.   A student has to read 5 books for his examination,
there are 400 pages in one book, 526 in the second, 74 in
the third, 129 in the fourth and 83 in the fifth   How many
pages is he to read altogether ?

24   The Bombay Mail carried yesterday 37 first class
passengers, 80 second class and 154 inter-class , how many
did it altogether carry ?

25   In the year 1932 A D , there were 31 days in
January, 29 in February, 31 in March, 30 in April, 31 in
May, 30 in June, 31 in July, 31 in August, 30 in September, 31 in October, 30 in November and 31 in December
How many days are there in the whole year ?

26   A man had some money in a Bank , he withdrew
Rs 476 and then again Rs 294 and then found that there
is a balance of Rs 300 left   How much had he in the Bank ?

27   In a certain town, there are 948230 Mohamedans,
8704213 Hindus, and 25107 other castes   What is the total
population of the town ?

28   In one of the divisions of a province, there are
4031490 Hindus in another 3920076, in a third 521076
and in the fourth 1234060   What is the total Hindu population in the whole province ?

29   A has Rs 125 , B has Rs 37 more than A, C has
Rs. 20 more than what A and B together have and D has
as much as the three others together have.   What is the
total sum that they have ?

30   In the High Department of a school there are
592 boys , in the middle 705 and in the Primary Department
1704.   What is the total numerical strength of the school ?

SUBTRACTION

§5 Subtraction is the method of finding what number is left, when a smaller number is taken from a greater one

The number thus left is called the **remainder**, the number to be subtracted is called the **subtrahend** and that from which it is subtracted, the **minuend.**

§7 The **sign of minus**   The sign '—' is read minus The sign when placed between two numbers signifies that the second number is to be subtracted from the first Thus '8 — 5' means that 5 is to be subtracted from 8   It is read as eight *minus* five

The symbol '⌐' is called the sign of difference. It indicates that the smaller number is to be subtracted from the greater one   Thus '5⌐8 means 3

§7   Like addition table, to effect the operation of subtraction the students should commit the following **subtraction table** also to memory

1 from	2 from	3 from	4 from	5 from	6 from	7 from	8 from	9 from
2= 1	3= 1	4= 1	5= 1	6= 1	7= 1	8= 1	9= 1	10= 1
3 „ 2	4 „ 2	5 „ 2	6 „ 2	7 „ 2	8 „ 2	9 „ 2	10 „ 2	11 „ 2
4 „ 3	5 „ 3	6 „ 3	7 „ 3	8 „ 3	9 „ 3	10 „ 3	11 „ 3	12 „ 3
5 „ 4	6 „ 4	7 „ 4	8 „ 4	9 „ 4	10 „ 4	11 „ 4	12 „ 4	13 „ 4
6 „ 5	7 „ 5	8 „ 5	9 „ 5	10 „ 5	11 „ 5	12 „ 5	13 „ 5	14 „ 5
7 „ 6	8 „ 6	9 „ 6	10 „ 6	11 „ 6	12 „ 6	13 „ 6	14 „ 6	15 „ 6
8 „ 7	9 „ 7	10 „ 7	11 „ 7	12 „ 7	13 „ 7	14 „ 7	15 „ 7	16 „ 7
9 „ 8	10 „ 8	11 „ 8	12 „ 8	13 „ 8	14 „ 8	15 „ 8	16 „ 8	17 „ 8
10 „ 9	11 „ 9	12 „ 9	13 „ 9	14 „ 9	15 „ 9	16 „ 9	17 „ 9	18 „ 9

§8.   A question of subtraction can be put in many different ways   Thus   "What is the difference between 9 and 4 ?" may be put in the following ways also —

1   Find the difference between 9 and 4

2   By how much is 4 less than 9 ?

3.   By how much is 9 greater than 4 ?

4   What must be added to 4 to make 9 ?

5   What must be the remainder when 4 is taken from 9 ?

6.   By how much does 9 exceed 4 ?

7.   The sum of two numbers is 9   One number is 4, find the other.

§9   The following is the rule for subtraction :—

Rule   *Write down the smaller number under the greater as in the process of addition and draw a line underneath   Begin at the right hand and subtract each digit of the subtrahend from the corresponding digit of the minuend and put down the result below the line, but if any lower digit be greater than the upper, add ten to the upper digit and then take the lower digit from the upper number thus formed , put down the remainder as before   In this case add 1 to the next lower digit before carrying on the process of subtraction further*

Example 1   Subtract 2135 from 9849.

Sol   Arrange the numbers according to the rule given above and proceed thus —

Explanation

9849 minuend,
2135 subtrahend,   { $9-5=4$, put down 4, $4-3=1$, put down 1
7714 rem **Ans**   { $8-1=7$ „   7, $9-2=7$,   „   7

Example 2

Subtract 5639 from 8254

Sol.   *Explanation*

8254 minuend,
5639 subtrahend,
2615 remainder.
Ans

9 cannot be taken from 4, we add 10 to the 4 making it 14, $14-9=5$, put down 5   Now add 1 to the next lower figure 3 making it 4 , $5-4=1$, put down 1   Similarly add 10 to the 2 making it 12, $12-6=6$, put down 6   Now add 1 to the next lower figure 5 making it 6, $8-6=2$, put down 2.

Test of correctness   Add   subtrahend   and   the remainder, the sum will correspond with the minuend

Or,

'Cast out nines as in addition   The difference of the remainders of the *minuend* and *subtrahend* will correspond with the remainder of the *remainder*.

Observe the test of the second example

8254 } $8+2+5+4=19=18+1$, 1 is the remainder.
5639 } $5+6+3+9=23=18+5$, 5 is the remainder.
——
2615 } $2+6+1+5=14=9+5$, 5 is the remainder

Here add 9 to the remainder of the minuend because it is less than the remainder of the subtrahend. $9+1=10$, $10-5=5$ which corresponds with the remainder of the remainder.

§10 A number preceded by the sign $+$ is called a positive number and that preceded by the sign $-$, is a negative number When no sign is affixed to a number, it is considered as positive

§11 **Expressions and terms** An **expression** is one in which two or more numbers are connected by the sign $+$ or $-$ and the numbers thus connected are called **terms** Thus $8+6-4-2$ is an *expression* and 8, 6, -4, -2 are its *terms*, 8, 6 are *positive* 4, 2 are *negative*

Example 3 Find the value of—

$$1329-576+504-302-29$$

Sol. Sum of positive terms $=1329+504=1833$
Sum of negative terms $=576+302+29=907$.
Value required $=1833-907=926$. **Ans.**

Hence the following

Rule *Subtract the sum of all the negative terms from the sum of all the positive terms*

EXERCISE 2.

Perform the following subtractions—

1.	9534	2.	8956	3	6359	4	8937
	2313		3524		2048		2513

5	85629	6.	34956	7	83569	8	20001
	49393		21099		64397		17354

9.	201319	10.	301001	11.	71935	12	900090
	189352		89599		49296		499938

Simplify —

13. $265302-189543$ 14 $169356-89354$.
15. $290093-109989$. 16 $793590-189056$.
17 $825+576+328+21-675$.
18 $1817-936-1021+676-75$
19. $15012+21-21025+80023-65233$.
20 $1002+2512-3409+89+185$.

Complete the following sums of addition by filling in the digits whose places are marked by asterisks —

21	5132	22	3'256	23	83256	24	19516
	3 *2		29' *5		*2'3*		*3*
	*894		*4731		*756*5		*8753

Complete the following sums of subtraction by filling in the digits whose places are marked by asterisks —

25.	7895	26	2835	27	25329	28	10000
	3 *		* 9		' 98		*35
	+568		14 '9		'62 *		**

29. The sum of two numbers is 1893609 and one of them is 930994 , find the other

30 Minuend is 803563 and the remainder 15102 Find the subtrahend

31 A deposited Rs 576 in the bank on Monday, Rs 708 on Tuesday, Rs 2105 on Thursday B deposited Rs 376 in another bank on Monday, Rs 728 on Tuesday, Rs 1689 on Thursday By how much does A's money exceed B's money in the bank ?

32 By how much does 1002490 exceed 704050 ?

33 What is the excess of 9497605 above 8888888 ?

34 What must be added to the sum of 432, 507, 78, 710 and 803 in order to get the number 3000 ?

35 A man earns Rs 4000 in a month and he spends Rs 3702 What is his saving in a month ?

36 In the year 1932 A D , I was 57 years old, when was I born ?

37 Hari was born in 1875 A D , what was his age in 1932 ?

38 The population of a town is 17254120, out of which there are 8624022 males Find the number of females.

39. I bought a house at Rs 8249 and spent Rs. 1200 on its repairs I then sold it for Rs 9000. How much did I lose or gain ?

40 My son was born in the year 1902 When would he have been as old as I was in the year 1909, my year of birth being 1875 ? How old would I then have been ?

41. One mountain is 13572 ft high and another is 9024 ft. high How much is the former higher than the latter ?

42 I had Rs. 682 with me   I gave Rs 205 to a friend, Rs 82 on charity and spent Rs. 349 on my own account   Find how much money is left with me.

43. I spent Rs. 13562 for a house and some land , the land alone was worth Rs 8024 , what was the value of the house ?

44 A man had one lac of rupees at the time of his death. He gave Rs 3120 to his son , Rs 2090 to his daughter and the remainder to his wife. How much did the wife get ?

MULTIPLICATION

§12   Multiplication is the short method of finding the sum of a given number of repetition of a certain number. Thus to find the sum of six times 12, it will be

$$12 + 12 + 12 + 12 + 12 + 12 = 72$$

The number to be repeated is called the **multiplicand** and the other which indicates the number of repetitions is the **multiplier**, and the sum obtained the **product**   Thus in the above solution 12 is the *multiplicand*, 6 is the *multiplier* and 72 is the *product*.

§13   The sign of multiplication is ×. It is read as **into**   It is used to indicate that the numbers between which it is placed are to be multiplied together. Thus 12 × 6 which is read *twelve into six* indicates that 12 is to be multiplied by 6

Each of the numbers multiplied together is called a **factor**   Thus 12 and 6 are the factors of 72.

Note   Sometimes the operation of multiplication is expressed by a dot   Thus 12 × 6 may be written 12 6

§14   Rapidity and accuracy in multiplication depends upon the complete familiarity with **multiplication tables** For the sake of convenience we subjoin the table here

Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	16	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

⁋15. To prove that four times six＝six times four

four times six＝6＋6＋6＋6＝24,
six times four＝4＋4＋4＋4＋4＋4＝24

Or,

Let 24 counters be arranged as in the figure

Now, No of counters＝No of horizontal
rows × No of counters in each row＝4×6
    Also, No, No of counters＝No of vertical
rows × No. of counters in each row＝6×4
    four times six＝six times four.

It follows therefore that the *factors of a product may
be taken in any order   Or the multiplicand and the mul-
tiplier may be interchanged without altering the product.*

§16   Short Multiplication

When the multiplier does not exceed 20, the multi-
plication is called short multiplication

    Rule   *Write down the multiplier under the multipli-
cand and draw a line underneath   Begin at the unit's
figure of the multiplicand and multiply each figure by the
multiplier, putting down and carrying precisely as in
addition.*

Example 1.   Multiply 45321 by 9   45321
    Exp   9
(i)   9 times 1＝9, put down 9   ——————
(ii)   9 times 2＝18, put down 8 and carry 1   407889
(iii)   9 times 3＝27, 27＋1＝28, put down 8 and carry 2.
(vi)   9 times 5＝45, 45＋2＝47, put down 7 and carry 4.
(v)   9 times 4＝36, 36＋4＝40, put down 40

    The product ＝407889 Ans

§17. Multiplication by a simple number followed
by zeroes

    Rule. *Multiply the multiplicand by the simple num-
ber and add as many zeroes to the right of the product as
there are zeroes to the right of the multiplier.*

Example 2. Multiply 5374 by 900.

Sol   5374 × 900 = 5374 × 9 hundreds
                                    = 48366 hundreds
                                    = 4836600 **Ans**

Note   In multiplication when one factor is 0, the product is also 0   Thus 165 × 0 = 0 and 0 × 165 = 0

EXERCISE 3

Multiply—

1.	892 by	7	2	5603 by	8.	3	5302 by	12.
4	5023 by	13	5	7089 by	14	6	8972 by	15
7.	6305 by	16	8.	8925 by	17	9	12315 by	18.
10	5605 by	10	11	6325 by	20	12	7321 by	100
13.	6329 by	500	14	8935 by	6000	15.	21526 by	8000

§18   Long Multiplication

When the multiplier exceeds 20, the multiplication is called long multiplication

Rule   *Write the multiplier below the multiplicand and draw a line underneath   First multiply the multiplicand by the unit's figure of the multiplier and then multiply the multiplicand by ten's figure of the multiplier and put the product under the first product so that the unit's figure of this product may fall under the ten's figure of the first. Repeat this operation with hundreds, thousands, etc.*

Example 3   Multiply 8556 by 237.

Sol.   Proceeding by the rule given above

$$
\begin{array}{r}
8556 \\
237 \\
\hline
\end{array}
$$

8556 × 7 = 59892   59892
8556 × 3 = 25668   2566
8556 × 2 = 17112   17112

                                    2027772 **Ans**

Test of correctness (*i*) Interchange the multiplicand and the multiplier , the product will be the same

(*ii*) Cast out nines from the multiplicand and the multiplier , multiply the remainders and again cast out nines The remainder now obtained will correspond with the

remainder which remains after casting out nines from the product

8556	$8+5+5+6=24=18+6$, **6** is the remainder.
237	$2+3+7\quad =12=9\ +3$, **3** is the remainder.
59892	$6\times 3\qquad =18=18+0$, **0** is the remainder
25668	
17112	
2027772	$2+0+2+7+7+7+2=27=27+0$, **0** is the rem

multiplicand
&
multiplier

multiplicand 6 $\times$ multiplier 3

0

product
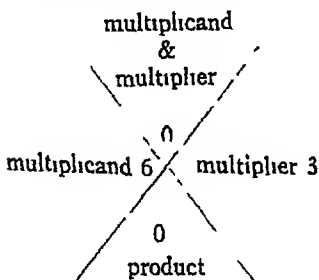
Or

$$\begin{array}{r} 237 \\ 8556 \\ \hline 1422 \\ 1185 \\ 1185 \\ 1896 \\ \hline 2027772 \end{array}$$

Example 4   Multiply 53728 by 2005
Sol.   Proceeding by the rule given above

$$\begin{array}{r} 53728 \\ 2005 \\ \hline \end{array}$$

$53728\times 5 \qquad\qquad =268640$ 268640
53728×2 thousands $=107456$ thousands 10745600

107724640 **Ans.**

§19.   If there are zeroes to the right of the multiplicand or multiplier or both, the zeroes may be neglected and the remaining figures may be multiplied together and the neglected zeroes may be annexed in the product.

Example 5.   Multiply 85300 by 19200.
Sol

$$\begin{array}{r} 853 \\ 192 \\ \hline 1706 \\ 7677 \\ 853 \\ \hline 163776 \end{array}$$

Neglecting zeroes, the product$=163776$

∴ the req product$=1637760000$ **Ans.**

F 2

§20   Multiplication by factors

Example 6   Multiply 728 by 192.

Sol $192 = 16 \times 12$

$728 \times 12 = 8736$
$8736 \times 16 = 139776$

$$\begin{array}{r} 728 \\ 12 \\ \hline 8736 \\ 16 \\ \hline 139776 \end{array} \text{ Ans}$$

§21   Continued product

Multiplication of more than two numbers is called their **continued product**   Thus $728 \times 12 \times 16$ is an example of continued product, which has been solved above

§22   When a number is multiplied by itself once, twice, thrice, etc , the product is called second, third, fourth, etc , **power** of that number respectively   Thus

6×6 is called the **second power** of 6,
$6 \times 6 \times 6$ is called the **third power** of 6,
$6 \times 6 \times 6 \times 6$ is called the **fourth power** of 6,

and so on

6×6 is written 6^2 and is read as "6 squared"
$6 \times 6 \times 6$ „ 6^3 „   „   "6 cubed"
$6 \times 6 \times 6 \times 6$ „ 6^4 „   „   "6 to the fourth power"
Similarly 6^5 is read as   "6 to the fifth power",
as so on

§23   **Index**   The small figures which indicate how many times the number is multiplied is called the **index** or **exponent** of the power   Thus in 6^5, 5 is the index

§24   If the three signs $+$, $-$, $\times$, are used in an expression, the operation of *Multiplication* is to be performed first and then that of *Addition* or *Subtraction*

Thus, $5 \times 2 - 3 \times 4 + 2 \times 4 = 10 - 12 + 8$
$= 18 - 12 = 6$ Ans

EXERCISE 4

Multiply by factors —

1	5635 by 156.	2	5621 by 108
3.	8551 by 117.	4	25301 by 256.
5	36205 by 208,	6	21935 by 132.

Find the continued product of —

7	$65 \times 18 \times 16$	8.	$56 \times 25 \times 36$
9	$76 \times 21 \times 49$	10	$45 \times 35 \times 25$
11.	$28 \times 38 \times 48$	12	$55 \times 45 \times 65$

Multiply and test the correctness of the following —

13.	8935	by 1890.	14	56389	by 15600
15	56000	by 27000	16	51800	by 45000
17	50009	by 25005	18	20359	by 250025
19	189725	by 41935	20.	283561	by 283562.

Find the squares of —

    21.   45, 71, 121, 125 and 256.

Find the cubes of —

    22   11, 21, 25, 36 and 40

Find the value of —

    23    $9 \times 3 - 5 \times 5 + 7 \times 8 + 121$.

    24    $25 \times 6 + 36 \times 4 - 28 \times 8 + 51$

    25    $37 \times 9 - 28 \times 12 + 35 - 17 \times 21 + 576$.

    26   192 pies make one rupee. How many pies are there in Rs 374 ?

    27   A book contains 579 pages and each page contains 3749 letters. How many letters are there in the whole book ?

    28   The price of one Bigha of land is Rs. 904. Find the price of 98 Bighas

    29.   8634 men pass over the Ravi Bridge every day How many men will pass over it in 706 days ?

    30   The cost of one elephant is Rs 3479 and that of a horse is Rs 765 How much will 6 elephants and 16 horses cost ?

    31   There is a tank of water with a hole in it. 78 gallons of water leak out by the hole in each hour. When the tank is full of water, it is emptied in 48 hours Find the capacity of the tank

    32   The population of a town is 127600 It increases at the rate of 320 persons each year. What will be the population 10 years hence ?

    33.   A Railway train runs at an average speed of 22 miles an hour It takes 42 hours to run from Calcutta to Delhi Find the distance between Calcutta and Delhi.

34. The rays of light come from the sun to the earth in 448 seconds. Light moves at the rate of 186000 miles per second. Find the distance of the earth from the sun

DIVISION.

§25 Division is the method of finding how many times one number is contained in another. The former of these numbers is called the **divisor** and the latter the **dividend** and the number of times found the **quotient**. That which is left after the operation is finished is the **remainder**

Suppose we want to know how many times 6 is contained in 25. By the help of the multiplication table, we can at once tell "4 times, with a remainder 1"

$$
\begin{array}{ll}
25 & \\
\underline{6} & \text{once} \\
19 & \\
\underline{6} & \text{twice} \\
13 & \\
\underline{6} & \text{thrice.} \\
7 & \\
\underline{6} & \text{4 times.} \\
1 &
\end{array}
$$

The operation at the side also shows the same result.

Here 25 is the *dividend*, 6 is the *divisor*, 4 is the *quotient* and 1 is the *remainder*

From above it follows that

Divisor × quotient + remainder = dividend

§26 The sign of division is — It is read 'divided by'. 25—6 denotes 25 divided by 6

§27. Short Division

When the divisor does not exceed 20, it is called **short division**. The operation in this case is done mentally. The accuracy depends upon the complete familiarity with the multiplication table

Example 1. Divide 8972 by 12

Sol. Here 12 goes no time in 8, but it goes 7 times in 89 and 5 over. Put down 7 under the 9

$$12)\underline{8972}$$
$$747—8 \text{ rem}$$

Then 12 goes 4 times in 57 and 9 over. Put down 4 under the 7

Lastly 12 in 92 goes 7 times and 8 over.

The quotient is 747 and the remainder 8. **Ans**

§28 Long division.

When the divisor exceeds 20, it is called long division
The rule for operation is as under .—

> **Rule.** *Write the divisor and the dividend thus*
> *divisor) dividend (*

*Now, try to find how many times the one or two
figures on the left hand of the divisor contain in the one
or two figures on the left hand of the dividend and write
the result as the figure (on the left) of the quotient.
Multiply this partial quotient by the divisor and put down
the product under the dividend on the left hand side
Subtract it and write down the remainder. Annex to the
right of this remainder the next figure of the dividend to
form the next partial dividend Proceed as before-and
continue the process till all the figures of the dividend
have been brought down If at any stage the divisor is
found greater than the partial dividend, put a zero to the
quotient and bring down the next figure of the dividend.*

Example 2 Divide 537089 by 328

Sol *Explanation.*

328)537089(1637
   328
  2090
  1968
   12_8
   984
  2449
  2296
   153

Here 3 goes once in 5   Put 1 as the
first figure of the quotient. Multiply this
one by 328 and put down the product
under the dividend as shown in the process.
The remainder is 209. Bring down the
next figure 0 to form the partial dividend
2090 Now 3 goes 6 times in 20   Put 6
as the second figure of the quotient Mul-
tiply this 6 by 328 and put the product
under the partial dividend The next remainder is 122 Bring
down the next figure 8 to from the next partial dividend
1228 Again 3 goes 4 times in 12, but if we put 4 as the next
figure of the quotient and multiply this 4 by 328 the product
will exceed the partial divended 1228, therefore put 3 as the
next figure of the quotient Multiply this 3 by 328 and put
down the product under 1228 The remainder is 244 Bring
down the next figure 9 to form the last partial dividend

2449   3 goes 8 times in 24, but the product will exceed the dividend, therefore put 7 as the last figure of the quotient. Multiply this 7 by 328 and put the product under the last partial dividend   The remainder is 153

the whole quotient is 1637 with a remainder 153. **Ans.**

Test of Correctness (i) Multiply the divisor by the quotient and add the remainder to the product. If the result corresponds with the dividend the answer is correct. Thus, $1637 \times 328 = 536936$ Adding the remainder to it ($536936 + 153 = 537089$) the result corresponds with the dividend. Hence the answer is correct

(ii) Cast out nines from the *divisor* and *quotient* and write down the remainders   Multiply these remainders and again cast out nines. The remainder now left will correspond with the remainder obtained by casting out nines from the difference of the *dividend* and the *remainder*

$$328)537089(1637$$
$$\underline{\text{Rem.}\quad 153}$$

Diff. of the Dividend and the Rem.$=$ 536936

$3 + 2 + 8$ $= 13 = 9 + 4$, 4 is the remainder of Divisor.
$1 + 6 + 3 + 7$ $= 17 = 9 + 8$, 8 is the remainder of Quotient
4×8 $= 32 = 27 + 5$, 5 is the final remainder.
$5 + 3 + 6 + 9 + 3 + 6 = 32 = 27 + 5$, 5 is the rem of Difference

. the final remainder corresponds with the remainder of the difference

. the answer is correct.

Example 3. Divide 368002 by 892.

Sol   *Explanation.*

$892)368002(412$
$\underline{3568}$
1120
$\underline{892}$
2282
$\underline{1784}$
498

Here 8 in 3 goes no time. There-fore take two figures on the left hand of the dividend. Now 8 in 36 goes 4 times
Now proceed as explained in Ex 2
The quotient is 412 and the remainder is 498. **Ans**

§29   **Division by 10, 100, 1000, and so on.**

Rule.   *To divide a number by 10, 100, 1000 and so*

on, cut off as many figures from the right of the dividend as there are zeroes in the divisor, the figures thus cut off will be the remainder and the remaining figures will form the quotient

Example 4   18693÷10   =quotient 1869, rem 3
Example 5   18693÷100   =quotient 186, rem 93.
Example 6   18693÷1000 =quotient 18, rem 693.
Example 7.   18693÷10000 =quotient 1, rem 8693

§30   Division by factors

This method is very important The student is advised to note the solution very carefully and learn how to find out the complete remainder.

Example 8. Divide 85357 by 120, using factors $4 \times 5 \times 6$

Sol.   4| 85357
       5| 21339, 1 unit   = 1
       6| 4267, 4 groups of 4 units   = 16
             711, 1 group of 4×5 units each = 20
                    remainder   = 37

quotient=711 , remainder=37. **Ans**

From the above solution we deduce the following

Rule. *Complete remainder = 1st rem. + (2nd rem × 1st divisor) + (3rd rem × 1st divisor × 2nd divisor) + etc*

Example 9. Divide 415635 by 180 using factors $2 \times 3 \times 5 \times 6$

Sol. 2|415635
      3|207817, 1 unit   = 1
      5| 69272, 1 group of 2 units   = 2
      6| 13854, 2 groups of 2×3 units each   = 12
          2309, no group of $2 \times 3 \times 5$ units   = 0
                      remainder   = 15

quotient=2309 , remainder=15. **Ans**

EXERCISE 5.

(Divide (1—12) —

1	56352 by 12	2	36432 by 18
3	106799 by 19	4.	118456 by 221
5	105795 by 521	6	535001 by 625
7	285359 by 824	8.	910025 by 728
9.	565902 by 756.	10	889956 by 1625
11	215623 by 9535.	12	853699 by 9999

13   Divide 56535698 separately by 10, 100, 1000, 10000

14   Divide 89300022 separately by 10, 100, 1000, 10000

15.   Divide 48300987 separately by 100, 1000, 10000, 100000

Divide by factors and find the complete remainders in (16—19) —

16	1563557 by 144	17.	4800935 by 288
18.	56359281 by 256	19.	3790289 by 132

20.   Divide 5630298 by 5, 6, 7 in succession and find the complete remainder

21   Divide 21908956 by 7, 9, 6 in succession and find the complete remainder

22.   A number is divided by 5, 6 and 7 in succession and the remainders are 3, 2, and 5 respectively.   Find the remainder had it been divided by 210

23   A number is divided by 9, 8, 12 in succession and the remainders are 7, 4 and 2 respectively   Find the remainder had the number been divided by 864

24   A number is divided by 5, 6, 9 in succession and the remainders are 2, 3, 4 respectively   Find the remainder had the number been divided by 270.

----

§31   If all the four signs +, —, ×, ÷ occur in an expression, the operation of *Division* is to be performed first, and then of *Multiplication* and next those of *Addition* and *Subtraction*

· ·   Thus $9-3+2 \times 12-4+5-3 \times 2$
$$= 3 + 2 \times 3 + 5 - 6$$
$$= 3 + 6 + 5 - 6 = 8.   \text{Ans.}$$

EXERCISE 6

Simplify the following expressions :—

1. $12 \times 6 - 3 + 2.$ 2. $15 - 5 \times 4 - 1$
3. $21 \div 7 \times 3 - 4$ 4. $25 \times 15 - 3 - 25.$
5. $28 - 25 \div 5 \times 2.$ 6. $45 + 51 - 17 \times 3$
7. $56 \times 81 \div 27 - 168$ 8. $55 - 11 - 88 \div 44.$
9. $45 \times 15 - 121 \div 11.$ 10. $144 \div 18 \times 3 - 14$
11. $196 - 28 \times 3 - 45 \times 15 \div 5 + 125.$
12. $196 \div 28 \times 5 + 56 \times 21 \div 7 - 150$
13. $169 \div 13 + 168 - 14 + 144 - 18 \times 3.$
14. $226 \times 16 \div 8 + 5 \times 38 - 324 - 81.$
15. $117 \div 13 \times 9 - 81 + 104 - 26 \times 4$

Abbreviated Methods of Multiplication.

§32   To multiply a number by 10, 100, 1000, etc.

Rule   *Add as many zeroes to the multiplicand as there are zeroes in the multiplier*

Example 1   Multiply 852 separately by 10 and 1000.

Sol $852 \times 10 = 8520$, $852 \times 1000 = 852000.$ Ans

§33.   To multiply a number by 20, 30, 40, etc

Rule   *Add as many zeroes to the multiplicand as there are in the multiplier and then multiply the resulting number by the remaining digit or digits in the multiplier.*

Example 2   Multiply 5025 separately by 30 and 700.

Sol $5025 \times 30 = 50250 \times 3 = 150750$
$5025 \times 700 = 502500 \times 7 = 3517500$ } Ans.

§34.   To multiply a number by 11, 101, 1001, etc

Rule.   *Add 1, 2, 3, zeroes respectively to the multiplicand and then add the multiplicand to the resulting number.*

Example 3.   Multiply 5023 separately by 11 and 1001.

Sol. $5023 \times 11 = 50230 + 5023 = 55253$
$5023 \times 1001 = 5023000 + 5023 = 5028023$ } Ans.

§35.   To multiply a number by 5.

Rule   *Annex a zero to the right of the multiplicand and then divide it by 2,*   . $5 = 10 \div 2.$

Example 4.   Multiply 89356 by 5.

Sol. $89356 \times 5 = 893560 \div 2.$

' the req product=446780 Ans

§36 To multiply a number by 15.

Rule. *Annex a zero to the right of the multiplicand and to it add its half*

Example 5 Multiply 56359 by 15
Sol 56359×15 $=563590 + 281795$
    the req product=845385. Ans

§37 To multiply a number by 25

Rule. *Annex two zeroes to the right of the multipli cand and then divide it by 4,* $25 = 100 - 4.$

Example 6 Multiply 890023 by 25
Sol $890023 \times 25 = 89002300 - 4$
    the req product = 22250575. Ans.

§38. To multiply a number by 125

Rule *Affix three zeroes to the right of the multipli cand and then divide it by 8,* $1000 - 8 = 125$

Example 7 Multiply 5670809 by 125.
Sol $5670809 \times 125 = 5670809000 - 8$
    . the req. product $= 708851125$ Ans

§39. To multiply a number by 625

Rule. *Affix four zeroes to the right of the multipli cand and then divide it by 16,* . $625 = 10000 - 16.$

Example 8 Multiply 2102563 by 625.
Sol. 2102563×625 $= 21025630000 - 16$
    ., the req. product = 1314101875 Ans.

§40. To multiply a number by 15, 35, 45, 55 and 65.

Rule. *Multiply the multiplicand by 30, 70, 90, 110, 130 [Art 33] and divide the result by 2.*

Example 9. Multiply 5625 separately by 15, 45, 65.
Sol. $5625 \times 15 = (5625 \times 30) - 2 = 168750 - 2 = 84375.$
    $5625 \times 45 = (5625 \times 90) - 2 = 506250 - 2 = 253125$
    $5625 \times 65 = (5625 \times 130) - 2 = 731250 - 2 = 365625$

§41. To multiply a number by 75, 175, 275, etc.
Rule. *Multiply the multiplicand by 300, 700, 1100 and then divide the result by 4 (Art 40)*

§42. To multiply a number by a number all the figures of which are nines, *i e*, by 9, 99, 999, etc.

$9 = 10 - 1$, $99 = 100 - 1$, $999 = 1000 - 1$ and so on, we have the following

Rule *Place as many zeroes to the right of the multiplicand as there are nines in the multiplier, and from the result subtract the multiplicand*

Example 10 Multiply 895023 by 999.

Sol $895023 \times 999 = 895023000 - 895023$
∴ the req product $= 894127977$ **Ans**

§43 To multiply a number by a number all the figures of which are one *i e*, by 111, 1111, etc

Rule *First put down the unit's digit of the multiplicand as the unit's digit of the product and then add each figure of the multiplicand to its next digit up to as many figures on its left as the number of digits in the multiplier, to get the other digits of the product carrying the number if any The number thus formed is the required product.*

Example 11. Multiply 8972 by 111,

Sol   8972   *Explanation,*
          111   (1) Put down 2 as the unit's figure
         ─────   of the product.
          8972
          8972   (2) $2 + 7 = 9$, put down 9 as the
          8972   next figure of the product
         ─────
         995892 Ans.(3) $2 + 7 + 9 = 18$, put down 8 and
                        carry 1 Since the multiplier
                        consists of 3 digits we shall
                        now omit 2, and add 7, 9, 8
                        then omit 7 and add $9 + 8 + ..$
                        and so on,

§44. To multiply a number by a number a little less or greater than any multiple of 10.

Example 12 Multiply 89536 by 9995, 10008.

Sol (1)   9995 $= 10000 - 5.$
         89536 × 9995 $= 895360000 - 89536 \times 5$
                         $= 895360000 - 447680.$
        ∴ the req product $= 894912320.$ **Ans,**

(2) '   10008 = 10000 + 8
        89536 × 10008 = 895360000 + 89536 × 8
                      = 895360000 + 716288
' the req product = 8960766288   **Ans**

§45. To multiply in a given number of lines.

Sometimes the digits of the multiplier are so related to each other that the operation of multiplication may be considerably shortened by using some peculiar artifices.

The following examples should be carefully studied :—

Example 13   Multiply 28536 by 9144 in two lines

Sol   28536
            9144
        ――――――
        256824᙮᙮᙮
        4109184
        ――――――
        260933184
            Ans

Explanation. First multiply by 9 leaving unit's, ten's and hundred's places blank, then multiply the result by 16 to get the product of 144 at once   This product may be so arranged that units may come under units, tens under tens and so on.

Example 14   Multiply 856252 by 2096 in two lines

Sol   856252
            2096
        ――――――
        13700032
        1781004160
        ――――――
        1794704192 **Ans.**

Explanation Since 2096 = 2080 + 16, therefore first multiply by 16, then multiply the result by 130 to get the product of 2080 at once and then add the results

Example 15   Multiply 3025396 by 1441872 in three lines.

Sol   3025396
            1441872
        ――――――
        54457128
    ' 217828512
    435657024 ᙮᙮᙮
    ――――――――
    4362233781312
            Ans

Explanation First multiply by 18, then multiply the result by 4 to get the product of 72 at once and then multiply the product of 72 by 2 to get the product of 144 Arrange these products correctly and add.

EXERCISE 7

Using the shortest method multiply —

1.   59634 by 100, 1000   2   75624 by 30, 110
3   42502 by 1001, 10001. 4.   263891 by 5
5.   263895 by 15   6   369538 by 25

7.	378951 by 35.	8	498359 by 75.
9	361575 by 125.	10	219357 by 125.
11	279583 by 625	12.	359387 by 625
13	569908 by 99	14,	379002 by 999.
15.	453001 by 9999.	16	378350 by 9999
17	56735 by 111	18	863542 by 1111.
19	163508 by 9992.	20	470025 by 9998
21	350259 by 10004	22	480902 by 10009.

Multiply in two lines —

23	53902 by 488	24	370892 by 726
25	163005 by 1449	26	359302 by 1048.
27.	219035 by 8128.	28	459392 by 9135.
29.	569287 by 18012	30	193086 by 16256.
31	453095 by 13195	32	319084 by 38019.
33	897564 by 1936	34.	763542 by 2415

Multiply in three lines —

35	479356 by 192246	36.	289209 by 2081048.
37	519410 by 15810135	38	219083 by 128016256
39.	483592 by 210189014	40	352092 by 1800108012

Abbreviated Methods of Division.

§46   To divide a number by 10, 100, 1000, etc.

Rule *Cut off as many digits from the right side of the dividend as there are zeroes in the divisor. This will form the remainder and the remaining digits, the quotient*

Example 1 Divide 85624 separately by 100 and 1000

Sol   85624—100 =Q 856, rem 24 ⎫
       85624—1000=Q 85, rem. 624. ⎬ **Ans**

§47   To divide a number by 30, 700, 11000, etc.
Rule   [Apply Art. 46, 30]

§48.   To divide a number by 5
Rule. *Multiply the dividend by 2 and divide the product by 10   Now omit the last figure , the omitted figure—2 will be the remainder.*

Example 2.   Divide 89356 by 5,
Sol   89356—5=(89356×2)—10
      =178712   —10

the req. quotient = 17871 , remainder = 2−2 = 1. **Ans.**

§49   To divide a number by 25

Rule   *Multiply the dividend by 4, and divide the product by 100   Now omit the last two figures , the omitted figures—4 will be the remainder*

Example 3   Divide 562359 by 25

Sol   562359−25 = (562359 × 4)−100
$$= 2249436−100$$
the req quotient = 22494, remainder = 36−4 = 9 **Ans.**

§50   To divide a number by 125

Rule   *Multiply the dividend by 8, and divide the product by 1000   Now omit the last three figures , the omitted figures—8 will be the remainder*

Example 4   Divide 159356 by 125

Sol   159356−125 = (159356 × 8)−1000
$$= 1274848−1000$$
the req. quotient = 1274, rem = 848−8 = 106. **Ans.**

§51.   To divide a number by 625

Rule.   *Multiply the dividend by 16 and divide the product by 10000   Now omit the last four figures , the omitted figures—16 will be the remainder*

Example 5   Divide 219356 by 625

Sol.   219356−625 = (219356 × 16)−10000
$$= 3509696−10000$$
. the req. quotient = 350 , rem = 9696−16 = 606 **Ans.**

§52   To divide a number by 15, 35, 45, etc

Rule.   *Multiply the dividend by 2 and divide the product by 30, 70 and 90, etc respectively. The remainders —2 are the real remainders*

Example 6   Divide 8935 by 15
Sol 8935−15   = (8935 × 2)−30
$$= 17870−30$$
$$= 595, 20 \text{ rem.}   [Art 30]$$
` the req quotient = 595 , remainder = 20−2 = 10. **Ans.**
Example 7.   Divide 8356 by 35.

Sol   8356—35=(8356×2)—70
                    =16712—70
                    =238 , 52 rem
.. the req quotient=238, remainder=52—2=26 **Ans.**

Example 8 Divide 15938 by 45
Sol. 15938—45=(15938×2)—90
                    =31876—90
                    =354, 16 rem
. the req quotient=354, remainder=16—2=8. **Ans.**

§53   To divide a number by 75, 175, 275, etc

Rule *Multiply the dividend by 4 and divide the product separately by 300, 700 and 1100 The remainder —4 is the real remainder*

Example 9 Divide 15936 separately by 75, 175, 275.
Sol   (1) 15936—75=(15936×4)—300
                        =63744—300
                        =212, 144 rem.
' the req   quotient=212 , rem.=144—4=36 Ans
(2)   15936—175=(15936×4)—700
                        =63744—700
                        =91, rem 44
the req   quotient=91 , rem.=44—4=11   **Ans.**
(3)   15936— 275=(15936×4)—1100
                        =63744—1100
                        =57, rem. 1044
the req. quotient=57 , rem =1044—4=261 **Ans.**

§57   To divide a number by 375 and 875.

Rule *Multiply the dividend by 8 and divide the product respectively by 3000 or 7000 The remainders —8 are the real remainders*

Example 10 Divide 897563 separately by 375 and 875.
Sol   (1) 897563—375=(897563×8)—3000
                        =7180504—3000
                        =2393, 1504 rem
the req. quotient=2393, rem =1504—8=188 **Ans.**
(2) 897563—875=(897563×8)—7000
                        =7180504—7000
                        =1025, 5504 rem.

˙ the req quotient=1025, rem.=5504—8=688. **Ans.**

§55 To divide a number by a number the figures of which are all nines, *i. e* , **by 99, 999, 9999, etc.**

Rule *Add one to the divisor to get a new divisor and divide the dividend by the divisor thus obtained , divide the quotient again by the same divisor and thus proceed as long as possible. Now add together all the quotients and all the remainders, carrying (if any) from the sum of the remainders to the sum of the quotient and adding the same to the sum of the remainders   The results thus obtained will be the required quotient and remainder.*

Example 11 Divide 129893568 by 99
Sol. Since 99+1=100

```
                        quotient   rem
. 129893568—100=1298935,     68
    1298935—100=   12989,     35
      12989—100=     129,     89
        129—100=       1,     29
          1—100=       0,      1
                   ————————    ——
          Total  =1312054,     22
                        2,      2  [The carried No ]
                   ————————    ——
                   1312056,     24   Ans
```

The process may be abbreviated thus —

quotients	rem	
1298935	68	*Explanation*
12989	35	2 was carried from remainder to
129	89	quotient and adding the same to re-
1	29	mainder the req quotient=1312056
0	1	and remainder=24. **Ans.**
1312056	22	

§56 To divide a number by **998, 9997, etc.**

Rule *If the divisor is less than the number made of all nines by 1, 2, 3, etc , then add 2, 3, 4, etc , as the case may be, to get the new divisor a number consisting of 1 and zeroes.   Divide the dividend by the new divisor*

multiply the quotient by the number added and then divide the result again by the new divisor Thus proceed as long as possible. Now add together all the remainders and quotients carrying, if any, from the sum of the remainders to the sum of the quotients and also adding the added number to the remainder

Example 12   Divide 563425 by 997

Sol · $997 + 3 = 1000$

		Q	rem
∴	$563425 \div 1000$	$= 563,$	425
Now	$(563 \times 3) \div 1000$	$= 1,$	689
again	$(1 \times 3) \div 1000$	$= 0,$	3
		$565,$	117

Q $= 565$, rem.$= 117 + 3$ or 120 **Ans**

EXERCISE 8

Find the value of the following by using the shortest method —

1	$153956 \div 5$	2	$895634 \div 5$
3	$9593898 \div 25$	4.	$2195639 \div 25$
5	$8893596 \div 15$	6	$8893854 \div 15$
7	$2195683 \div 35$	8	$9935610 \div 55$
9	$49900256 \div 45.$	10	$31935682 \div 65$
11	$569002863 \div 125$	12	$389356881 \div 125$
13	$563789502 \div 75$	14	$889021566 \div 375.$
15	$9635002056 \div 625$	16	$8935683561 \div 875$
17	$5689356954 \div 997$	18	$5156380242 \div 995$
19	$78502349835 \div 999.$	20	$3190805621854 \div 9999$

§57 **Brackets.**

( ), ⊣ ⊢, [ ] are called **brackets** They denote that all quantities enclosed within them are to be treated as one quantity Thus $6 \times (3 + 2)$ means that we first add 2 to 3 and then multiply the result by 6 Hence $6 \times (3 + 2) = 6 \times 5 = 30$ The above brackets are named thus

( ) is called a circular bracket

⊣ ⊢ is called a curly   bracket

[ ] is called a square bracket.

F 3

Sometimes a line is placed above the terms as in $\overline{5+7}\times4$; it is called **Vinculum** It has also the force of a bracket   Hence $\overline{5+7}\times4$ means 12×4 and *not* $5+28$

§58 When a bracket is used within a bracket, the expression within the innermost brackets is first simplified. then the expression of the next innermost brackets, and so on

Example 1 Simplify :—
$15+[25+\{35-8+(7-\overline{2+3})\}]$.

Sol The expression $=15+[25+\{35-8+(7-5)\}]$
$$=15+[25+\{35-8+2\}]$$
$$=15+[25+29]$$
$$=15+54=69.\quad\textbf{Ans.}$$

EXERCISE 9.

Simplify :—

1	$144-(7+9)$	**2**	$1872-(18+8)$.
3	$728-(\underline{20+8})\times25$	**4**	$[1580-(21+19)]-140$.
5	$256-70-20-50$	**6.**	$521-325+171-16$
7	$15^2+(5+11)(8+4)$.	**8**	$(21)^2-(16-9)(21-10)$.
9	$15(26+14)-21\times16$	**10**	$122-35+17-13$

11 $35-\{27-[28-21]+10\}$.
12 $84-\{28+[27-(15-10)]\}$
13 $182-\{73+[27-(21+\overline{4-3})]\}$
14 $58-[58-\{58-(58-\overline{58\ \ 18})\}]$.
15 $225-[164-\{75+(86-28-16)\}]$
16 $(194+65)\times7+(352-220)-11-952-(91-35)$.
17 $6+8[3\times6+\{3+7-(8+3-6)-(2\times6-3+$
$$3-2)\}]$$

§59   Some Important Typical Examples

Example 1 What number is that which if I divide by 5, to the quotient I add 21, from the sum I take 36 and multiply the remainder by 6 then the product is 72 ?

Sol The required number$=(72\div6+36-21)\times5$
$$=(12+36-21)\times5$$
$$=27\times5=135.\quad\textbf{Ans.}$$

This is a backward process   In this process addition is changed into subtraction and *vice versa*   Multiplication is changed into division and *vice versa.*

Example 2.   What least number must be subtracted from 893564 to make it exactly divisible by 144 ?

Sol.

```
144)893564(6205
    864
    ───
    295
    288
    ───
     764
     720
     ───
      44
```

Evidently the number to be subtracted is 44   **Ans**

Example 3   What least number must be added to 5638956 to make it divisible by 256 ?

Sol

44+212=256

Rem =44

Evidently the number to be added is

256−44=212   **Ans.**

```
256)5638956(22027
    512
    ───
    518
    512
    ───
    695
    512
    ────
    1836
    1792
    ────
      44
```

Note   The new quotient will evidently be 22028

Example 4.   Find the greatest number of 6 digits which is exactly divisible by 1224

Sol   The greatest number of 6 digits is 999999.   On dividing this by 1224, the remainder is 1215

Evidently the reqd No is 999999−1215=998784 **Ans.**

Example 5   Find the least number of six digits which is exactly divisible by 1224

Sol

```
1224)100000(81
     9792
     ────
     2080
     1224
     ────
      856
```

Explanation

The least   number   of 6 digits is 100000   On dividing this by 1224, the remainder is 856   If we add 1224−856 =368 to 100000 we shall get the required number of 6 digits

the reqd No =100368   **Ans**

Example 6   In a division sum, the divisor is 125, the quotient is 24 and the remainder is 61 , find the dividend.

Sol   Dividend $=$ divisor $\times$ quotient $+$ remainder
                  $=125\times24+61$
                  $=3061$ Ans.

Example 7   In a division sum, the dividend is 1976, the quotient is 79 and the remainder is 1, find the divisor

Sol   Dividend $=$ divisor $\times$ quotient $+$ remainder
                  dividend $-$ remainder $=$ divisor $\times$ quotient
          Or $1976-1=1975$ $=$ divisor $\times$ quotient
                  the divisor is $1975\div79=25$ **Ans**

Example 8   A number when divided successively by 7, 8, 9 leaves 2, 5 and 3 as remainders respectively   What are the remainders if the order of divisors is reversed ?

Sol   Complete remainder is equal to $2+7\times5+7\times8\times3=205$

Now suppose the final quotient is x, therefore the dividend $=7\times8\times9\times x+205$

7	
8	, rem 2
9	, rem 5
	, rem. 3

Now divide this dividend by 9, 8, 7 successively

The remainders are 7, 6 and 2   Ans

9	$7\times8\times9\times x+205$
8	$7\times8x+22,$ rem 7
7	$7x+2,$ rem 6
	$x,$ rem. 2

Test of correctness   The complete remainder in this case is also equal to $7+9\times6+9\times8\times2=205$

Example 9   A boy multiplied 623 by a certain number and obtained 78852 as his answer   He was told that both the eights in the product were wrong , find the multiplier and the correct product

Sol   *Explanation*
   623   (i) Since the unit's digit of the product is
   124   2, therefore the unit's digit of the multiplier
  2492   must be 4, multiplying 623 by 4 we get 2492
  1246   (ii) Since the ten's digit of the product
   623   is 5, therefore there must be 6 under 9 of the
 ─────   first row of the product   To get 6 we must
 77252   multiply 623 by 2, $623\times2=1246$.

(*iii*) Since the last digit 7 in the product is also correct, therefore to get 7 we must multiply 623 by 1   Hence the multiplier is 124, the product is 77252   Ans

Example 10   The sum of two numbers is 98 and their difference is 32   find the numbers

Sol   The greater number $=(98+32)-2=130-2=65$
The smaller number $=(98-32)-2=\;\;66-2=33$.
The reqd numbers are 65 and 33   Ans

Example 11   A man spending at the rate of Rs 60 p m finds after 8 months that he is exceeding his income, he then reduces his expenditure to Rs 45 p m   at the end of 4 months he finds that he is just out of debt   Find his income

Sol   Expenses for 8 months $=$ Rs $60 \times 8 =$ Rs 480
Expenses for 4 months $=$ Rs $45 \times 4 =$ Rs 180
Income of one year $= 480 + 180 =$ Rs. 660
   income per mensem $=$ Rs. $660 - 12$
                             $=$ Rs 55   Ans.

EXERCISE 10

1   What number is that which if I divide by 7 , if to the quotient I add 14, from the sum if I take 20 and multiply the remainder by 8 then the product is 80 ?

2   What number is that which if I divide by 16, if to the quotient I add 10, if from the sum I take 12 and then multiply the remainder by 13, the product is 52 ?

3   What least number   must be subtracted from 3593809 to make it exactly divisible by 256 ?

4   What is the least number which when subtracted from 8935063 makes the remainder divisible by 196 ?

5   What least number must be added to 210359 to make it divisible by 117 ?

6   What is the least number which when added to 4809356 make the sum exactly divisible by 245 ?

7.   Find the greatest number of 5 digits which is exactly divisible by 265

8   Find the greatest number of 6 digits which is exactly divisible by 315.

9. Find the least number of 6 digits which is exactly divisible by 216

10. Find the least number of 5 digits which is exactly divisible by 520

11. Find the nearest number to 21935 which is exactly divisible by 516

12 Find the nearest number to 26572 which is exactly divisible by 320

13 In a division sum the divisor is 124, the quotient is 21 and the remainder is 51, find the dividend

14 In a division sum the dividend is 2523, the quotient is 21 and the remainder is 24, find the divisor.

15 A number is divided by 541 and the successive partial dividends are 1239, 1573 and 4914   Find the dividend, quotient and remainder

16. A number is divided by 321 and the successive partial dividends are 359, 383, 624 and 3038   Find the dividend, quotient and remainder.

17. The sum of two numbers is 854 and their difference is 198, find the numbers.

18. The sum of two numbers is 1547 and the larger number exceeds the smaller by 105, find the numbers.

19. The difference of two numbers is 415 and the greater is 956, what is the smaller number ?

20 In a division sum the quotient is 5 times the divisor and 15 times the remainder   If the remainder be 6, find the dividend

21 A certain number when divided successively by 5, 4, 6 leaves 1, 3, 3 as remainders respectively   What are the remainders if the order of divisors be reversed ?

22 A number is successively divided by 5, 6, 8 leaving remainders 3, 4, 7 respectively   What will be the remainders if the same number be divided by 8, 6, 5 ?

23. A number is divided thus —

$x)$ ‾ ‾ ‾ ‾

$y)$ ‾‾‾‾‾‾‾, remainder 1

‾‾‾‾‾, remainder 3

The complete remainder is 19, find the value of x.

24   A number is divided thus —

) ‾ ` ‾ ᴿ ‾ `
       7) ‾ˣ ' ‾ˣ ‾ , remainder 5
          1 0 1 8 2, remainder 4

If the true remainder be 23, find the dividend

25   A number is successively divided by 5, 6 and 8 leaving remainders 2, 3 and 4 respectively. If the true quotient be 21, find the dividend.

26   A number is successively divided by 7, 5 and 4 leaving remainders 3, 1 and 2 respectively If the true quotient be 24, find the dividend.

27   Find the missing figures (denoted by stars) in the following examples on multiplication —

(a)   8 9 7 2   (b)   5 9 3 8
               4 * *   . .

       ‾‾‾‾‾‾‾‾‾   ‾‾‾‾‾‾‾‾‾
       * * * 4 3 2   ‾ ‾ 3 3 4

28   A boy multiplies a number by 243 and obtains the product 1933036 If both the first threes are wrong but the other figures are right, find the correct answer and also, find the multiplicand.

29.   A boy multiplies 1526 by a certain number and obtains the product 477424. If both the sevens are wrong but the other figures are right find the correct answer and also the multiplier.

30   At a game of cricket, A and B together score 32 runs, B and C together score 37 runs, and A and C together score 25 runs, find the number of runs scored by each of them.

31.   A and B together earn Rs 130 in a month, B and C together earn Rs 170, and A and C together earn Rs 150, find the amount of rupees earned by each of them.

32   Divide Rs 98 among A, B and C so that A may receive Rs 11 more than B and B Rs 6 more than C

33   Divide Rs 86 among *A*, *B* and *C* so that *A* may receive Rs 7 more than *B* and *B* may receive Rs 5 more than *C*

34   A man spending at the rate of Rs 820 a year finds after 5 years that he is exceeding his income, he then reduces his expenditure to Rs 540 a year   After 2 years he clears off his debt , find his income

35   A clerk living at the rate of Rs 75 p m for a year finds that he is running into debt , he then reduces his expenditure to Rs 40 p m for a year and then finds that he has cleared off his debt and saved Rs 60 , find his salary

36   A farmer exchanged 25 cows for 125 sheep worth Rs 10 per head , what did he receive for a cow ?

37.   A father left his eldest son Rs 400 more than his second son , and he left his second son Rs 300 more than his third , his whole estate was Rs 6,400 , what did each son receive ?

38   Divide 4,680 rupees, after giving away 180 rupees to the poor, between *A*, *B* and *C*, giving *B* 216 rupees more than *A*, and *C* 336 rupees more than *B*

39   The quotient = 5 times divisor = 7 times remainder = 105 , find the dividend

40   The Duke of Wellington died in the year 1852 aged 83 , Napoleon was born in the same year as the Duke and died in 1821 , what was Napoleon's age at the time of his death ?

41   Divide Rs 1,800 among *A*, *B* and *C*, so that for every 2 rupees *A* gets, *B* shall get 3 and *C* 4

42   Divide 5248 rupees among *A*, *B* and *C*, so that for every 5 rupees given to *A*, *B* may get 11 and *C* 16

43   The price of a carriage with horse is 1920 rupees and the price of the carriage is 5 times that of the horse. Find the price of the horse

44   If 23 men earn Rs 1610 in a month, how many men will earn Rs 1750 in the same time ?

45   A gentleman left Rs. 112500 to be divided amongst his 4 sons and 3 daughters in such a way that

each son might get three times as much as each daughter.
How much did each son receive ?

46   The sum of two numbers is 15678 and the larger
number exceeds the smaller by 1234   Find the numbers

47   What number multiplied by 1256 will give the
same product as (*i*) 314 by 476 , (*ii*) 7536 by 378 ?

48   What least number must be added to 40398, so
that the sum may be divisible by 2073 ?

49   The quotient which is obtained by dividing
39875365 by a certain number is 4607 and the remainder
is 6387   Find the divisor

50   The product of two numbers is 20405426 and
half of one of them is 1739 , find the other number

51   A man bought 81 seers of ghee at Rs. 3 a seer
and 127 seers of an inferior type at Rs 2 a seer   He
mixed the two and sold the whole for Rs 500. How much
did he gain or lose ?

52   *A* is 21 years older than *B* and 8 years younger
than *C*, who is 60 years of age   *D* is as old as the ages of
A and *B* together. Is *C* older or younger than *D* ? and by
how much ?

53   A man divided his property worth Rs 12765
among his 4 sons in such a manner that the eldest received
Rs 145 more than the second, the second Rs 130 more
than the third and the third Rs 120 more than the fourth.
How much did each receive ?

54   A gentleman divided Rs 123600 among two
sons, four daughters and one sister in such a way that each
daughter received twice as much as the sister and each son
one-half of what the three daughters received. How much
did the sister get ?

55   Two men walk towards each other at the rate
of 8 and 9 miles per hour. If the distance between them
be 153 miles, when will they meet ?

CHAPTER III.

COMPOUND QUANTITIES

§1.   Tables The following tables should be committed
to memory by a student of Arithmetic —

British Indian Money.

3 Pies (p.)	make	1 Pice
4 Pice or 12 Pies	,	1 Anna (a)
16 Annas	,,	1 Rupee (Re)
15 Rupees	,,	1 Pound or Sovereign (£)

English Money.

4 Farthings (f or q.)	make	1 Penny (d)
12 Pence	,,	1 Shilling (s)
20 Shillings	,,	1 Pound or Sovereign(£)

2 Shillings	make	1 Florin
2 Shillings 6 Pence	,,	1 Half Crown
5 Shillings	,,	1 Crown
10 Shillings	,,	1 Half Sovereign
21 Shillings	,,	1 Guinea

Indian Weight

8 Chawals	make	1 Rati
8 Ratis	,,	1 Masha
12 Mashas	,,	1 Tola
5 Tolas	,	1 Chatak (ch.)
16 Chataks	,,	1 Seer (sr)
40 Seers	,,	1 Maund (md.)

Avoirdupois Weight
(For weighing common goods)

16 Drams (dr)	make	1 Ounce (oz.)
16 Ounces	,,	1 Pound (lb)
28 Pounds	,,	1 Quarter (qr.)
4 Quarters	,,	1 Hundred-weight (cwt)
20 Hundred-weights	,,	1 Ton (ton)

Note —14 lbs make 1 stone (st)

Troy Weight

(For weighing gold, silver, jewellery.)

24 Grains (grs)	make	1 Penny weight (dwt)
20 Penny weights	,,	1 Ounce (oz)
12 Ounces	,,	1 Pound (lb)

Apothecary's Weight.

(Used in mixing medicines.)

20 Grains	make	1 Scruple (scr.)
3 Scruples	,,	1 Dram (dr)
8 Drams	,,	1 Ounce (oz)
12 Ounces	,,	1 Pound (lb.)

Linear Measures

12 Inches (in)	make	1 Foot (ft)
3 Feet	,,	1 Yard (yd.)
5½ Yards	,,	1 Pole (po)
40 poles or 220 Yards	,,	1 Furlong (fur)
8 Furlongs or 1760 yards	,,	1 Mile (mi)
3 Miles	,,	1 League (lea)

Note —Gunter's chain is used in land surveying
1 chain = 100 links = 22 yds
80 chains = 1 mile

Cloth Measures

2¼ Inches (in.)	make	1 Nail (nl.)
4 Nails	,,	1 Quarter (qr) or span
2 Quarters or spans	,,	1 Cubit
4 Quarters or 2 cubits	,,	1 Yard
3 Quarters	,,	1 Flemish ell
5 Quarters	,,	1 English ell.
6 Quarters	,,	1 French ell

British Square Measures.

144 Square Inches (sq. in.)	make	1 Square Foot (sq. ft.)
9 Square Feet	,,	1 Square Yard (sq yd)
30¼ Square Yards	,,	1 Square Pole (sq. po.)
40 Square Poles	,,	1 Rood (ro)
4 Roods or 4840 sq yds	,,	1 Acre (a.)
640 Acres	,,	1 Square Mile

Note —10 Square chains or 4840 sq yds make 1 acre

Punjabi Square Measures

1 Square Karm	make	1 Sersai
9 Sersais	,,	1 Marla
20 Marlas	,,	1 Kanal
4 Kanals	,,	1 Bigha
2 Bighas	,,	1 Ghumaon
324 Square Yards	,,	1 Kanal

Cubic Measures

1728 Cubic Inches	make	1 cubic Foot (cub. ft.)
27 Cubic Feet	,,	1 Cubic Yard (c yd)

Measure of Time.

60 Seconds (sec)	make	1 Minute (m)
60 Minutes	,,	1 Hour (hr)
24 Hours	,,	1 Day
7 days	,,	1 Week
4 Weeks	,,	1 Month
12 Months	,,	1 Year
100 years	,,	1 Century

Note —365 days make 1 common year and 366 days make 1 Leap year   But reducing a number of days to the fraction of such a year is not customary to add 1 day to 365

Measure of Number

12 Units	make	1 Dozen
12 Dozen	,,	1 Gross
20 Units	,,	1 Score

Paper Measures

24 Sheets of paper	make	1 Quire
20 Quires	,,	1 Ream
10 Reams	,,	1 Bale

Measure of Capacity

20 Ounces	make	1 Pint (pt)
2 Pints	,,	1 Quart (qt)
4 Quarts	,,	1 Gallon (gal)
2 Gallons	,,	1 Peck (pk)
4 Pecks	,,	1 Bushel (bu)
8 Bushels	,,	1 Quarter (qr.)

Angular Measures

60 Seconds (60″)	make	1 Minute (1′)

60 Minutes	make	1 Degree (1°)
90 Degrees	,,	1 Right angle.

REDUCTION.

§2. A quantity when expressed in one denomination is called a **simple quantity** and when is is expressed in more than one denomination, it is called **compound quantity**, as, Rs 2 is a simple quantity and Rs 2 3a 4p is a compound quantity.

§3 Reduction is the process by which a compound quantity is expressed as a simple quantity or *vice versa*

The following solved examples will best illustrate the process —

Descending Reduction

Example 1   Reduce Rs 5. 6a 9p to pies

Sol   Rs a p
```
     5  6  9
    16
    ──
    80
     6
    ───
    86 a
    12
   ────
   1032     .
      9
   ─────
   1041 p  Ans
```

Hence the following

Rule *Multiply the number in the highest denomination by the number of units of the next inferior denomination contained in a unit of the higher   Add the number of the inferior denomination if any and continue this process for each succeeding denomination till the required result is obtained*

Example 2. Reduce £15 10s 8d to pence

Ascending Reduction.

Example 1 Reduce 1041 pies to Rs. a p

Sol   12)1041 p
```
      ────────────
      16)86a + 9p
      ──────────
       Rs 5  +6a
    Rs  5  6a  9p. Ans.
```

Hence the following

Rule. *Divide the given number by the number of units which make a unit of the next higher denomination putting down the remainder if any   This remainder will be of the same denomination as its dividend   Continue this process till the required result is obtained*

Example 2 Reduce 3728 pence to £ s. d

Sol.

£	s	d
15	10	8

$$\begin{array}{c} 20 \\ \hline 300 \\ 10 \\ \hline 310s \\ 12 \\ \hline 3720 \\ 8 \\ \hline 3728d \quad \textbf{Ans} \end{array}$$

Example 3 Reduce 15 mds 37 sr 8 ch. to chataks.

Sol.

mds	sr	ch
15	37	8

$$\begin{array}{c} 40 \\ \hline 600 \\ 37 \\ \hline 637 \text{ sr} \\ 16 \\ \hline 10192 \\ 8 \\ \hline 10200 \text{ ch} \quad \textbf{Ans} \end{array}$$

Example 4. Reduce 15 tons 8 cwt 3 qr 16 lbs to lbs

Sol

tons	cwt	qr	lb
15	8	3	16

$$\begin{array}{c} 20 \\ \hline 300 \\ 8 \\ \hline 308 \text{ cwt.} \\ 4 \\ \hline 1232 \\ 3 \\ \hline 1235 \text{ qr} \\ 28 \\ \hline 9880 \\ 2470 \\ \hline 34580 \\ 16 \\ \hline 34596 \text{ lbs. } \textbf{Ans} \end{array}$$

Sol.

$$\begin{array}{r} 12)\overline{3728\ d} \\ 20)\ \overline{310s.+8d} \\ \hline £15+10s \end{array}$$

£15. 10s 8d **Ans**

Example 3 Reduce 10200 chataks to maunds etc

Sol.

$$\begin{array}{r} 16)\overline{10200\ ch} \\ 40)\overline{637\ sr+8\ ch.} \\ \hline 15\ mds+37\ sr. \end{array}$$

15 mds. 37 sr 8 ch. **Ans**

Example 4 Reduce 34596 lbs to tons etc

Sol 28 $\left\{ \begin{array}{l} 4)\overline{34596\ lbs} \\ 7)\ \overline{8649} \end{array} \right.$

$$\begin{array}{r} 4)\ \overline{1235qr+16lbs} \\ 20)\ \overline{308\ cwt+3qr.} \\ \hline 15\ tons+8cwt \end{array}$$

∴ 15 tons 8 cwt. 3 qr 16 lbs
                                    Ans.

Example 5. Reduce 10 miles 116 yds 2 ft 3 in. to inches.

Sol

Miles	yds	ft	in
10	116	2	3

```
        1760
       17600
        116 ·
       ───────
       17716 yds
           3
       ───────
       53148
           2
       ───────
       53150 ft
          12
       ───────
       637800
           3      ·
       ───────
       637803 in    Ans.
```

Example 5. Reduce 637803 inches to miles etc.

Sol 12)637803 in.

```
         3) 53150 ft   ÷ 3 in
       ⎧ 10) 17716 yds + 2 ft.
  1760 ⎨ 11)  1771 + 6 rem
       ⎩ 16)   161
                10 mi + 1 rem.
```

Rem. = 6 + 10 × 0 + 10 × 11 × 1
= 6 + 110 = 116 yds.

∴ 10 miles 116 yds. 2 ft.
    3 in   **Ans**

EXERCISE 11

Reduce to annas —

  1   Rs 15 7a **2**   Rs 21 4a **3**   Rs 37. 6a
  4   Rs 35 9a **5**   Rs 47 15a **6**   Rs. 55. 10a.

Reduce to pies —

  7   Rs 21. 5a 4p   **8**   Rs. 23 6a 9p.
  9   Rs. 35 6a 9p   **10**   Rs. 28 13a 4p
11.   Rs 26. 5a 9p   **12.**   Rs 28 5a 10p

Reduce to pice —

13   Rs 31 9a 6p   **14**   Rs 31. 12a 9p
15   Rs 51 9a 3p   **16**   Rs 28. 10a 9p
17   Rs 28 13a 3p.   **18.**   Rs. 41 15a. 9p

Reduce to shillings —

19.   £21 9s   **20**   £48. 15s   **21.**   £31 18s.
22   £45 16s.   **23.**   £56 19s   **24**   £37. 6s.

Reduce to pence —

25   £27 15s 6d   **26**   £ 99 15s. 9d
27   £21 9s 7d   **28**   £ 45 13s 3d
29   £31 14s. 10d   **30**   £137. 10s 6d

Reduce to chataks —

31	45 mds	8 sr	9 ch.	32.	48 mds	9 sr.	15 ch
33	35 mds	15 sr	10 ch	34	85 mds	37 sr	10 ch.
35	89 mds.	21 sr	15 ch	36	78 mds	35 sr	12 ch

Reduce to ratis —

37	8 tolas	9 mashas	6 ratis
38	9 tolas	10 mashas	7 ratis
39	18 tolas	10 mashas	7 ratis
40	21 tolas	11 mashas	3 ratis
41	98 tolas	11 mashas	4 ratis
42	85 tolas	7 mashas	7 ratis

Reduce to inches —

43	38 yds.	2 ft	9 in	44.	31 yds	2 ft.	5 in
45.	46 yds	1 ft	8 in.	46	84 yds	1 ft	11 in
47	87 yds	2 ft	11 in	48.	21 yds	2 ft	4 in

Reduce to seconds —

49	5 days	3 hrs	5 min	35 sec
50	26 days	18 hrs	8 min	40 sec
51	21 days	20 hrs	35 min	30 sec
52	47 days	12 hrs	27 min	35 sec
53	45 days	22 hrs	18 min	45 sec
54	25 days	18 hrs	45 min	36 sec.

Reduce to lbs —

55	7 tons	15 cwt	2 qr	21 lb
56	22 tons	18 cwt	3 qr	11 lb
57	22 tons	16 cwt	3 qr	7 lb
58	16 tons	12 cwt	2 qr	11 lb
59	35 tons	12 cwt	1 qr	26 lb
60.	22 tons	13 cwt	3 qr	24 lb.

Inverse Operations

Reduce to Rs a p —

61	28972 pies		62	23575 pies
63	13787 pies		64	378356 pies,
65	383092 pies		66.	483569 pies
67.	383542 pice		68	318302 pice
69	359352 pice		70	110001 pice

Reduce to £ s d

71	215356 pence		72	350204 pence
73	893596 pence		74	832029 pence
75.	310983 pence		76	889354 pence.

77	123859 farthings	78	8356 guineas.
79.	5635 guineas	80	1608 crowns
81	11508 half crowns.	82.	1823 florins

Reduce to mds. srs. chks etc —

83	283059 chks	84	350093 chks
85	478309 chks	86	2683505 tolas
87	3108050 tolas	88	4780506 tolas
89	2185028 mashas	90	4780291 mashas.

Reduce to tons etc —

91	835935 qr	92	3783594 lb
93	1835694 lb.	94	3802564 lb
95	2100093 lb	96	4702356 lb

Reduce to weeks, days etc —

97	893020842 minutes.	98	450208354 minutes
99	435635942 seconds	100	485283591 seconds

Reduce to miles, yds etc —

101.	563056935 inches	102	856389354 inches.
103	583015925 inches	104	410205935 inches

Reduce to right angles, degrees etc —

105	853428'	106	215028'
107	473590"	108	990259"

Reduce to acres, sq yds etc —

109	7856937 sq in	110	9999999 sq in
111	7777777 sq in	112	87000001 sq in

COMPOUND ADDITION

§4   Compound Addition is the method of finding the sum of several quantities (of the same kind) expressed in more than one denomination

Rule   *Arrange the quantities so that the units of the same denomination may be under one another, then draw a line underneath, add the numbers of the lowest denomination and reduce the sum to the next higher denomination, put down the remainder and carry the quotient   Repeat this operation in all other subsequent denominations.*

Example   Add together Rs 15 8a 9p., Rs. 21. 9a. 10p and Rs 45 7a. 6p

Sol.	Rs	a	p	Explanation
	15	8	9	$9p + 10p + 6p = 25p = 2a \quad 1p.$, put
	21	9	10	down $1p$ and carry $2a$, $2a + 8a + 9a.$
	45	7	6	$+ 7a = 26a = $ Rs 1 $10a$, put down
	82	10	1	$10a$ and carry Re. 1.

Now add Rs. and put down the sum.

EXERCISE 12

Add the following —

	Rs	a	p			Rs	a	p			Rs	a	$p.$
1.	21	7	8		2	35	9	8		3	17	5	4
	35	8	9			56	14	4			21	9	7
	45	6	3			39	15	7			35	13	4
	21	4	2			21	4	9			47	15	9

	Rs	$a.$	p			Rs.	a	p			Rs	a	$p.$
4	501	7	3		5	201	11	3		6	805	14	7
	288	12	6			88	7	2			87	11	8
	85	9	11			45	12	9			203	10	5
	37	10	9			113	13	8			86	8	9

	£	s	d			£	s	d			£	s	d
7	121	15	9		8.	1139	7	8		9.	1625	7	4
	2035	8	11			521	9	7			838	11	4
	335	17	10			326	14	2			325	13	9
	556	12	8			8972	18	7			35	7	8
	4356	19	9			25	13	11			16	18	9
	437	8	9			7	14	9			321	9	8

	£	s	d			£	s	d			£	s	d
10	728	15	8		11	87	10	3		12	256	8	3
	156	12	9			523	9	7			57	12	7
	27	16	7			1124	4	11			1359	7	11
	8356	7	11			534	7	2			326	4	8
	356	19	8			302	9	8			256	9	10
	28	16	7			27	19	11			28	16	6

	mds	srs	chs.			mds.	srs	chs			mds	srs	chs
13	88	37	9		14.	47	21	9		15.	38	14	12
	47	15	15			521	8	15			219	12	9
	137	18	14			3569	26	11			5139	7	15
	8356	28	12			728	37	7			374	29	8
	431	9	9			3526	25	9			88	37	15
	126	35	10			76	18	11			12	17	10

	yds	ft	in			yds	ft	in.			yds	ft	in
16.	21	2	8	**17.**		21	1	5	**18.**		17	2	9
	576	1	9			35	2	9			85	1	11
	728	2	11			356	2	7			382	2	9
	135	1	10			156	1	11			83	1	10
	325	2	10			28	2	9			295	2	9
	266	1	9			27	1	7			85	1	7

	tolas	mashas	ratis			tolas	mashas	ratis			tolas	mashas	ratis
19.	7	8	4	**20**		85	7	5	**21**		55	4	3
	12	9	7			21	9	4			88	9	7
	8	11	6			121	4	6			55	7	6
	85	7	4			86	10	7			44	4	2
	32	10	2			46	5	4			21	5	9

	mi	fur	po			mi	fur	po.			mi	fur	po.
22	19	6	15	**23.**		21	7	4	**24.**		49	6	18
	28	7	35			39	4	15			21	2	23
	35	4	21			235	5	35			56	4	18
	39	2	28			37	2	18			23	2	21
	47	5	39			45	3	29			328	7	34

	hr	min	sec			hr	min	sec.			hr	min	sec.
25.	15	35	40	**26**		18	21	9	**27**		99	25	45
	21	29	30			21	18	24			45	36	25
	18	45	35			45	35	49			75	21	49
	19	25	21			56	28	48			28	35	55
	24	28	45			48	52	28			27	26	48

	sq yds	sq ft	sq in			sq yds.	sq ft	sq in.			sq yds	sq ft	sq in.
28	45	7	121	**29**		145	4	128	**30**		156	4	29
	36	8	135			116	7	115			134	6	99
	40	6	134			156	4	75			21	3	121
	121	4	79			28	7	28			128	2	142
	18	2	16			45	6	16			75	8	17

	ac	r	sq po			ac	r	sq po			ac	r	sq po
31	17	2	18	**32.**	156	0	35	**33.**	156	2	17
	28	3	26		117	1	34		77	1	39
	121	1	36		85	2	5		18	3	7
	32	0	39		56	3	27		78	2	29
	16	2	7		8	2	9		7	3	35

	wk	da	hr		wk	da	hr		wk	da.	hr
34	7	6	23	**35**	9	6	15	**36.**	21	4	16
	9	4	17		45	3	8		15	6	4
	17	3	12		17	4	3		8	5	21
	14	2	21		5	2	22		85	2	22
	7	1	8		4	3	16		6	3	18

	deg	min	sec		deg	min	sec	rt ang.	deg	min
37	28	21	35	**38**	45	16	35	**39**	45	70	45
	17	35	45		29	8	27		24	21	55
	21	9	35		21	15	9		28	17	28
	17	28	9		17	8	47		9	8	45
	8	36	16		25	19	28		21	25	8

COMPOUND SUBTRACTION

§5   Compound Subtraction is the method of finding the difference of two quantities (of the same kind) expressed in more than one denomination

Rule *Write down the smaller quantity below the greater so that the units of the same denomination may be under one another and draw a line underneath   Begin from the right and subtract each number of the lower line from the corresponding number of the upper line.   If the number of the lower line be greater than the corresponding number of the upper line, then add to the upper number as many numbers of the same denomination as make a unit of the next higher denomination, also add 1 to the next preceding number of the lower line*

Example   Subtract Rs 8 3a 4p from Rs. 15 1a 2p

Sol   Rs   a   p   *Explanation*   Add 12p.
       15   1   2   to 2p making 14p., 14p −4p
        8   3   4   =10p , put down 10p   Now
       6 13 10   add 1 to 3 making 4   Again
                        4a cannot be subtracted from

1a add 16a to 1a making 17a , 17a −4a =13a   Put down 13a and add 1 to 8 making Rs 9 Rs 15−Rs. 9= Rs 6 , put down Rs 6

the difference is Rs 6 13a 10p   **Ans.**

EXERCISE 13

Perform the operation of subtraction in the following:—

	Rs	a.	p.		Rs	a	p		Rs.	a.	p
1	28	7	9	2	21	9	7	3	56	7	2
	24	6	5		16	8	5		29	9	8

	Rs	a.	p		Rs	a	p.		Rs	a	p.
4	35	11	7	5	126	12	8	6	218	4	3
	29	15	11		87	13	9		159	9	7

	£	s	d.		£	s	d.		£	s	d
7	128	5	9	8	156	17	7	9	111	2	4
	57	14	10		148	9	11		59	14	11

	£	s.	d.		£	s	d.		£	s.	d.
10	129	11	4	11.	143	5	7	12	118	2	3
	128	13	7		119	12	9		117	11	7

	ton	cwt.	qr.		ton	cwt	qr		ton	cwt	qr.
13	21	16	2	14.	124	4	1	15.	121	2	3
	18	19	3		56	18	2		89	16	2

	mds	srs	ch		mds	srs	ch.		mds	srs	ch.
16	33	27	8	17	45	17	3	18.	127	19	12
	17	37	9		29	39	5		89	38	15

	yds	ft	in.		yds.	ft	in.		yds.	ft.	in.
19	35	2	4	20.	41	1	8	21.	36	1	7
	29	1	11		37	2	9		35	2	11

Subtract (22—30) —

22. Rs 1585 6a. 8p. from Rs. 2912 2a. 3p.

23. Rs. 2109. 2a 7p from Rs. 3509 1a. 2p

24 £1385 8s. 9d from £1936 7s. 5d

25 £2139. 17s 10d from £4521 12s. 7d.

26. 136 mds 28 srs. 15 chks. from 247 mds 21 srs 12 chks.

27 128 mds 29 srs 12 chks from 318 mds. 28 srs. 9 chks

28 121 yds 1 ft. 9 in. from 156 yds. 2 ft 3 in

29 86 tons 12 cwt 2 qr from 151 tons 7 cwt 1 qr.

30 128 tons 18 cwt 3 qr. from 159 tons 6 cwt. 2 qr

31   What must be added to Rs 128   3*a* 9*p* to make Rs 156 1*a* 2*p*. ?

32   What must be subtracted from £121 2*s* 6*d* to get the remainder £89 17*s* 9*d* ?

Find the value of the following —

33. Rs 41   3*a*   4*p* + Rs 21   2*a* 9*p* − Rs 55. 11*a* 3*p* − Rs 17 12*a* 2*p* + Rs 16 8*a* 3*p*

34   £41. 17*s* 6*d* + £17 15*s* 2*d* − £121 8*s* 2*d* − £2. 3*s* 11*d* + £78 2*s* 7*d*

35   21 mds 28 srs 10 chks + 27 mds 24 srs 12 chks. + 28 mds 12 srs 13 chks − 22 mds 3 srs 9 chks − 29 mds 29 srs 3 chks

COMPOUND MULTIPLICATION.

§6   Compound Multiplication is the method by which the sum of a given number of repetitions of a compound quantity is found.

The multiplicand in compound multiplication is a compound quantity and the multiplier an abstract number

§7   When the multiplier is a number not greater than 20, we have the following

Rule   *Write the multiplier under the lowest denomination of the multiplicand and draw a line underneath Multiply the lowest denomination by the multiplier and reduce the product to the next higher denomination, put down the remainder and carry the quotient to the next product   Repeat the process till all the denominations are multiplied.*

Example 1.   Multiply Rs. 5 6*a* 8*p* by 9

Sol   Rs *a* *p*   *Explanation* (*i*) 8*p* × 9 = 72*p* = 6*a*.
          5   6   8   0*p* , carry 6*a*. (*ii*) 6*a* × 9 = 54*a*
                      9   54*a* + 6*a* = 60*a* = Rs 3. 12*a* ,
Rs   48 12   0 **Ans.**   put down 12*a* and carry Rs. 3
                                        (*iii*) Rs 5 × 9 = Rs 45, Rs 45
                                        + Rs. 3 = Rs 48 , put down Rs 48.

§8   When the multiplier is a number greater than 20 but can be split up into factors, each factor being less than 20, we have the following

Rule   *Multiply the multiplicand by one of the factors as in Art 7 and then multiply the product by another and so on.*

Example 2   Multiply £15 13s. 4d by 132

Sol   £   s   d.   *Explanation*   ∵   132=12×11,
     15   13   4   therefore, first multiply by 12 and
           12   then by 11   The result is the
    ‾‾‾‾‾‾‾‾‾   required product.
    188   0   0
           11
   ‾‾‾‾‾‾‾‾‾‾‾
   £2068   0   0   **Ans.**

§9. When the multiplier cannot be split up into convenient factors, the process is a combination of multiplication and addition or multiplication and subtraction; thus :—

Example 3.   Multiply Rs 5   8*a*. 9*p*. by 109 and 83.

Sol.   109=12×9+1 ,   83=12×7−1 ,

(*i*)   Rs   *a*. *p*   (*ii*)   Rs. *a*. *p*
     5   8   9   5   8   9
         12   12
   ‾‾‾‾‾‾‾‾‾   ‾‾‾‾‾‾‾‾‾
   66   9   0   66   9   0
        9   7
   ‾‾‾‾‾‾‾‾‾   ‾‾‾‾‾‾‾‾‾‾
Add ⎰   599   1   0   465   15   0
   ⎱   5   8   9   Subtract ⎰   5   8   9
   ‾‾‾‾‾‾‾‾‾‾   ⎱ ‾‾‾‾‾‾‾‾‾‾‾
   604   9   9   **Ans.**   460   6   3   **Ans.**

Aliter   Rs. *a* *p*.   *Explanation.*
         5   8   9   (*i*)   9*p*.×109=981*p*.=81*a*   9*p*.,
           109   put down 9*p* and carry 81*a*.
   ‾‾‾‾‾‾‾‾‾‾‾
   604   9   9   (*ii*)   8*a*×109=872*a*.,   872*a* +
                    81*a* =953*a*.=Rs. 59. 9*a*.,
put down 9*a*. and carry Rs. 59. (*iii*) Rs. 5×109=Rs. 545
Rs 545+Rs 59=Rs. 604, put down Rs. 604

the reqd product is Rs. 604 9*a* 9*p*.   **Ans**
Similarly we can solve the (*ii*) part.

EXERCISE 14

Multiply (1—20) —

1	Rs 8 5a 4p by 12		2	Rs 21 10a 8p. by 15.
3	Rs. 12 2a 8p. by 18.		4	Rs 15. 1a 4p by 16
5	Rs 21. 5a 6p by 21		6	Rs 26 4a 8p. by 25
7	£ 17 13s 4d by 24		8	£ 12 17s 6d by 36.
9	£ 24 17s. 8d by 40		10	£ 16. 19s 10d by 42.
11	£ 42 18s 6d by 56		12	£ 58 12s 8d by 64.
13	£ 156. 17s 8d by 133		14	£ 148. 19s. 6d by 139.

15   15 mds 37 srs 8 chks   separately by 117, 119.
16   21 mds 28 srs 12 chks   „   by 128, 113
17   15 tons 16 cwt 2 qr 12 lb.   „   by 104, 93
18   45 ton, 12 cwt. 3 qr 21 lb   „   by 135, 149
19   35 sq yds 8 sq ft. 116 sq in ,,   by 121, 129
20   36 sq yds. 7 sq. ft 120 sq in ,,   by 124, 141.

21   What is the cost of 145 bags of wheat at Rs 25. 3a. 8p per bag ?

22   Find the value of 156 bags of rice at Rs. 38. 11a. 9p per bag.

23   Find the value of 1156 mds. of wheat at Rs 5. 10a. 6p per md

24   Find the wages of 97 work men for 15 days at Re. 1. 2a 6p per day for each man

25   A sum of money was divided among 257 men , each man got £ 27. 13s. 9d. , find the sum divided.

COMPOUND DIVISION.

§10. Compound Division is the method of dividing a compound quantity by a given number and to find the value of one of the parts It is also the method of finding how often one compound quantity is contained in another compound quantity of the same kind

Thus in compound division we consider two cases In the first case the dividend is a compound quantity, the divisor is an abstract number and the quotient is a compound quantity This method is called **Partition**

In the second case the dividend and the divisor are both compound quantities of the same kind and the quotient is an abstract number This method is called **Quotition**

§11　Case 1.　Partition

Rule.　*First find out how often the divisor is contained in the highest denomination of the given quantity, put down the quotient and reduce the remainder to the next inferior denomination.　Add to it the number of the same denomination in the dividend and repeat the operation till all the denominations are divided by the divisor*

Example 1　Divide Rs 547 4a 3p by 15

Sol　　　Rs　a　p　　　*Explanation*
　　　15)547　4　3　　　(i) Rs 547—15=Q　Rs 36,
　　　　　36　7　9　　Rem 7　Put down Rs 36 and
　　　　　　　　　　　reduce Rs. 7 to annas.

(ii) 7 × 16=112, 112+4=116a, 116a —15=Q　7a,
Rem 11a　Put down 7a and reduce 11a to pies

(iii)　11 × 12=132, 132+3=135p, 135p —15=9p.
Put down 9p.　.　Rs 36 7a 9p　Ans.

Example 2　Divide Rs. 1451. 10a. 3p by 51 and 75.

	Rs.	a	p		Rs	a.	p
Sol.	51)1451	10	3(Rs 28		75)1451	10	3(Rs 19
	102				75		
	431				701		
	408				675		
	23				26		
	16				16		
	368				416		
	10				10		
	51)378(7a				75) 426(5a		
	357				375		
	21				51		
	12				12		
	252				612		
	3				3		
	51)255(5p				75)615 (8p		
	255				600		
					15		

.　the required quotients are

(i) Rs. 28. 7a 5p, (ii) Rs 19. 5a 8p and 15p over　Ans

§12.　To divide by 10, 100, 1000, etc.

Rule　*Cutt off from the right of each dividend as*

many figures as there are z toes in the divisor. The figures on the left will denote the quotient and figures to the right the remainder.

Example 3   Divide £3371. 13s 4d. by 100 and 700.

	£	s	d.		£	s.	d.
Sol	100)33,71	13	4		7)3371	13	4
	20				100) 4,81	13	4
	1420		.		20		
	13				1620		
	14,33				13·	.	
	12				16,33		
	396				12		
	4	.			396		.
	4,00				4		
					4,00		

£33. 14s. 4d   **Ans.**   £4 16s 4d   **Ans**

§13.   Quotient to the nearest pie or penny

Rule   *Add 1p or 1d to the quotient if the remainder is half or greater than half the divisor, otherwise neglect it*

Example 4   Divide Rs 990 1a 6p by 35 and 41

	Rs	a	p		Rs	a	p
Sol	35)990	1	6(Rs 28		41)990	1	6(Rs 24
	70	.			82		
	290	:			170		
	280	'			164		
	10	,			6		
	16	;			16	.	
	160	.			96	.	
	1				1		
	35)161(4a.	'			41)97(2a.		
	140	.			82		
	21	.			15		
	12				12		
	252				180		
	6	, ·			6	.	
	35)258(7p				41)186(4p		
	245				164		
	13				22		

In the first case the remainder is 13p., which is less than half the divisor, therefore neglect it   The required quotient, in this case is Rs 28. 4a 7p **Ans**

In the second case the remainder is 22p which is greater than half the divisor, therefore add 1p to the quotient. The reqd quotient in this case is Rs. 24 2a 5p **Ans**

§14   Case II   Quotition

Rule   *Reduce the dividend and the divisor to the same denomination and then divide.*

Example 5   How many times is Rs 5. 7a. 7p contained in Rs. 147 12a. 9p

Sol	Rs.	a	p.		Rs	a	p
	147	12	9		5	7	7
	16				16		
	2352				80	:	
	12				7··	:	
	2364 a.	.			87 a		
	12				12		
	28368				1044	:	
	9·				7·		
	28377 p				1051 p		

∴ the required quotient is 28377−1051=27.   **Ans**

EXERCISE 15.

Divide —

1	Rs.	526.	6a.	4p	by 11.
2	Rs.	738.	10a	1p	by 13
3	Rs	1129	12a	4p.	by 14.
4	Rs	1375.	13a	4p	by 16
5.	£	2028.	12s	6d.	by 15.
6	£	1945	10s	8d.	by 16.
7.	£	1929	3s	5d	by 11
8	£	1821.	7s.	1d.	by 13.
9.	Rs	1576	5a	8p.	by 10.
10	Rs	2178	6a	2p	by 10.
11	£	1753	6s.	8d.	by 10.
12	£	2156	8s	4d	by 10.

Divide to the nearest pie or penny —

13	Rs 2156.	7a	8p.	by 15.

14	Rs. 5156.	9a	7p	by	16.
15.	£ 2948	13s	7d.	by	13
16	£ 8835.	15s	4d	by	14.
17	Rs. 3856.	15a.	5p	by	100.
18	Rs 5159	14a	10p.	by	100.
19	£ 4839.	19s.	7d	by	100
20	£ 5139	17s.	3d.	by	100
21	£ 15389	17s	11d	1y	1000
22	£ 49556	18s	4d.	by	1000

Divide by factors and find the complete remainder :—

23	Rs 2157.	13a.	5p	by	56.
24	Rs 8917.	12a	9p	by	72
25	Rs 5675.	11a.	7p	by	108.
26	Rs 5815.	7a	4p.	by	132
27	£ 5159.	3s	11d.	by	156.
28.	£ 7728.	12s	9d	by	160.
29	£ 2756	12s.	2d.	by	90.
30	£ 5159	2s	10d	by	140
31	£ 5785.	17s	11d.	by	88
32	£ 6939	15s	7d.	by	256.

Divide each of the following as nearly as possible, into the given number of equal parts —

33	Rs 563893.	3a	8p	into 728 parts
34	Rs 383028	15a	6p.	into 531 parts.
35	£ 193502	17s.	8d.	into 551 parts
36	£ 280291	18s	3d.	into 929 parts.
37.	51356 mds.	38 srs	15 chks	into 123 parts
38	48501 mds	27 srs	11 chks	into 129 parts.
39	15481 tons	17 cwt	2qr 21 lb	into 161 parts
40	65321 tons	12 cwt.	3 qr 24 lb.	into 181 parts.

How many times is

41   Rs. 7   8a   9p   contained in Rs 113 3a. 3p.?
42   Rs 28. 13a   7p   contained in Rs. 461 9a 4p.?
43   £ 17 14s   5d.   contained in   £ 372 2s. 9d ?
44   £ 16 17s 11d   contained in   £ 760 6s 3d ?
45.   121 mds. 28 srs 13 chks contained in 3043 mds. 5 chks.?
46   156 mds. 37 srs. 8 chks contained in 6434 mds 17 srs 8 chks.?
47   56 tons 7 cwt. 2 qr 8 lb. contained in 281. tons 17 cwt 3 qr 12 lb ?

48   41 tons 3 cwt 1 qr 21 lb contained in 329 tons 7 cwt 2 qr ?

49   The cost of 156 tables is Rs 835 4*a*.   What is the cost of each table ?

50   How many chairs each worth Rs 5 8*a* 6*p* can I buy for Rs 71 14*a* 6*p* ?

51   How many horses at Rs 95 7*a* each can I buy for Rs 1336. 2*a* ?

52   How many days must a labourer work at 14*a* 6*p*. per day to earn Rs 19 6*p* ?

53   A man pays Rs 318. 4*a* 6*p* as income-tax which is 5*p* for every rupee in his income , what is his income ?

54   A person completes a journey of 298 miles 2 fur 28 po in Feb 1936 What distance does he travel per day?

55   How many lengths each equal to 7 fur 22 po 4 yds will make up 52 miles 10 poles ?

§15 Some Important Problems in Compound Rules

Example 1   *A, B* and *C* have Rs 1012. 6*a* between them, *B* and *C* have together Rs. 810 12*a*. and *A* and *C* have together Rs 708 2*a*.   How much has *C* ?

   Sol   *A, B* and *C* together have Rs 1012 6*a*.
       and *B, C* together have Rs 810 12*a*
       . *A* alone has Rs 1012 6*a*.—Rs 801 12*a*.
                   =Rs 201. 10*a*
       Also *A* and *C* together have Rs 708 2*a*
           C alone has Rs 708 2*a*.—Rs 201 10*a*.
                   =Rs 506 8*a*. **Ans**

Example 2.   Divide Rs 151. 14*a*. among *A, B* and *C* so that for every Rs 3 2*a*. that *A* gets, *B* gets Rs 4 6*a* and *C* gets Rs. 2 10*a*

   Sol   Rs 3. 2*a* + Rs 4 6*a*. + Rs. 2 10*a* = Rs 10. 2*a*

If Rs. 10 2*a* are divided *A* gets Rs 3 2*a*., *B* gets Rs 4. 6*a* , *C* gets Rs 2 10*a* , but Rs 10. 2*a* is contained 15 times in Rs 151 14*a*

       A gets Rs 3   2*a* ×15=Rs. 46 14*a* ⎫
       B gets Rs 4   6*a* ×15=Rs 65. 10*a*   ⎬ **Ans**
       C gets Rs 2 10*a*. × 15=Rs 39   6*a* ⎭

Example 3   A labourer was employed, on the condi-
tion that he would get Re 1 4a on every day he is
present and he would pay a fine of 5a on every day he is
absent. After a month he received Rs 29. 11a How
many days was he absent ?

   Sol   His remuneration for 30 days = Re. 1 4a × 30
                                    = Rs 37 8a
                     But he was paid = Rs 29 11a

   he lost his remuneration and paid as fine for the
days he was absent Rs 37 8a — Rs 29 11a = Rs 7. 13a

   he lost Re 1 4a + 5a. = Re 1 9a. daily in case he
is absent

   he lost Rs 7 13a in 5 days   **Ans.**

   Example 4.   The total expenses of a family when
wheat is at Rs 5 2a per maund are Rs 96 2a , when
wheat is at Rs 5 4a per maund they are Rs 97 4a. (other
expenses remaining the same)   Find the total expenses of
the family when wheat is at Rs 4 14a per maund.

   Sol   An increase of (Rs 5 4a — Rs 5 2a) i e, 2a.
per maund in the price of wheat makes an increase of
(Rs 97 4a — Rs 96 2a) = 18a in the family expenses

   It follows therefore that the quantity of wheat con
sumed by the family   = 18—2 = 9 maunds

      the expenses on wheat = Rs 5 2a × 9 = Rs 46 2a

   and   the other expenses = Rs 96. 2a — Rs 46 2a.
                     = Rs 50

   Now the price of 9 mds of wheat = Rs 4 14a × 9
                     = Rs. 43. 14a

   Hence the required expenses = Rs 43. 14a + Rs 50
                     = Rs. 93. 14a **Ans.**

   Example 5   A purse contains an equal number of
pounds, shillings and pence.   If the total amount be £26.
7s. 1d , how many coins of each kind are there ?

   Sol   1 pound + 1 shilling + 1 penny = 253d

   .. number of each kind of coin = £26 7s 1d ÷ 253d.
                     = 6325d ÷ 253d
                     = 25.   **Ans**

Note   In problems on coins the *number* of coins and the *value* of coins should be carefully distinguished

Example 6   A man having lived at the rate of £1253 a year for 4 years finds himself in debt, and then reduced his expenditure to £1120 a year, he is just out of debt in 3 years   What is his income ?

Sol.   Clearly 4 yrs ' expenditure + 3 yrs ' expenditure
                                    = 7 yrs ' income

7 yrs ' income = (1253 × 4 + 1120 × 3) or £8372.

Annual income = £8372 ÷ 7 = £1196   Ans.

EXERCISE 16

1   A, B and C have Rs 1058 5a between them   A and B have together Rs 809 12a. and A and C have together Rs. 749. 13a   How much has A ?

2   A, B and C have Rs 1030 6a between them   B and C have together Rs 818 1a , A and C have together Rs 707. 14a   How much has C ?

3   A, B and C were partners in a business   After a year they distributed the profit amounting to Rs 1296 14a between them   A and B got together Rs 784. 5a., B and C got together Rs 940 11a. How much did each get ?

4.   A and B have together Rs 1265. 5a. 3p , B and C have together Rs 857 15a 9p , and A and C have together Rs 1050. 4a   How much has each ?

5   Divide Rs. 117 among A, B and C so that for every Rs 3. 5a that A gets, B gets Rs 4 6a and C gets Rs 2. 1a.

6   Divide Rs 183 6a among A, B and C so that for every Rs 2. 12a that A gets, B gets Rs 3 4a and C gets Rs 4 3a

7   How many yards of velvet at Rs. 5 2a a yard must a cloth merchant give in exchange for 41 mds of wheat at Rs 4 10a per maund ?

8   How many sheep at Rs 9 14a a head must a farmer give in exchange for 16 bullocks at Rs 24 11a. per bullock ?

9   How many times can you give away 2 florins and a farthing from 45 guineas and 4 crowns ?

10   A carriage and a horse were purchased for Rs 321 4a What did each cost if the horse was worth 4 times as much as the carriage ?

11   A house and its furniture were purchased for Rs 3891 15a. What did each cost if the price of the house was 8 times as much as the furniture ?

12   Divide Rs 57 12a. among A, B and C so that for every rupee that A gets, B gets an eight anna piece and C gets a four anna piece

13   Divide Rs 530 4a. among A, B and C so that for every anna that A gets, B gets 4 annas and C gets one rupee

14   What is the least sum which should be added to Rs. 556 7a 8p. to divide it equally amongst 35 persons ?

15   What is the least sum which should be subtracted from Rs. 728 7a 9p to divide it equally amongst 24 persons

16   How often would a cart wheel which is 4 yards 2 feet round, revolve in going over 4 miles 3 furlongs 8 yards 6 inches ?

17   The fore wheel of a carriage is 8 ft 6 in in circumference and the hind wheel 4 ft 10 in more. How many more turns will the former make than the latter in a distance of 4 miles 2 furlongs ?

18   A person bought 121 sheep at Rs 7. 8a 9p per sheep, 21 of them died At what price per sheep must he sell now so as to gain Rs 25 14a 3p on the whole ?

19   A person bought 21 bags of wheat each containing 2 mds 20 srs at Rs 8 8a per md Two of them were stolen At what price per bag must he sell the rest so as to gain Rs 4 12a 10p on the whole ?

20   A man gained Rs 37 15a. 6p by selling 15 cows at Rs 566 11a 6p , what was the cost price of each cow ?

21   A man gained Rs. 29 6a by selling 20 chairs at Rs 10 per chair , what was the cost price of each chair ?

22.   A labourer was engaged for a month at 12a 6p per day on the condition that he will pay a fine of 2a 6p in case he will come late After the stipulated time he received Rs 21. 1a 6p. How often was he late ?

23 A peon whose pay for a week is Rs. 2 13*a* 4*p* is fined 1*a* 3*p* if he comes late. At the end of 21 weeks he received Rs. 57 15*a* How often was he late ?

24 A labourer was employed on the condition that he would get 14*a* 6*p* for every day he is present and would lose 4*a*. 6*p*. for every day he is absent After 25 days he received Rs. 14 5*a*. 6*p*. How many days was he absent ?

25. A servant entered into an agreement with a gentleman that he would get Re 1 2*a* for each day he is present and would pay a fine of 5*a* 6*p*. for each day he is absent After a month he received Rs 19. 1*a* only How many days was he present ?

26 A certain number of sovereigns, twice as many crowns, 5 times as many half crowns, 8 times as many shillings and 12 times as many six-pences together amount to £28. 5*s*., find the number of each coin

27 A certain number of two-anna pieces, four times as many four-anna pieces, 6 times as many eight-anna pieces and 8 times as many rupees together amount to Rs. 121. 4*a* Find the number of rupees

28 If 5 balls cost as much as 21 bundles of pencils worth 14*a* a bundle, how many balls can be bought for Rs 165 6*a* ?

29 If 10 tables cost as much as 18 chairs worth Rs 5. 8*a* each, how many tables can be bought for Rs 297 ?

30 A grocer mixes 20 lbs of tea costing 1*s* 2*d*. per lb. with 12 lbs costing 2*s* 1*d* per lb , find the cost to the nearest penny of the mixture per lb

31 A shopkeeper mixes 15 mds of sugar at Rs 11 4*a* a maund with 8 mds at Rs 15 8*a* a maund Find to the nearest pie the cost of the mixture per maund

32. How much water must be mixed with 20 seers of milk worth 3*a*. per seer in order to reduce its price to 2*a* 6*p* per seer ?

33 How much water must be mixed with 25 gallons of wine worth Rs 4. 8*a* per gallon to reduce its price to Rs. 3 2*a* per gallon ?

F 5

34. Divide Rs 1989 8*a* into three such parts that the first part shall be Rs 44 2*a* more than the second and the second part Rs 107. 15*a*. more than the third

35 Divide Rs 1537 11*a* into three such parts that the first part shall be Rs 127 3*a* more than the sum of the second and third, and the second Rs 7 14*a* more than the third

36. I divide £50. 3s. 4*d* among 4 men, 5 women and 6 children, such that each man receives thrice as much as each child and each woman twice as much as each child Find how much each man, woman and child receives.

37. An equal number of men, women and boys earn Rs 56 4*a* in 5 days. A man earns 7*a* , a woman 6*a* and a boy 5*a*, daily   Find the number of men.

38. A number of boys, twice as many women and thrice as many men earn Rs. 255 in 8 days   A boy earns 5*a* , a woman 8*a* and a man 10*a* daily , find the number of boys

39 The total expenses of a family when wheat is at Rs 3 12*a* per maund are Rs. 53 12*a*., when wheat is Rs 4. 2*a* per maund they are Rs 55 10*a* (other expenses remaining the same), find the total expenses of the family when wheat is at Rs. 4 6*a*. per maund.

40 The total expenses of a family when rice is at Rs 4. 2*a* per maund are Rs. 64 12*a*. and when rice is at Rs 4 per maund they are Rs 64, (other expenses not changing), find the total expenses of the family when rice is at Rs 3. 12*a* per maund.

41 A person having lived at the rate of Rs 2526 a year for 6 years finds himself in debt, and then reduced his expenditure to Rs 2016 a year, he is just out of debt in 4 years. Find his annual income.

42 The Calcutta rupee is worth 1*s* 11*d*. 3*f* each, how many must be given for £9895. 16*s* 8*d* ?

CHAPTER IV

FACTORS AND PRIME NUMBERS

§1   Factors.

$8 \times 3 = 24$

Here 24 is the product of 8 and 3,

and 8 and 3 are the factors of 24

But 24 has other factors also, i $e.$, $6 \times 4 = 24$,

$$\text{and } 12 \times 2 = 24$$

Definition.   A number which can divide a given number without a remainder is called a **factor** or **measure** of that number

EXERCISE 17.

Split up into factors —

1	15	2.	25	3	39.	4.	42
5.	55	6	65.	7	78	8	85.
9	49	10	70				

Can you split the following into factors —

11.	13.	12	19.	13.	21	14	35
15.	41	16	23.	17	34.	18	29
19.	31	20	32	21.	37	22	72

§2   Prime numbers

In the previous exercise, you have noticed that some of the numbers could not be split up into factors, they could only be divided by themselves and unity.   Such numbers are called **prime numbers**

Definition   A number which is divisible only by itself and unity is called a **prime number**

§3   A number which can be split up into factors is named a **composite number**

§4.   It may also be noted that numbers are either even or odd   A number which can be divided by 2 exactly is called an *even* number and a number which cannot be so divided is called an *odd* number.   Thus 2, 4, 6, 8, etc., are even and, 1, 3, 5, 7, etc , are odd numbers.

§5   Two numbers are *prime* to each other, when their only common factor is 1, thus 5 and 7 are prime to each other.

§6   One number is *divisible* by another when it can be divided by that number exactly, *i e.*, without any remainder. Thus 20 is divisible by 5 because when 20 is divided by 5, there is no remainder left

§7   Criteria of divisibility

The following facts will help us to resolve a composite number into its elementary factors —

(*i*)   A number is divisible by 2 if its first right hand digit is zero or is divisible by 2   Thus 120 and 156 are divisible by 2.

(*ii*)   A number is divisible by 3 if the sum of the digits is divisible by 3.   Thus 2562 is divisible by 3,

. $2 + 5 + 6 + 2 = 15$ is divisible by 3

(*iii*)   A number is divisible by 4 if its last two digits are zeroes or are divisible by 4   Thus 2500, 5736 are divisible by 4

(*iv*)   A number is divisible by 5 if its last digit is 0 or 5.   Thus 1620 and 15925 are divisible by 5.

(*v*)   A number is divisible by 6 if it is divisible by both 2 and 3   Thus 516 and 354 are divisible by 6

(*vi*)   A number is divisible by 8 if its last three digits are zeroes or are divisible by 8   Thus 51000 and 125256 are divisible by 8

(*vii*)   A number is divisible by 9 if the sum of its digits is divisible by 9   Thus 12375 is divisible by 9,

$1 + 2 + 3 + 7 + 5 = 18$ is divisible by 9.

Note   But if a number is not divisible by 9 the *remainder* in that case is the same as when the sum of its digits is divided by 9,

'   Let the number be 563425

$100000 - 1$ is divisible by 9,			$500000 - 5$ is so too	
$10000 - 1$	,,	9,	$60000 - 6$,, ,, ,,	
$1000 - 1$	,,	9,	$3000 - 3$,, ,, ,,	
$100 - 1$	,,	9,	$400 - 4$,, ,, ,,	
$10 - 1$	,,	9,	$20 - 2$,, ,, ,,	

Hence $563425 - (5+6+3+4+2+5)$ is divisible by 9

Hence 563425 when divided by 9 gives the same remainder, as $5+6+3+4+2+5$, i e , 25 when divided by 9

$25-9$ gives rem 7 $563425-9$ will also give rem 7

(viii) A number is divisible by 10 if its last digit is zero. Thus 1260, 1200 are divisible by 10

(ix) A number is divisible by 11 if the difference between the sum of its digits in the odd and in the even places is 0 or is divisible by 11   Thus 4950 and 9185 are divisible by 11, $(4+5)-(9+0)$ and $(9+8)-(1+5)$ are 0, 11 respectively

Note   But if a number is not divisible by 11, and also if the sum of the digits in the odd places is greater than the sum of the digits in the even places, then the remainder, when the number is divided by 11 is the same as when this difference is divided by 11 But if the sum of the digits in the even places is greater than the sum of the digits in the odd places, then the remainder obtained by dividing *this* difference by 11 is subtracted from 11 to get the real remainder

Let the numbers be (i) 895632 and (ii) 985632

(i) The sum of the digits in the odd places $=2+6+9=17$
and the sum of the ,,   ,, even ,, $=3+5+8=16$
difference $=17-16=1$
the remainder by dividing this 1 by 11 $=1$
the remainder by dividing 895632 by 11 $=1$

(ii) The sum of the digits in the even places $=3+5+9=17$
and the ,,   ,,   ,,   odd ,, $=2+6+8=16$
difference $=17-16=1$
the remainder by dividing this 1 by 11$=1$
the remainder by dividing 985632 by $11=11-1=10$

Proof

$100000+1$ or $99990+11$ is divisible by 11, $800000+8$ is so too
$10000-1$ or 9999,   ,,   ,, 11, $90000-9$
$1000+1$ or $990+11$,,   ,, 11, $5000+5$,,
$100-1$ or 99,   ,,   ,, 11, $600-6$,,
$10+1$ or 11,   ,,   ,, 11, $30+3$,,

Hence $895632+8-9+5-6+3-2$ is divisible by 11

Hence 895632 when divided by 11 gives the same remainder as $8-9+5-6+3-2$ when divided by 11

$8-9+5-6+3-2$ means that the sum of the digits in the odd places is greater than the sum of the digits in the even places by 1

The remainder by dividing this 1 by 11 is 1,
the remainder in 895632 is also 1
Similarly we can prove the second part

(x) A number is divisible by 12 if it is divisible by both 3 and 4   Thus 1608 and 2904 are divisible by 12.

(xi)   To divide a number by 7, 11 and 13 we have the following

Rule. *Divide the figures of the number into groups containing three each, as far as possible, counting from right to left   Add the alternate groups, and find their difference   If the difference is 0 or is divisible by 7 or 11 or 13, the number itself is also divisible by them.*

Thus 89563796 is divisible by 7, 48566254 is divisible by 11 and 563569357 is divisible by 13

For (i) 89563796 has 3 groups counting from right to left, i e , 89,563,796, and, the difference of alternate groups=(796 + 89) — 563=322

this difference 322 is divisible by 7,

·   the number is also divisible by 7

Similarly 48566254 and 563569357 are divisible by 11 and 13 respectively

Note   But if a number is not divisible by 7 or 11 or 13 and also if the sum of the groups in the *odd* places is greater than the sum of the groups, in the *even* places then the remainder, when the number is divided by 7 or 11 or 13 is the same as when the difference of the groups is divided by 7 or 11 or 13   But if the sum of the groups in the *even* places is greater than the sum of the groups in the *odd* places then the remainder obtained by dividing the difference of the groups by 7 or 11 or 13 is subtracted from 7 or 11 or 13 as the case may be to get the real remainder

Let the number be (i) 56325792569 and (ii) 56325932569

The numbers after dividing into groups (from right to left) are 56,325,732 569 and 56,325,932,569

(i)   The sum of the groups in the odd places=569 + 325=894
The sum of the groups in the even places=732 + 56 =788

Difference = 894 — 788 = 106

the rems by dividing 106 by 7 11 and 13 are 1, 7   2 respectively,

the remainders by dividing 56325732569 separately by 7, 11 and 13 are 1, 7, 2 respectively

(ii)   In 56,325,932,569 —

ᴉ The sum of the groups in the even places = 932 + 56 = 988

The sum of the groups in the odd places = 569 + 325 = 894

Difference = 988 — 894 = 94

the rems by dividing 94 by 7, 11 and 13 are 3,6,3 respectively,

the remainders by dividing 56325932569 separately by 7, 11, and 13 are 7 — 3 = 4, 11 — 6 = 5 and 13 — 3 = 10 respectively

§8   The following facts also may be noted carefully·—

1   A number of two digits made of the same figures as 11, 22, 33, etc , is always divisible by 11,

2   A number of three digits made of the same figure as 111, 222, 333, etc is always divisible by 3 37

3   A number of 2 figures once repeated as 6767, 8989 is always divisible by 101

4   A number of 3 figures once repeated as 256256   754754 is always divisible by 7 11 and 13

5   A number of 4 figures once repeated as 23452345 is always divisible by 73 and 137

6   A number of 5 figures once repeated as 2563425634 is always divisible by 11,

7   A number of 6 figures once repeated as 256342256342 is always divisible by 101

8   A number of 7 figures once repeated as 25603492560349 is always divisible by 11

9   A number of 8 figures once repeated as 1502345615023456 is always divisible by 17

10   A number of 9 figures once repeated as 156034289156034289 is always divisible by 7, 11 13, and 19

EXERCISE 18.

Without actual division find which of the following numbers are divisible by 2 —

| 1. 576. | 2. 7280 | 3. 859 | 4 725 |
| 5 963. | 6 524. | 7 832. | 8. 941. |

Without actual division find which of the following numbers are divisible by 4 —

| 9 936. | 10. 842 | 11. 1228 | 12. 1556. |
| 13 822 | 14 914 | 15. 1518 | 16. 2196. |

Without actual division find which of the following numbers are divisible by 8 —

| 17 1528 | 18 2176 | 19 2514 | 20 3572. |
| 21 15424. | 22. 21596 | 23. 16536. | 24. 15932. |

Without actual division find which of the following numbers are divisible by 3 —

| 25 8972 | 26. 15234. | 27. 22632. | 28. 16103. |
| 29 25836 | 30 21537. | 31. 19552 | 32. 25932. |

Without actual division find which of the following numbers are divisible by 9 —

| 33 18936 | 34 25654 | 35 56934. | 36. 38952. |
| 37 103569 | 38. 123043. | 39 220563 | 40 185409. |

Without actual division find which of the following numbers are divisible by 11 —

41   189567   **42**   663564   **43**   285835
44   2156389   **45**   5685669   **46**   839382918

Supply the missing digits (denoted by stars) so that the following numbers may be divisible by 8 —

47   185362*.   **48**   215635*   **49**   219355
50   2856⁴2   **51.**   56856'6   **52**   3858*4

Supply the missing digits (denoted by stars) so that the following numbers may be divisible by 9 —

53   26534 5   **54**   2~86358   **55**   21563 89
56   218 7356   **57**   563*8935   **58.**   359356⁴6.

Supply the missing digits (denoted by stars) so that the following numbers may be divisible by 11 —

59   152'5   **60**   9835*5   **61.**   5060*1
62   25*351.   **63**   78542*   **64**   56' 502

Without actual division write down the remainders when the following numbers are divided by 9 and 11 —

65.   85638   **66**   363596   **67.**   5935685
68   4835652   **69**   5730052   **70**   8125005

---

§9   There is no direct method of determining primes We give below a list of the prime numbers from 1 to 227

1	11	29	47	71	97	113	149	173	197
2	13	31	53	73	101	127	151	179	199
3	17	37	59	79	103	131	157	181	211
5	19	41	61	83	107	137	163	191	223
7	23	43	67	89	109	139	167	193	227

§10   To ascertain which numbers are prime.

(i)   A number whose last digit is 0, 2, 4, 6, 8, i. e., an even number is divisible by 2 and hence every even number except 2 is not a prime. A number whose last digit is 0 or 5 is divisible by 5 and hence every such number except 5 is not a prime   Hence the last digit of every prime number except 2 and 5 must be 1, 3, 7, 9

(ii)   If the last digit be 1, 3, 7 or 9, try as divisors the primes 3, 7, 11, 13 etc., if there is a remainder in each case, the number is a prime   It is not necessary to try a divisor whose square is greater than the given number.

§11   To resolve or decompose a composite number into its prime factors is to find those prime numbers whose product is equal to the given number   Thus

$$280 = 2 \times 2 \times 2 \times 5 \times 7 = 2^3 \times 5 \times 7.$$

When the factors obtained are all primes, the number is said to be resolved into prime or elementary factors

Note   No number can be resolved into prime factors in more than one way

§12   **To resolve a composite number into its prime factors**

Rule   *Divide the given number by a prime number which will divide it exactly   Then divide the quotient by another and so on till you obtain a quotient which is a prime number   The divisors and the last quotient are the prime factors of the given composite number*

Example 1.   Resolve 41580 into its elementary factors

Sol

```
2)41580
2)20790
3)10395
3) 3465
3) 1155
5)  385
11)  77
      7
```

Explanation.

(i)   the last figure is 0,
       the No is divisible by 2 and 5
(ii) $4+1+5+8 = 18 = 3 \times 6$,
       the No is divisible by 3
(iii.) $(4+5+0)-(1+8) = 0$,
       the No is divisible by 11.
Now divide the number successively by 2, 3, 5 and 11

2, 2, 3, 3, 3, 5, 7, 11 are the prime factors   **Ans**

Example 2   Divide 95040 into its prime factors.

Sol.
```
10| 95040
11|  9504
 8|  864
 9|  108
     12
```
ten   test applies
eleven test applies
eight test applies
nine   test applies

number $= 10 \times 11 \times 8 \times 9 \times 12$
$= 2 \times 5 \times 11 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 2 \times 2$
$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 11$.**Ans.**

Example 3   A number 1568*x*35*y* is divisible by 88 , find *x* and *y*

Sol.   the number is divisible by 8 × 11,

the last three digits of the number must be divisible by 8

∴ 35*y* must be 352 or *y*=2.

Substituting 2 for *y* the number=1568*x*352.

To be divisible by 11 the difference of the sum of its digits in the even and odd places must be either 0 or a multiple of 11, therefore add the digits in the even and odd places,

2+3+8+5=18 and 5+*x*+6+1=12+*x*

To make the difference 0, *x* is evidently=6

x=6 and *y*=2.   **Ans.**

§13   To find the number of different divisors of any composite number.

Rule   *Find the prime factors of the given number and add 1 to the index of each factor The product of the increased indices is the required result, unity and the number itself being included*

Example 4 How many different integers besides unity will divide 37800 without remainder.

Sol.   37800 $=2\times2\times2\times3\times3\times3\times5\times5\times7$
$=2^3\times3^3\times5^2\times7.$

No. of divisors$=(3+1)(3+1)(2+1)(1+1)=96$

the number of divisors besides unity $=96-1=95$ **Ans**

Note   The index of 7 is 1

EXERCISE 19.

Resolve into elementary or prime factors :—

1	728.	2	536.	3	327.
4.	5145.	5	3892	6	5148.
7.	5445.	8	5436.	9	6754.
10.	8970.	11	5841.	12.	7865.
13	7680	14	8430.	15	7315.
16	27720	17	999999.	18.	725760.
19	1666665	20.	2393160	21.	11918907.

22   Write down all the prime numbers lying between 15 and 81

23   Find the digits x and y when the number $15x8351y$ is divisible by 72

24.   Find the digits x and y when the number $1565x562y$ is divisible by 88

25   Find the digits x and y when the number $2856354xy$ is divisible by 99

26   What is the smallest number of which the alternate figures are zeroes that is divisible both by 9 and 11 ?

27   By what prime numbers may 1157 be divided so that the remainder may be 2 in each case ?

28   By what prime numbers may 21946 be divided so that the remainder may be 1 in each case ?

29.   There are four prime numbers , the product of the first three is 1683 and that of the last three is 3553, find them.

30   Show that 3001 has no divisor.

31.   Find how many different integers, besides unity, divide 14175.

32.   Arrange 1, 2, 5, 5, 6, 8 so as to form a number divisible by 2475   Enunciate the principle on which you proceed

     'Hint   Find the prime factors of 2475 ,

33.   How many different integers, besides unity, will divide 42336 without remainder ?

34.   The product of 495 with a certain other number gave $2875ab$, find a, b

35.   Without performing the operations of divisions prove that 19434492 is divisible by 99.

CHAPTER V

GREATEST COMMON MEASURE (G C M)

§1 Definition. If one number divides another number without remainder, it is called a **measure**, as, 2 is a measure of 8.

If that *measure* divides more than one number without remainder that measure is called the **Common Measure**, as, 2 is the common measure of 8, 12 and 16

But 8, 12 and 16 are also divisible by another common measure 4 and there is no other common measure greater than 4 which can divide 8, 12 and 16 exactly 4 is, therefore, called the **Greatest Common Measure** (G. C M). Hence *the Greatest Common Measure is the greatest measure which divides two or more given numbers without a remainder*

The following are the methods for finding the G C M of the given numbers —

§2 First method. To find the G C M by factors
Example. Find the G C M of 176, 286 and 88.

Sol. $176 = 11 \times 2 \times 2 \times 2 \times 2$ } 11 and 2 are the common
$ 286 = 11 \times 13 \times 2$ } measures
$ 88 = 11 \times 2 \times 2 \times 2$ } the G C M $= 11 \times 2 = 22$ Ans.

Rule *Find out the prime factors of the given numbers, select and multiply together all the common measures. The product will be the G. C. M. of those numbers*

EXERCISE 20

Write down the G C M of —

1	15, 25	**2**	16, 24	**3**	25, 30.	**4**	21, 27.
5	28, 35	**6**	30, 40.	**7**	48, 64.	**8**	26, 65
9.	28, 84	**10**	16, 72.	**11**	42, 48	**12**	16, 36
13	64, 84	**14**	35, 45.	**15**	51, 85		

16 28, 35, 49   **17** 56, 108, 132.   **18** 144, 176, 208.
19 192, 216, 360   **20** 525, 300, 625   **21** 576, 432, 720.
22 1500, 1600, 450.   **23** 336, 420, 630.
24. 370, 666, 962   **25** 5120, 7200, 9600
26. 12500, 57000, 3040

§3 Second Method To find the G.C M by division

G C M. of 45 and 25=5
Difference of 45 and 25=20.

45 and 25 are divisible by 5 and you also observe that their difference is also divisible by 5 Hence *if two numbers are divisible by a number their difference is also divisible by that number* This method depends upon this law Hence the following

Rule *Divide the greater number by the less and bring down the remainder Put the remainder as the divisor and the original divisor as the dividend Continue the process till there is no remainder left. The last divisor would be the required G. C. M.*

Example Find the G. C M of 285 and 465.

Sol.   285)465(1
                      285
                      180)285(1
                          180
                          105)180(1
                              105
                              75)105(1
                                  75
                                  30)75(2
                                      60
                                      15)30(2
                                          30

The required G. C M = 15 **Ans.**

Explanation Here 285 is less than 465. Therefore divide 465 by 285   180 is the remainder , according to the law stated above the G C M of 465 and 285 would be the same as the G C M. of 285 and 180   Therefore make 180 the divisor and the original divisor 285 the dividend and continue the process till there is no remainder left. 15 is the last divisor. Therefore 15 is the G C. M required

Note If the G C M of two numbers be *unity* the numbers are prime to one another

§4. To find the G C M of more than two numbers.

Rule *First find the G C. M of any two numbers Then find the G C M of the G. C. M. thus obtained and of another of the given numbers and so on The last G C M will be the G C. M required.*

Example   Find the G. C. M. of 585, 819, 1404 and 702.

Sol   585)819(1   117)1404(12   117)702(6
           585   117_   702
           ‾‾‾   ‾‾‾   ‾
           234)585(2   234
           468   234
           ‾‾‾   ‾
           117)234(2
           234
           ‾‾‾

Exp.   First find the G C. M. of 585 and 819 which is 117 , then find the G C M of 117 and 1404 which is 117, again find the G. C M of 117 and 702.

the reqd G. C. M. is 117   Ans

EXERCISE 21

Find the G C. M. of —

1 54, 126	**2** 272, 304	**3.** 275, 450.
4 364, 576.	**5** 381, 891	**6** 715, 871
7 689, 1573.	**8** 872, 986	**9** 225, 625
10 2145, 2235	**11** 1625, 2625.	**12** 1875, 2425
13 1348, 1568.	**14** 1568, 1897	**15.** 3015, 5040
16. 56340, 753480	**17.** 62100, 59040	
18 525, 625, 375.	**19.** 576, 600, 384	
20. 6435, 8970, 7235	**21.** 6161, 2440, 3111	
22. 71500, 51300, 89720	**23.** 10353, 8925, 9180	
24. 56700, 89730, 56730	**25** 18750, 16950, 19320.	

§5. Some Important Typical Examples on G C M.

Example 1   What is the largest sum of money which will divide Rs 7. 4a. 8p. and Rs. 6 11a. 4p exactly ?

Sol   Rs. 7. 4a. 8p =1400p
           Rs 6 11a 4p.=1288p

G C.M. of $1400p$ and $1288p$ will divide both the sums
their G C M $=56p$ the reqd sum $=4a$ $8p$ **Ans**

Example 2   Find the greatest number that will divide
2629 and 2483 leaving remainders 4 and 8 respectively.

Sol.   On dividing 2629, the remainder is 4,
. $2629 - 4 = 2625$ is exactly divisible by that
number

Similarly, $2483 - 8 = 2475$ is exactly divisible by that
number.

Hence the greatest number is the G C M of 2625
and 2475 which is 75

the reqd number $= 75$ **Ans**

Example 3   Find the greatest number that will
divide 578, 503, and 528 leaving the same remainder in
each case

Sol. Let r be the remainder.

Then $578 - r$, $503 - r$, $528 - r$ are exactly divisible by
that number

If two numbers are divisible by a number, their differ-
ence is also divisible by that number   [Art 3]

$(578 - r) - (503 - r)$, $(528 - r) - (503 - r)$ and $(578 -
r) - (528 - r)$

or $(578 - r - 503 + r)$, $(528 - r - 503 + r)$ and $(578 -
r - 528 + r)$

or $(578 - 503)$, $(528 - 503)$ and $(578 - 528)$

or 75, 25 and 50 are also divisible by that number.

G C M of 75, 25 and $50 = 25$

Hence the reqd. number $= 25$ **Ans.**

Note the line asterisked and deduce a rule from it

Example 4. The sum of two numbers is 144 and
their G C M is 24 , find them

Sol Let the quotient by dividing the first No. by $24 = x$
and   „   „   „   second „   „ $24 = y$
then the first number $= 24x$
and the second number $= 24y$.
. $24x + 24y$ $= 144$
or $24(x + y)$ $= 144$
or $x + y$ $= 6$

The numbers whose sum is 6 are (1, 5) , (2, 4) , (3, 3)

We reject the 2nd and 3rd pairs because they are not prime to each other.

   only possible pair is 1, 5

   multiplying by 24 the numbers are 24×1 and 24×5

∴ 24 and 120 are the required numbers.   **Ans**

Example 5   The product of two numbers is 2560 and their G C M is 16 , find all the possible pairs of such numbers.

Sol   The first number $=16x$ [*Ex.* 4]

             and the second number $=16y$

                  ∴ $16x \times 16y = 2560$

or $256xy = 2560$

or $xy = 10$

The possible pairs whose product is 10 are 1, 10 and 2, 5

   multiplying by 16 the numbers are either 16, 160 or 32, 80   **Ans.**

Example 6   What number is that which when divided by 5 and the quotient again by 6 and the quotient again by 8, will give the G C M of 121 and 143 ?

Sol.   The G. C M of 121 and $143 = 11$

Now the question is what number is that which when divided by 5, the quotient again by 6 and that quotient again by 8, will give 11 ?

   5, 6 and 8 are the three divisors and 11 the last quotient, therefore the reqd. No. $= 11 \times (5 \times 6 \times 8) = 2640$ **Ans**

Example 7   5635 and 3915 are divided by a certain number of three digits and the remainder is the same in both cases.   Find the number and the remainder.

Sol   Let d be the divisor and q' and q the quotients.

      $5635 =$ divisor × quotient + remainder

      $3915 =$ divisor × quotient + remainder

∴ $1720 = d(q - q')$ [*by subtraction*]

∴ d is a factor of 1720

but $1720 = 172 \times 10$

∴   the number (d) is of three digits,

∴ it is 172 and the rem. is 131   **Ans.**

$$
\begin{array}{r}
32 \\
172)\overline{5635} \\
516 \\
\hline
475 \\
344 \\
\hline
131
\end{array}
$$

Note. $1720 = 344 \times 5$ and 860×2

         ∴ 344 and 860 can also be the reqd. numbers.

Example 8   In solving a division sum, the dividend
is 158728 and the successive remainders are 351, 216 and
108   Find the divisor and the quotient

Sol   On subtracting the remainders 351, 216
and 108 from the partial dividends the
successive partial products will be 1587
$-351=1236$, $3512-216=3296$ and 2168
$-108=2060$.   Hence the divisor must
be a common factor of these products
G C M of 1236, 3296 and 2060 is 412
the divisor is 412 and quotient is
$(158728-108)\div412=385$.   **Ans**

```
        ***
     )158728
      *  * *
      ─────
       3512
       *** 
       ─────
       2168
       ****
       ─────
       108
```

Example 9   In working out a question on the G C M
of two numbers the quotients are 1, 3, 5   The last divisor
is 132 , find the numbers

Sol   the last divisor is 132   2112)2772(1
               and quotient 5,   2112
               dividend$=132\times5=660$ ─────
            Now divisor$=660$, quotient   660)2112(3
               3 and remainder 132   1980
               dividend$=660\times3+132=2112$ ─────
                                                132)660(5
                                                 660
                                                 ───

Now again divisor is 2112, quotient 1 and rem 660
        dividend$=2112\times1+660=2772$

Hence the numbers are 2112 and 2772. **Ans**

Example 10   Find the numbers between 1800, 2700
that have 323 as their G C M.

Sol   Evidently the numbers required are multiples
of 323

The smallest possible number is 323×6 since 323×5
is less than 1800 and the largest number is 323×8 since
323×9 is greater than 2700

               Nos must be 323×6, 323×7, 323×8
                    or 1938, 2261, 2584   **Ans**

F 6.

EXERCISE 22

1 Find the greatest number which will exactly divide 1561 and 2453

2 What is the largest sum of money which will divide both Rs 8 12*a* and Rs. 15 4*a* exactly ?

3 Find the largest sum of money which will divide both Rs 10 10*a* 8*p* and Rs 16 5*a* 4*p* exactly.

4 Two masses of silver weighing 85 and 95 tolas respectively are each to be made into coins of the same weight   Find the weight of the largest possible coin.

5 Two masses of gold weighing 2825 and 2275 tolas respectively are each to be made into ornaments of the same weight   What is the weight of the largest possible ornament ?

6 Two bills of Rs 6075 and Rs. 8505 respectively are each to be paid by cheques of the same amount   What is the largest possible amount of each cheque ?

7. Find the greatest number that will divide 213, 241 and 297, leaving remainder 3 in each case

8 Find the greatest number that will divide 365, 512 and 323 so as to leave a remainder 8 in each case

9 Find the greatest number that will divide 364 and 532 leaving remainders 4 and 7 respectively.

10 Find the greatest number that will divide 653 and 679 leaving remainders 5 and 4 respectively

11. Find the greatest number which will divide 362, 633 and 310 leaving 2, 3 and 4 as remainders respectively

12 Find the greatest number which will divide 1742, 3723 and 1843 leaving 4, 5 and 6 as remainders respectively.

13 Is there any number that will divide 520 and 728 leaving remainder 7 in each case ?

14 Is there any number that will divide 719 and 524 leaving remainders 3 and 5 respectively ?

15 Is there any number that will divide 515 and 321 leaving remainders 3 and 7 respectively ?

16   Find the greatest number that will divide 151, 175 and 235 leaving the same remainder in each case

17   Find the greatest number that will divide 221, 263 and 326 leaving the same remainder in each case

18   The sum of two numbers is 180 and their G C M is 15   Find the numbers

19   The sum of two numbers is 192 and their G C M is 24   Find all the possible pairs of such numbers

20   The sum of two numbers is 162 and their G C M is 18   Find all the possible pairs of such numbers

21   The product of two numbers is 2700 and their G.C M is 15   Find all the possible pairs of such numbers.

22.   The product of two numbers is 8640 and their G C M is 24   Find all the possible pairs of such numbers

23   The two topmost classes of a school are to be divided into sections having equal number of boys in each. Find the maximum numbers of boys in each section, there being 225 and 315 boys in the classes

24   Find the greatest number that will divide 964, 1238 and 1400 leaving remainders 41, 31 and 51 respectively

25   Find the greatest number that will divide 295, 436 and 622 leaving remainders 15, 16 and 27 respectively

26   A labourer was engaged for a certain number of days for Rs 12 8a   But being absent on some of those days he was paid only Rs 9 1a , prove that his daily wages could not be more than 5 annas

27   A person bought a certain number of oranges for Re 1 8a 6p and sold some of them without profit for 12a, 6p , show that he had still left *at least* 24 oranges

28   Find the two numbers nearest to 10000 that have 169 for their G C. M

29   Find the greatest number of 4 digits and the least number of 5 digits that have 144 for their G C M

30   Find the greatest and the least numbers of five digits that have 85 for their common measure. What is their G C M ?

31   What number is that which when divided by 6, the quotient again by 7 and the quotient again by 8, will give the G C M of 144 and 156 ?

32.   Two numbers 5824 and 7256 when divided by the least number of three digits give the same remainder , find the number and the remainder

33   Two numbers 65636 and 78956 are divided by the greatest number of four digits and the remainder is the same in both cases.   Find the number and the remainder.

34   In solving a division sum, the dividend is 28556 and the successive remainders are 142, 138 and 99   Find the divisor and the quotient.

35   In a division question, the dividend is 56529 and the successive remainders are 37, 108 and 33   Find the divisor and the quotient.

36   In working out a question in the G. C M of two numbers, the quotients are 3, 10 and 2   The last divisor is 45   Find the numbers

37   In working out a question, in the G C M of two numbers the last divisor is 25 and the quotients in order are 10, 3, 2, 2, 1 and 2 , find the numbers

38   In solving a question, in the G C. M. of two numbers the different remainders were 315, 165, 150 and 15 and the first two quotients 1 and 40 , find the numbers and the last two quotients

39   Find the greatest and the least numbers of 6 digits each so that they have 251 for their common measure Also find their G C M

40   Prove that 4157 and 24727 are prime to one another.

41   Show that 85242 and 656361 are not prime to one another

42   Is it possible to divide 5239 into two parts such that their G C M may be 26 ?

43   264 oranges and 693 mangoes are to be distributed among some girls so that each girl may get as many mangoes and as many oranges as another girl   Find the largest possible number of girls and the least possible number of fruits of each kind which a girl gets

CHAPTER VI

LOWEST COMMON MULTIPLE (L. C. M.)

§1 Definition If a number is divisible by another number without remainder, the former is called a **multiple** of the latter, but if it is exactly divisible by several numbers it is called a **common multiple** of them. For instance, 15 is a multiple of 3 because it is divisible by 3 without remainder, but 15 is also exactly divisible by 5 so it is a common multiple of both these numbers

Besides 15 there are other numbers also as 30, 45, 60, 75, etc etc, which are common multiples of 3 and 5 Since 15 is the least number, therefore it is called the **lowest common multiple (L C M.)**

Hence the lowest common multiple of the given numbers is the least number which is exactly divisible by each of these numbers

Note Every common multiple of the given numbers is also divisible by their Lowest Common Multiple (L C M.)

The following are the methods for finding the L C M. of the given numbers —

§2 First method By use of Multiplication Table.

Example Find the L C M of 8 and 12.

Sol Multiples of 8 are 16, 24, 32, 48, 64, 72 and so on,

Multiples of 12 are 24, 36, 48, 60, 72 and so on,

24, 48, 72 etc are common multiples,

24 is the lowest common multiple (L C M). Ans

EXERCISE 23

Find the L C M of the following numbers by use of multiplication table —

1.	4, 6	2	6, 9	3	9, 12.	4	12, 15.
5	12, 16.	6	16, 20	7	15, 20.	8.	6, 15
9	12, 18.	10	8, 10	11	10, 20	12	6, 8.

§3　Second Method.　By use of G. C. M

Let us take the first example once again and find out the G. C. M and L C M of those numbers

G C M. of 8 and $12 = 4$

L C M of 8 and $12 = 24$.

Product of the numbers $= 12 \times 8 = 96$

Product of the G. C. M and L C M $= 24 \times 4 = 96$

The example shows that *the product of two numbers is equal to the product of their G. C M and L C M.*

Proof　Let A and B be any two numbers, P their G. C M and Q their L C M

If we, therefore, divide A and B by P, the quotients M and N (say) must be prime to each other ,

$$A = M P , B = N P ,$$
$$Q = M.N P$$

Now　$A B = M P N P$

$$= P M N P$$

$$= P \times Q \text{ and hence the theorem is proved}$$

Thus for finding the L C M of two numbers we have the following

Rule.　*Divide the product of the given numbers by their G C M　The quotient is the L C. M required.*

Or　*Divide one of the numbers by their G. C M. and multiply the quotient by the other.　The result is the L C M required.*

Example　Find the L. C M. of 81 and 99.

Sol.　The G C M of 81 and $99 = 9$,

L. C. M $= (99 \times 81) - 9 = 891$.　**Ans.**

Or,　　　　$99 - 9 = 11 , 81 - 9 = 9$

.　the L C. M $= 11 \times 81$ or $99 \times 9 = 891$.　**Ans.**

EXERCISE 24

Find the L C. M. of the following by use of G. C M —

1	36, 48.	2	54, 72.	3.	75, 125.
4.	121, 165	5	176, 256.	6	150, 225
7.	256, 400	8	224, 336.	9.	222, 333.
10	625, 500.	11	528, 792	12	1210, 1430.
13.	448, 672	14	630, 840.	15.	690, 920.
16.	780, 910.	17.	620, 744.	18.	975, 1625.

§4   Third Method   By use of Factors.

The following is the rule for finding the L C M of the given numbers when the numbers can be readily separated into Prime Factors —

Rule   *Split the given numbers into Prime Factors and write down the continued product of all the different ones   If a Prime Factor occurs twice or thrice in any of them it must occur the same number of times in the product also*

Example   Find the L C M of 12, 15, 18 and 21.
Sol   Prime Factors of $12 = 2 \times 2 \times 3$
     „   „   „ $15 = 5 \times 3$
     „   ,   „ $18 = 2 \times 3 \times 3$
     „   „   „ $21 = 3 \times 7$

∴ the reqd L C.M $= 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$ **Ans.**

Explanation   The L C M must contain the factors 2, 3, 5, and 7. 2 occurs twice in 12, 3 occurs twice in 18, 5 and 7 occur only once Therefore there must be *two* factors 2, *two* factors 3, *one* factor 5, *one* factor 7 in the L C M

EXERCISE 25.

Find the L C M of the following by factors,—

1   12, 18   2   16, 24   3   21, 27
4   9, 12, 15.   5   12, 18, 24   6   21, 27, 36
7   27, 36, 45.   8   45, 84, 90.   9   72, 120, 108
10   33, 55, 121   11   51, 85, 120   12   45, 60, 75
13   54, 81, 63   14   36, 60, 72   15   64, 96, 112
16   121, 132, 143. 17   77, 88, 176   18   132, 108, 72
19   21, 24, 27, 36. 20.   32, 36, 48, 56   21   28, 56, 63, 84.

§5   Fourth Method   By Ordinary Division

When several numbers are given, their L C M may be more easily found by ordinary division   The following examples will illustrate the method —

Example 1.   Find the L C M. of 16, 24, 36 and 48.
Sol.

∴ the L C. M.
$= 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$= 144$ **Ans**

2)	16,	24,	36,	48
2)	8,	12,	18,	24
2)	4,	6,	9,	12
2)	2,	3,	9,	6
3)	1,	3,	9,	3
	1,	1,	3,	1

Rule *Place all the given numbers in a line with a comma placed between every two of them as shown on page 87 Divide the numbers by a factor which is common to at least two of them Place the quotients and the undivided numbers in a line below Continue the process till all numbers prime to one another are left The continued product of all the divisors and the numbers in the last line will be the L C M. required.*

Note The process may be shortened by rejecting, at any stage of the work, any number which can exactly divide another number in the same line As in the preceding example 16 and 24 might be rejected at the beginning since each of them is a submultiple of 48 in the same line

Example 2 Find the L C M of 4, 6, 8, 12, 16, 20 30

Sol.

$$
\begin{array}{r}
2)\quad \underline{4,\ 6,\ 8,\ 12,\ 16,\ 20,\ 30} \\
2)\quad \underline{\qquad\qquad 6,\ 8,\ 10,\ 15} \\
3,\ 4,\ 5,\ 15
\end{array}
$$

. the L C M $=2\times2\times4\times15=240$ **Ans.**

EXERCISE 26.

Find the L. C. M of the following by division —

1 6, 9, 12	**2** 8, 12, 20.	**3** 4, 10, 15
4 15, 20, 25.	**5** 15, 25, 40	**6** 16, 24, 32
7 6, 8, 12, 15.	**8** 3, 4, 10, 12.	**9** 5, 15, 20, 30.
10 6, 9, 15, 21	**11** 24, 32, 20, 45.	
12. 12, 18, 36, 45, 48	**13** 8, 12, 15, 18, 21, 30	
14 5, 10, 15, 25, 35, 45.	**15** 10, 20, 30, 40, 50, 60.	
16 3, 4, 5, 6, 7, 8, 9, 10	**17.** 4, 6, 8, 10, 12, 14, 16.	
18 3, 5, 7, 9, 11, 13, 15	**19.** 20, 25, 30, 40, 35, 50.	
20. 12, 24, 36, 72, 84, 144	**21** 9, 15, 18, 24, 30, 45.	
22 21, 28, 35, 42, 49	**23** 55, 65, 88, 78, 143	
24 7, 8, 9, 14, 16, 18.	**25.** 88, 99, 132, 143, 130.	

§6 Some Important Typical Examples on L C M

Example 1 Find the least number which when divided by 8, 9, 12 and 15 will leave a remainder 3 in each case

Sol. L. C. M of 8, 9, 12 and 15=360

Since remainder is 3 in each case

, the required number=360+3=363   **Ans.**

Example 2 Find the least number which when divided by 8, 12, 18 and 24 leaves the remainders 6, 10, 16 and 22 respectively

Sol Since the difference between the numbers and the corresponding remainders is 2 in each case,

if we add 2 to the required number, the sum will be exactly divisible by 8, 12, 18 and 24 ,

but L C M of 8, 12, 18 and 24=72

∴ the required number=72−2=70 **Ans**

Example 3 Find the least number of 5 digits which is exactly divisible by 10, 15 and 25.

Sol Least number of 5 digits=10000

L C M of 10, 15 and 25=150

Let us see if 10000 is divisible by 150

By dividing we find that the remainder is 100

If we subtract 100 from 10000 the remaining number will be exactly divisible by 150, i e , by 10, 15 and 25 but the number will reduce to one of 4 digits. Therefore we must add (150−100) to 10000.

```
150)10000(66
    900
   ————
   1000
    900
   ————
    100
```

∴ the required number=10050 **Ans.**

Note   Students should clearly understand the last step

Example 4 Find the greatest number of 6 digits which being divided by 8, 12, 16 and 20 leaves 5, 9, 13 and 17 as remainders respectively

Sol The L C M. of 8, 12, 16 and 20=240

Dividing the greatest number of six digits 999999 by 240 we get 159 as remainder   Hence the number divisible by 240 is 999999−159 or 999840.

∴ the remainder in each case is less than the divisor by 3

∴ the required number =999840−3=999837 **Ans.**

Example 5. What is the least number which is exactly divisible by 11, but when divided by 5, 6, 7 leaves the remainder 4 in each case ?

Sol   L. C. M of 5, 6 and 7=210.

That is, 210 is exactly divisible by 5, 6 and 7 but when 210 is divided by 11 it gives a remainder 1   Now multiply this remainder 1 by such a number that if we add 4 to the product the sum may be divisible by 11

Now $1 \times 7 + 4$ is divisible by 11.

i e , 7 is such a number

the required number $= 210 \times 7 + 4 = 1474$.   **Ans**

Example 6   The L. C M of two numbers is 2880 and their G C M. is 24.   How many pairs of such numbers can be formed ?

Sol   Suppose the numbers divided by their G C. M. give x and y as quotients.

the numbers are $24x$ and $24y$

the L. C. M is $24xy$

$24xy = 2880$

$xy = 120$ (See Note below)

Let (1) $x=1, y=120$ | (5) $x= 5, y=24$
(2) $x=2, y= 60$ | (6) $x= 6, y=20$
(3) $x=3, y= 40$ | (7) $x= 8, y=15$
(4) $x=4, y= 30$ | (8) $x=10, y=12$.

Of these pairs Nos. 2, 4, 6 and 8 are not prime to each other, hence they are rejected.

The admissible pairs are Nos 1, 3, 5 and 8, i.e.,
1, 120 , 3, 40 ; 5, 24 , 8, 15.

Hence the required numbers by multiplying the pairs by 24 are 24, 2880 , 72, 960 , 120, 576, 192, 360.   **Ans**

Note. Since L C M $\times$ G C M = the product of two numbers hence we can arrive at this result as in Example 5, page 80

Example 7   The G C M. of three numbers is 15 and L C M 450   If two of the numbers be 30 and 45, determine the value of the third number

Sol   Dividing the first two numbers by the G. C M. the quotients are 2 and 3.

Let the quotient by dividing the third number be x.

Then the L C M.$= 15 \times 2 \times 3 \times x = 90x$

$90x = 450$ or $x = 5$

the third number $= 5 \times 15$ $= 75$. **Ans**

Note   This is the least value of the number, but other values are also possible, e g ,

$$75 \times 2 = 150$$
$$75 \times 3 = 225$$
$$75 \times 4 = 300$$
$$75 \times 5 = 375$$
$$75 \times 6 = 450$$

Example 8   The G C M. of two numbers of 4 digits is 103 and their L C M. is 337840   determine the numbers.

Sol   By way of illustration take two numbers 75 and 125 and note their relation to each other

L C M. of 75 and $125 = 25 \times 5 \times 3$ 25) 125, 75

and their G C M. $= 25$ 5, 3

Now $\dfrac{L. C. M}{G C M}$ $= \dfrac{25 \times 5 \times 3}{25} = 5 \times 3.$

Similarly in the above question,

$$\frac{L \ C \ M}{G \ C \ M} = \frac{337840}{103} = 3280.$$

or $= 1 \times 3280$
or $= 2 \times 1640$
or $= 4 \times 820$
or $= 5 \times 656$
or $= 8 \times 410$
or $= 10 \times 328$
or $= 20 \times 164$
or $= 40 \times 82$
or $= 41 \times 80$

Now it is easy to pick out the pairs which, multiplied by 103, will give numbers of 4 digits.   Evidently they are 40, 82 and 41, 80.

Since 40, 82 are not prime to each other, hence they are rejected.

the reqd. numbers are 41×103, 80×103, i. e ,
4223, 8240. Ans.

Example 9   The sum of two numbers is 84, and their L. C. M is 144,   Find the numbers.

Sol.   Take two numbers and find their G. C. M , L C M and their sum   You will note that the G C M of these numbers is the same as the G. C M of their sum and L C.M.   This holds good in all cases

Hence the G C M of reqd   numbers＝the G. C M of 84, 144, *i e* , 12

G C M of two numbers＝12

and   their sum＝84

Now we can proceed as in Example 4, page 80, and find that the pairs of such numbers are 12, 72 , 24, 60 , 36, 48 Out of these pairs only the 3rd pair satisfy the given con‐ dition. Hence the reqd. numbers are 36, 48 **Ans.**

Example 10   The product of two numbers is 2688 and their L. C M. is 336 , find all pairs of such numbers

Sol   Since the product of two numbers is equal to the product of their G C.M and L C.M ,

G C M ＝2688—336＝8

G.C M ＝8 and Product＝2688.

Now proceed as in Ex 5, page 80.

The numbers are 8, 336 , 16, 168 , 24, 112 , 48, 56 **Ans**

EXERCISE 27

1. Find the least number which is exactly divisible by 16, 24 and 36.

2   Find the least number which is exactly divisible by 14, 21, 24, 30 and 42

3   Four bells toll after an interval of 8, 9, 12, and 15 seconds   When will they toll together ?

4   Find the least number which when divided by 18, 27 and 36 will leave remainder 7 in each case

5   Find the least number which leaves a remainder 3 in each case when it is divided by 15, 21, 27 and 30

6. Find the smallest number which being increased by 5 will be exactly divisible by 6, 15, 27, 35 and 45

7   What is the smallest number which when diminished by 3 is exactly divisible by 9, 12, 15, 18 and 21?

8   Find the least number which when divided by 5, 6, 7, 8 and 9 leaves remainders 3, 4, 5, 6 and 7 res‐ pectively

9   What is the smallest number which when divided by 12, 15, 18, 24 and 25 leaves remainders 8, 11, 14, 20 and 21 respectively ?

10. What is the least number which when divided by 39, 52 and 72 leaves remainders 36, 49 and 69 respectively ?

11 Find the least number of 5 digits which is exactly divisible by 5, 6, 7 and 8

12 Find the least number of 4 digits which when divided by 6, 9, 12 and 15 leaves remainders 1, 4, 7 and 10 respectively

13 Find the greatest number of 6 digits which being divided by 8, 12, 16, 20 and 24 leaves 6 as remainder in each case

14 Find the greatest number of 5 digits which when added to 2312, may make the sum exactly divisible by 5, 10, 15 and 20.

15 Find the least number which when added to 791 and then divided separately by 6, 7, 8 and 9 may leave remainders 3, 4, 5 and 6 respectively

16 What is the least number which when divided by 6, 7 and 9 leaves remainder 4 in each case, but is exactly divisible by 11 ?

17. (*a*) A number is exactly divisible by 7, but when divided by 2, 3, 4, 5 and 6 leaves remainders 1, 2, 3, 4 and 5 respectively. Find it.

(*b*) Find the least multiple of 17 which leaves a remainder 2 when divided by any of the six even integers 4, 6, 8, 10, 12 and 14.

18 Three men journey 24, 30 and 42 miles a day on cycles round a circular field, the circumference of which is 180 miles After how many days will they meet again ?

19. Find the nearest number to 100000 that can be divided exactly by 2, 3, 4, 5, 6 and 7 respectively

20. Find the nearest integer to 67281 which when divided by 3, 8, 11 and 16 leaves remainders 1, 6, 9 and 14 respectively

21 A heap of stones can be made up exactly into groups of 25, but when made up into groups of 18, 27 and 32, there are always 11 left. Find the least number of stones that may be contained in such a heap.

22 I have a certain number of oranges numbering between 600 and 900 If 2 oranges are taken away, the

remainder can be equally divided among 3, 4, 5, 6, 7 or 12 boys   Find the number of oranges I have

23   Three ponies are running round a race course of 5280 yards   The first runs 440 yards a minute, the second 352 yards and the third 264 yards   Find the time that elapses between their once being together and their coming all together next time again

24   The circumference of the wheels of a carriage are 7 ft 4 in and 11 ft   What is the least distance in which both the wheels will make an exact number of revolutions ?

25.   A vessel is to be exactly filled by any one of the measures 1 seer, 2 seers, 3 seers, 5 seers, 6 seers, 9 seers, or 15 seers.   Find the smallest possible vessel that will serve the purpose.

26   Find the numbers lying between 400 and 600 which are divisible by 5, 10, 15 and 20

27   Find numbers lying between 300 and 400 which when divided by 8, 12, 16 and 24 will leave a remainder 6 in each case

28   Find numbers lying between 500 and 800 which when divided by 12, 15, 18 and 20 will leave remainders 5, 8, 11 and 13 respectively

29.   Find the greatest number and the least number which being subtracted from 3000 will make the result divisible by 7, 11 and 13

30   Two numbers are 125 and 135 , prove that their product is equal to the product of their G C M and L C M

31.   The G. C. M of two numbers is 32 and their L C M is 7680 , if one of them is 512, find the other.

32   The L C M. of two numbers is 6328, their G C M is 113, one of the numbers is 791 , find the other

33   The L C M of two numbers is 1575 and their G C. M. is 25 , one of the numbers is between 200 and 300 , find the numbers   .

34   The product of two numbers is 34560 and their G. C M is 12   Find their L C M

35.   The product of two numbers is 8820 and their L. C M. is 1260   Find their G C M

36   Find the least number of 5 digits which can be divided exactly by all the *even* numbers up to 16 inclusive

37.   Find the greatest number of 5 digits which is exactly divisible by all the *odd* numbers up to 13 inclusive

38   Find the least number of 6 digits which when divided by the *even* numbers up to 20 inclusive may leave one as remainder in each case

39   The G C M. of three numbers is 12 and the L C M 360. If two of the numbers be 24 and 36, determine the value of the third number.

40   34, 51 and a third number have 17 as their G C M. and 510 as their L C M   What values can the third number have ?

41   The L C M of two numbers is 1260 and their G C M is 15   How many pairs of such numbers can be formed ?

42   The G C M of two numbers is 21 and their L C M is 2520   How many pairs of such numbers can be formed ?

43   Find two numbers of four digits each such that their G C M. is 75 and their L C M is 31500

44   The G C M. of two numbers of five digits each is 995 and their L C M. is 202980   Find the numbers

45   Find all the numbers between 250 and 600 that have 1728 as their L. C M

46   The sum of two numbers is 60 and their L C M is 72 , find the numbers

47.   The sum of two numbers is 126 and their L. C M is 216 Determine the numbers

48   The product of two numbers is 1728 and their L C M is 144.   Find all the pairs of such numbers

49   The L C M of two numbers is 168 and their product is 2352 , find the numbers.

50   Find the three largest numbers such that their G C M is 77 and their L. C. M. is 1155.

MISCELLANEOUS EXERCISES

I

1. Multiply 536025 by 1214496 by the shortest method known to you

2 Find the number such that, if it be added 16 times to 73921, the sum will be 75953

3. A grocer buys 20 lbs of tea at $10a$ a pound and 30 lbs of tea at $12a$ a pound and having mixed them sells 35 lbs of the mixture at $11a$, At what price per lb must he sell the remainder that he may neither gain nor lose ?

4 If a man started from New York at 9 o'clock this morning to walk to San Francisco, a distance of 300 miles and walked every day from 9 A M to 2 P M and from 3 P M to 7 P M at an average speed of 5 feet per second, on what day and at what hour would he arrive at San Francisco ?

5. A boy had to divide 64287 by 123   He copied a figure wrong in the divisor and obtained as his quotient 502 and remainder 31   What mistake did he make ?

6. Find the least number of five digits which is exactly divisible by 7, 8, 9 and 12

7 A bag contains £26 5s. in half crowns, florins and shillings, There are three times as many florins and four times as many shillings as half crowns Find how many coins of each kind the bag contains,

8. A man spends Rs 1200 annually for 5 years and runs into debt   He then reduces his expenses to Rs. 650 a year, and in 6 years just clears off his debt , what is his yearly income ?

II

1. Find the greatest number of six digits which is exactly divisible by 2037

2 A man spends Rs 750 a year for 6 years and saves some money , he then raises his expenditure during the next 10 years to Rs 1050 a year and finds all his savings spent, What is his yearly income ?

3   A man bought two heaps of mangoes, one for Rs 10 5*a* and the other for Rs 18 9*p*   If the price of each mango be the same, and neither less than three, nor more than four annas, find the total number of man goes he bought

4   Divide Rs 145 12*a* 6*p* between two persons giving twice as much to one of them as to the other

5   A number was divided by 7, 8 and 9 in succession and the remainders are 3, 5 and 4 respectively   Find the remainder had it been divided by 504

6.   What number multiplied by 238 gives the same result as 408 multiplied by 350 ?

7   Multiply 8350695 by 1921664 in three lines

8   A purse contains a certain number of sovereigns, four times as many shillings and six times as many four penny pieces , and the whole sum is £13.   Find how many sovereigns, shillings and four-pennies are there in the purse

III

1   Find the least number that can be divided by all the even numbers up to 20 inclusive

2   Multiply 3509856 by 1215672 in three lines.

3   Find the dividend and the remainder in

$$
\begin{array}{l}
5)\ *\ *\ *\ *\ \cdot \\
\hline
7)\ *\ \ *\ *\ * \\
\hline
8)\ \ \ *\ *\ \cdot\ * \\
\hline
\qquad 3\ 1\ 9,\ 1\ \text{remainder}
\end{array}
$$

1 remainder
0 remainder

4   An estate of 1416 acres 2 roods 16 sq poles was divided into allotments, each 4 acres 3 roods 27 sq poles in area , how many allotments were made ?

5   Three persons *A*, *B* and *C* are possessed of certain sums of money, such that *A* and *B* together have 170 rupees , *A* and *C* together have 200 rupees , and *B* and *C* together have 210 rupees   What is the sum possessed by each ?

F 7

6   A horse and its saddle together cost Rs 1250 and the horse is worth seven times the saddle   What is the value of each ?

7.   A number being successively divided by 6, 8 and 9 leaves remainders 3, 2 and 5 respectively   Find the remainders if the order of divisors be reversed

8 (*a*)   Find the greatest number of 4 digits and the least number of 5 digits that have 124 for their G C. M

(*b*)   Write in words 77665544332211.

IV

1   Find the least number which will, in each case leave a remainder 4 when divided by 15 and 18.

2   Find the greatest and the least numbers of 6 digits that have 251 for their common measure   What is their G C M. ?

3   A plot of land, measuring 7 acres 2 roods 12 sq yards, is sold for £907 16s   What would a field of 8 acres 16 sq poles fetch if sold at the same price per acre ?

4   A boy multiplies 533 by a certain number and obtains 70055 as his answer   If both the zeroes are wrong but the other figures are right, find the correct answer

5   A number is divided by 235 and the successive partial dividends are 1532, 1229 and 546   Find the dividend, quotient and the remainder

6.   Divide Rs 3954 between *A*, *B* and *C* so that *A* might get twice as much as *B* and *C* together and *B* one-third of *C*

7.   If 37 shares at £98. 17*s* 6*d*   each and 9 shares at another price per share, altogether cost £4350 2*s* 9*d*, how much did one of the latter shares cost ?

8   Find *x* in the following division question :—

$$\left. \begin{array}{l} x) \dfrac{*\ \dagger\ \ast}{} \\ y) \dfrac{\ast\ \dagger\ \dagger\ -\ \ast}{\ast\ \ast\ \ast\ \ast}, 3\ \text{rem} \\ \qquad\qquad, 4\ \text{rem} \end{array} \right\} \text{Complete remainder is 23}$$

V

1   What is the least number, which when divided by 16, 24, 36, 48 and 60 leaves remainders 9, 17, 29, 41 and 53 respectively ?

2   The sum of £363 1s is placed in three bags   In one is £78 3s, in another half as much again.   How much is in the third ?

3   By what number must 825 be multiplied so that when the product is subtracted from one million, the result is 484375 ?

4   Find the prime factors of 27573.

5   Find the quotient in the following division question —

$$
\begin{array}{l}
\,^{*})\;5\;3\;8\;9\;7\;8 \\
11)\overline{\text{* * * * *}},\,2\ \text{rem} \\
\overline{\text{* * *}},\,8\ \text{rem}
\end{array}
\left.\begin{array}{l}\\ \\ \end{array}\right\}\text{Complete remainder}=66
$$

6   A man living at the rate of Rs 350 a month for 10 months finds that his expenses are exceeding his income, and reduces his expenditure to Rs. 275 a month   At the end of 5 months he finds that he is just out of debt. What is his income ?

7   A man bought 144 oranges at the rate of 6 for four pice and exchanged them for mangoes at 8 pice a mango, how many mangoes did he receive ?

8   In solving a question in the G C M of two numbers, the quotients are 5, 1, 18, 1, 3, 1, and 2   The last divisor is 15.   Find the numbers.

VI

1.   Find the greatest number which will divide 1028, 1629 and 2130 leaving remainders 3, 4 and 5 respectively.

2   I buy 500 oranges at 2 for three half-pence, and again 500 oranges at 2 for three pence, after which 60 of the better sort are eaten, I then sell the remainder at five farthings each.   How much do I gain or lose ?

3   A man living at the rate of Rs 250 a month for 8 months finds that his expenses are exceeding his income, and reduces his expenditure to Rs 160 a month, at the end of 10 months he finds that he is not only out of debt but has saved Rs 180, what is his income ?

4   A number being successively divided by 5, 6 and 7 leaves 2, 0 and 5 as remainders respectively. What are the remainders if it is divided by 7, 6 and 5 ?

5   What least number must be added to 8356935 to make it exactly divisible by 2356 ?

6 A, B and C have Rs 973 $5a$ between them, B and C have together Rs 442 and A and C have Rs. 687, $12a$ together   How much has C ?

7   In a division sum the quotient is 6 times the divisor and thirty times the remainder. If the remainder be 3, find the dividend

8   Find the multiplier in the following examples on multiplication and complete the operation —

(a)   5 6 2   (b)   5 6 2 3
              ⁊ ✳ ˸   ⁊

          ‾ ⁊ ⁊ ˟ ⁊ ‾   ‾ ‾ ˟ ✳ ˟

              ⁊   ˟ ⁊   ✳ ⁊   ˥ ˺

          ✳   ⁊ ˅   ✳ ✳ ⁊ ˟

          ‾ ⁊ ˅ '8 2 2 ‾   ‾ ˅ ⁊ ‾ 2 5 2

VII

1.   Multiply 8356926 by 25616192 in three lines

2   A number when divided by 775 gives a remainder 188, what will be the remainder if the same number be divided by 31 ?

[Hint   Number = $775 \times$ Quotient + 188
                = $31 \times 25 \times$ Quotient + $31 \times 6 + 2$]

3   How many chairs worth Rs 2 $3a$ $6p$ each should be given in exchange for 71 dozen of pencils worth $7a$ $6p$. per dozen ?

4   How often is the sum of £19 11s 4d contained in the eleventh part of £1291. 8s ?

5 Divide £56 13s 4d between A, B and C so that A may have £3 6s 8d. more than B and B £1 3s 4d more than C

6 A certain number when divided successively by 7, 8 and 9 leaves 2, 0, 6 as remainders respectively. What are the remainders if the order of the divisors be reversed ?

7. Find the greatest number which will divide 116, 221 and 356 leaving the same remainder in each case.

8 Find the greatest and the least numbers of 5 digits exactly divisible by 15, 20, 25 and 30.

VIII

1. A number is divided thus —

$$
9)\overline{}, \text{ rem } 1.
$$
$$
\overline{}, \text{ rem } 3
$$

If the true remainder be 61, find the complete divisor.

2 Among how many persons can I distribute Rs. 103 5a 10p. if I give 15a 9p to each and how much will be left ?

3 A and B together earn Rs 371 2a a month , B and C together earn Rs. 466. 12a and A and C together earn Rs 536 14a. ; find the amount earned by each of them.

4 Find the least number which is exactly divisible by 7, but when divided by 3, 5, 6 12 may leave one as remainder in each case

5 Five bells begin to toll simultaneously and they toll at intervals of 4, 6, 8, 9 and 10 seconds. After what time will they again toll simultaneously ?

6. The product of two numbers is 864 and their quotient is 6, find the numbers

[Hint   Product÷quotient= (smaller number)^2]

7 The total expenses of a family when wheat is at Rs. 4 8a. per maund are Rs 45 , when wheat is at Rs 5 per maund, they are Rs 48 (other expenses remaining the same) , find his total expenses when wheat is at Rs 5 12a per maund.

8. Find the greatest number which will divide 53032, 118433 and 140349 leaving 7, 8 and 9 as remainders respectively.

IX

1   Simplify by the shortest method known to you —

  (a)   7535693 × 9999   (b)   2637535693—9999

2   Multiply 856935943 by 16218108 in three lines

3   A man bought 50 cows at Rs 45 each, 60 cows at
Rs. 49 each and 75 cows at Rs 52 each , at what price
per head must he sell the cows so as to gain Rs 160 by his
bargain ?

4   A purse contains a certain number of sovereigns,
five times as many shillings, and eight times as many pence
If the whole sum is £6 8s 4d , find how many sovereigns,
shillings and pence there are in the purse

5.   Find the least number which when divided by 36,
48, 60, 72 and 80 leaves remainders 20, 32, 44, 56 and 64
respectively

6.   The product of two numbers is 700 and their
G C. M is 5 , find all the possible pairs of such numbers

7   Find the remainder and the dividend in the follow-
ing question —

$$\begin{array}{r} 7) \overline{} \\ 8) \overline{} \text{, remainder 5,} \\ 4\ 3\ 5\ 6\ 9 \text{, remainder 2} \end{array}$$

8   What quantity of water must I add to a pipe of
wine which cost Rs 900 to reduce its price to Rs 5 a gallon ?
(126 gallons make 1 pipe)

X

1   A man at his death directed in his will that his
property worth Rs. 10800 is to be divided among his 4
sons, 3 daughters and 2 cousins such that each son would
receive four times as much as each cousin and each daughter
three times as much as each cousin , how much does each
son receive ?

2   Find the number which when divided by 25 gives
a quotient which if diminished by 9 and the result multiplie
by 12 gives a product 108

3   A man's yearly income is £825. 13s 4d. and his daily expenditure a guinea and a half   How much did he save in the year 1936 ?

4   The sum of two numbers is 135 and their G C. M is 15 , find all the possible pairs of such numbers

5   If the monthly expenses of a family be Rs. 62 8a. when rice is at Rs 3. 12a per maund and Rs. 65 4a. when rice is at Rs 4 2a per maund, what would the expenses be when rice would be at Rs. 4 5a. per maund, supposing other expenses remain the same ?

6   Find the sum of money which when multiplied by 12, Rs 14 added to the product, the sum divided by 15, and Rs. 2 10a 8p added to the quotient, the result is Rs 16.

7   Complete the following examples on multiplication —

(a)
```
        5 3 3 2
        * * *
       ---------
        * *   *
      >  * * *
    *  * * * *
   --------------
    * * * * 1 8 0
```

(b)
```
        8 5 1 7
        * * * *
       ----------
    * * * * * *
       * * *
    * *   * *
   ----------------
   * * * * 6 9 1
```

8.   (a) Supply the missing figures denoted by the stars in the following division sum .—

```
7**)  52**1 ( 2
       **1*
      ------
       1*1
      ****
      ------
       1*9
```

(b) A certain number when divided successively by 12, 15 and 8 leaves remainders 5, 3, 7 respectively.   Find the remainder if it were divided by 1440.

CHAPTER VII

VULGAR FRACTIONS.

§1. Definition A **fraction** denotes a part or parts of a unit

Illustrations (*i*) Let the straight line AB indicate a unit

| A | C | D | B |

Divide it into three equal parts as $AC=CD=DB$

AC is one third of the unit and AD two thirds of it. AC and AD are therefore fractions of AB

(*ii*) $8a$ is a fraction of Re 1, because it is one half of a rupee 10a. 8p is a fraction of Re 1 because it is two-thirds of a rupee 12a is a fraction of Re. 1, because it is three fourths of a rupee.

§2 Notation A fraction is expressed by two numbers placed one above the other with a horizontal line between them The lower number is called the **denominator** and the upper number is called the **numerator** The numerator and denominator are called the **terms** of a fraction Thus one half, two thirds, three fourths, etc, etc are expressed as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, in which the upper numbers 1, 2, 3, are the *numerators* and the lower numbers 2, 3 and 4 the *denominators*.

§3. The denominator shows the number of equal parts into which the unit is divided and the numerator shows the number of times such equal parts are taken Thus $\frac{3}{4}$ means that a unit is divided into four equal parts and three of those parts are taken

$\frac{1}{2}$, $\frac{3}{4}$, etc are also read as one over two, three over four etc Fractions expressed in the above notation are called **vulgar fractions**, *i.e*, *common* or *ordinary* fractions

§4. The following distinctions may be made in fractions —

(1) **Proper fraction** is one in which the numerator is less than the denominator, as $\frac{2}{3}$, $\frac{5}{7}$, $\frac{11}{15}$, etc, etc.

(2) **Improper fraction** is one in which the numerator is equal to or greater than the denominator as $\frac{5}{3}$, $\frac{7}{5}$, $\frac{11}{4}$, etc.

(3) **Simple fraction** is one in which the numerator and denominator are both integers, as $\frac{3}{5}$, $\frac{9}{4}$, etc

(4) **Compound fraction** is a fraction of a fraction as $\frac{3}{4}$ of $\frac{5}{7}$, $\frac{2}{3}$ of $\frac{4}{7}$ of $\frac{11}{9}$ etc

(5) **Complex fraction** is one in which the numerator or the denominator or both are not integers, i. e , they are themselves fractional as $\frac{\frac{5}{6}}{8}$, $\frac{9}{\frac{7}{8}}$, $\frac{\frac{8}{2}}{\frac{4}{5}}$, $\frac{2}{3\frac{1}{2}}$, $\frac{5\frac{1}{2}}{8\frac{1}{2}}$, $\frac{\frac{1}{2}\times\frac{3}{4}}{\frac{5}{6}\times\frac{7}{8}}$ etc

In the examples given here, we have used the whole numbers combined with fractions, as $3\frac{1}{2}$, $5\frac{1}{2}$, etc. Such quantities are called **mixed numbers**

EXERCISE 28 (Oral)

Express as fractions —

1   Two thirds   2. Three-fifths. 3. Five sevenths.
4   Nine-fourteenths   5   Six-elevenths
6   Twenty five *over* one hundred and twenty one
7   One hundred and twenty three *over* five hundred and six

Express in words —

8   $\frac{3}{11}$ 9. $\frac{8}{15}$ 10 $\frac{7}{22}$ 11 $\frac{26}{98}$
12   $\frac{78}{139}$ 13 $\frac{75}{140}$ 14 $\frac{75}{414}$ 15. $\frac{2}{521}$.

Write down the value of —

16   $\frac{1}{2}$ of Re 1   17. $\frac{1}{4}$ of Re 1   18 $\frac{1}{4}$ of Re. 1.
19   $\frac{7}{16}$ of Re 1. 20 $\frac{5}{16}$ of 1 seer   21 $\frac{5}{8}$ of 1 md
22   $\frac{2}{3}$ of 1 yard 23. $\frac{5}{36}$ of 1 yard   24 $\frac{111}{1760}$ of 1 mile.
25   $\frac{3}{4}$ of Rs 5 12a

REDUCTION OF FRACTIONS

§5   To reduce a fraction to its lowest terms.

When the numerator and the denominator of a fraction are prime to each other, it is said to be in its lowest terms. To reduce a fraction to its lowest terms depends on the following proposition :—

When the numerator and the denominator of a fraction are each divided by the same number, the value of the fraction is not altered Thus

$$\frac{15}{6} = \frac{15-3}{6-3} = \frac{5}{2}$$

There are two ways of reducing a fraction to its lowest terms. First by reducing the numerator and the denominator into their elementary factors and cancelling the common ones Secondly by finding the G C M of the numerator and the denominator and dividing both by this G C M

Example. Reduce $\frac{1155}{5005}$ to its lowest terms

Sol First method $\frac{1155}{5005} = \frac{3 \times 5 \times 7 \times 11}{5 \times 7 \times 11 \times 13} = \frac{3}{13}$ Ans

Note 1 It should be very carefully noted that when a factor is *cancelled*, it is replaced by 1 not by 0

Note 2 By cancelling the common factors we actually divide the numerator and the denominator by the same number The tests of divisibility are very useful in solving such questions

We shall solve the above question thus

$$\frac{1155}{5005} = \frac{231}{1001} \text{ [by dividing the terms by } 5$$
$$= \frac{33}{143} \text{ [by ,, ,, ,, ,, } 7$$
$$= \frac{3}{13} \text{ [by ,, ,, ,, ,, } 11$$

Second Method G. C. M of 1155 and 5005 = 385,

$$\therefore \frac{1155}{5005} = \frac{1155 \div 385}{5005 \div 385} = \frac{3}{13}. \text{ Ans.}$$

EXERCISE 29.

Reduce to their lowest terms :—

1. $\frac{15}{21}, \frac{18}{24}$. 2. $\frac{24}{72}, \frac{21}{56}$. 3. $\frac{270}{180}, \frac{120}{160}$.
4. $\frac{18}{27}, \frac{51}{68}$. 5. $\frac{35}{45}, \frac{55}{121}$. 6. $\frac{74}{198}, \frac{112}{144}$.
7. $\frac{121}{165}, \frac{135}{165}$. 8. $\frac{169}{273}, \frac{240}{315}$. 9. $\frac{192}{208}, \frac{808}{840}$.
10. $\frac{250}{388}, \frac{300}{480}$. 11. $\frac{676}{600}, \frac{728}{819}$. 12. $\frac{640}{704}, \frac{576}{720}$.

13 $\frac{416}{627}, \frac{625}{750}$ 14 $\frac{952}{1234}, \frac{1529}{8780}$ 15. $\frac{1452}{1980}, \frac{1624}{2275}$.

16 $\frac{2184}{1912}, \frac{2880}{4100}$. 17. $\frac{2240}{8600}, \frac{5485}{7205}$ 18 $\frac{2466}{2605}, \frac{3366}{4448}$

19 $\frac{2475}{2750}, \frac{2651}{7271}$ 20 $\frac{5841}{8485}, \frac{5005}{17017}$ 21. $\frac{5786}{11566}, \frac{11575}{12964}$.

22. $\frac{10584}{15624}, \frac{11479}{78088}$ 23 $\frac{13080}{13952}, \frac{29175}{33849}$ 24. $\frac{126852}{288368}, \frac{45576}{80762}$

25 $\frac{3297641}{4416965}$. 26 $\frac{1545600}{3675520}$ 27 $\frac{1699858}{4777170}$.

Reduce (by cancelling) the following to lowest terms :—

28 $\dfrac{16 \times 18}{27 \times 80}$ 29 $\dfrac{100 \times 48}{49 \times 175}$ 30. $\dfrac{21 \times 10 \times 16}{24 \times 18 \times 70}$.

31. $\dfrac{19 \times 23 \times 26}{57 \times 78 \times 92}$ 32. $\dfrac{17 \times 29 \times 56 \times 65}{51 \times 49 \times 87 \times 39}$.

---

§6 To reduce a mixed number to an improper fraction.

Rule *Multiply the whole number by the denominator of the fractional part and then add the numerator to the product. Put the result as the new numerator and the denominator of fractional part as the denominator.*

Thus $5\frac{1}{3} = \dfrac{5 \times 3 + 1}{3} = \dfrac{16}{3}$.

§7 To reduce an improper fraction as a whole or a mixed number

Rule. *Divide the numerator by the denominator, express the quotient as a whole number and the remainder as the numerator and the denominator of the improper fraction as the denominator of the fractional part.*

Thus $\frac{502}{21} = 502 \div 21 = (23 \times 21 + 19) \div 21 = 23\frac{19}{21}$.

EXERCISE 30.

Reduce the following mixed numbers to equivalent improper fractions :—

1. $3\frac{1}{2}$. 2 $3\frac{3}{4}$. 3. $5\frac{2}{3}$. 4 $7\frac{1}{8}$

5 $9\frac{3}{4}$ 6. $12\frac{7}{8}$ 7. $11\frac{5}{8}$. 8 $16\frac{5}{8}$.

9. $16\frac{7}{8}$ 10 $21\frac{9}{11}$ 11. $24\frac{9}{16}$. 12. $28\frac{7}{8}$.

13 $121\frac{8}{8}$. 14 $124\frac{7}{8}$. 15 $324\frac{8}{20}$. 16 $521\frac{5}{17}$.

17 $981\frac{5}{21}$. 18 $892\frac{20}{31}$. 19 $8312\frac{24}{35}$. 20. $5625\frac{21}{35}$.

Reduce the following improper fractions to mixed numbers —

21	$\frac{8}{7}$	22	$\frac{19}{4}$	23	$\frac{21}{5}$.	24	$\frac{28}{5}$.
25	$\frac{77}{9}$	26	$\frac{101}{11}$	27	$\frac{194}{14}$	28	$\frac{197}{16}$.
29	$\frac{802}{15}$.	30	$\frac{981}{10}$.	31.	$\frac{1224}{18}$	32.	$\frac{1685}{24}$.
33	$\frac{7218}{20}$	34.	$\frac{7024}{31}$	35.	$\frac{12829}{56}$	36	$\frac{16784}{40}$
37	$\frac{16329}{46}$.	38.	$\frac{21529}{89}$	39.	$\frac{16526}{78}$.	40.	$\frac{21730}{51}$.

---

§8   To reduce a whole number or a fraction to an equivalent fraction with any denominator

To reduce a fraction to an equivalent fraction with any denominator depends on the following proposition —

When the numerator and the denominator of a fraction are each multiplied by the same number, the value of the fraction is not altered   Thus :

$$\tfrac{3}{4} = \frac{3 \times 3}{4 \times 3} = \tfrac{9}{12}.$$

Illustration. $\frac{3}{4}$ of Re 1 = 12a ; $\frac{12}{16}$ of Re 1 = 12a.

`` $\frac{3}{4} = \frac{12}{16}$ { since the numerator and the denominator of $\frac{3}{4}$ are each multiplied by 4

Graphical Illustration   Take two pieces of squared paper each having equal number of squares of the same size as shown below,
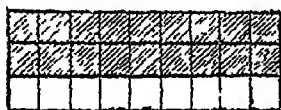
Fig I   Fig II.

In the first figure 18 squares are shaded and they express $\frac{6}{8}$ of the whole figure

In the second figure also 18 squares are shaded and they express $\frac{6}{9}$ of the whole figure.

. $\frac{6}{8} = \frac{6}{9}$ { since the numerator and the denominator of $\frac{2}{3}$ are each multiplied by 3

Example 1   Express 4 with denominator 12

Sol. $4 = \frac{4}{1} = \frac{4 \times 12}{1 \times 12} = \frac{48}{12}$ **Ans.**

Example 2   Express $\frac{5}{6}$ with denominator 96.
Sol. $96 = 6 \times 16$

$$\frac{5}{6} = \frac{5 \times 16}{6 \times 16} = \frac{80}{96}$$ **Ans**

§9.   To reduce fractions to equivalent ones with their least common denominator

Rule   *Find the L. C. M. of the denominators , this will be the least common denominator   Then divide the L C M so found by the denominator of each fraction and multiply the quotient so found by the numerator of the fraction which belongs to it for the new numerator of that fraction*

Note   Before applying the above Rules reduce mixed numbers to improper fractions and compound fractions to simple ones   also if he L C D be required   the given fractions should be first reduced to their lowest terms

Example 3   Reduce $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{5}{12}$ to equivalent fractions with the least common denominator

Sol   L C. M of the denominators 3, 6, 8 and 12=24.

$24 - 3 = 8$, $\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}$

$24 - 6 = 4$, $\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$

$24 - 8 = 3$, $\frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$

$24 - 12 = 2$, $\frac{5}{12} = \frac{5 \times 2}{12 \times 2} = \frac{10}{24}.$

the required fractions are $\frac{16}{24}$, $\frac{20}{24}$, $\frac{21}{24}$, and $\frac{10}{24}$ **Ans**

EXERCISE 31 (Oral)

Write down the missing numerators in the following fractions —

1   $\frac{1}{2} = \frac{}{6}$ 2. $\frac{3}{8} = \frac{}{12}$ 3 $\frac{3}{4} = \frac{}{16}.$

4   $\frac{2}{5} = \frac{}{15}$ 5 $\frac{3}{7} = \frac{}{35}$ 6. $\frac{4}{9} = \frac{}{18}.$

7   $\frac{2}{13}=\frac{}{52}$.   8 $\frac{5}{12}=\frac{}{60}$ 9 $\frac{7}{16}=\frac{}{80}$

10   $\frac{5}{18}=\frac{}{90}$ 11 $\frac{8}{19}=\frac{}{76}$ 12 $\frac{13}{18}=\frac{}{156}$

Write down the missing denominators in the following fractions —

13. $\frac{3}{4}=\frac{9}{}$ 14 $\frac{5}{6}=\frac{30}{}$ 15 $\frac{7}{8}=\frac{63}{}$

16   $\frac{13}{19}=\frac{60}{}$.   17. $\frac{15}{16}=\frac{75}{}$ 18 $\frac{16}{25}=\frac{64}{}$

19   $\frac{24}{25}=\frac{120}{}$ 20. $\frac{11}{34}=\frac{99}{}$.   21 $\frac{9}{14}=\frac{81}{}$

22   Express the whole numbers 12, 15, 16 and 20 as fractions with denominators 13

23   Express the whole numbers 11, 24, 25 and 45 as fractions with denominators 20

24   Express $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{9}{16}$ as equivalent fractions with denominators 96

25   Express $\frac{5}{8}$, $\frac{7}{16}$, $\frac{9}{32}$ and $\frac{17}{64}$ as equivalent fractions with denominators 128.

Reduce to equivalent fractions having the least common denominator —

26. $\frac{3}{4}$, $\frac{5}{6}$	**27** $\frac{5}{12}$, $\frac{7}{16}$	**28.** $\frac{5}{6}$, $\frac{7}{12}$.
29 $\frac{7}{8}$, $\frac{11}{12}$.	**30.** $\frac{5}{14}$, $\frac{11}{21}$	**31.** $\frac{6}{21}$, $\frac{9}{14}$
32 $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$.	**33** $\frac{3}{5}$, $\frac{9}{10}$, $\frac{7}{15}$	**34** $\frac{5}{8}$, $\frac{8}{15}$, $\frac{7}{80}$
35. $\frac{1}{10}$, $\frac{1}{15}$, $\frac{5}{12}$, $\frac{7}{18}$	**36** $\frac{5}{19}$, $\frac{3}{38}$, $\frac{4}{57}$, $\frac{5}{76}$	
37. $\frac{5}{14}$, $\frac{9}{28}$, $\frac{6}{35}$, $\frac{9}{42}$	**38** $\frac{5}{13}$, $\frac{7}{16}$, $\frac{9}{32}$, $\frac{17}{34}$	
39. $\frac{3}{7}$, $1\frac{4}{9}$, $\frac{5}{14}$, $\frac{8}{27}$.	**40** $\frac{2}{5}$, $1\frac{2}{5}$, $\frac{7}{8}$, $2\frac{1}{15}$	

§10   Comparison of fractions

Compare the fractions $\frac{5}{7}$ and $\frac{3}{7}$

Exp.   A unit is divided into 7 equal parts in both cases, but in the first case 5 parts are taken and in the second case 3 parts are taken   Therefore, $\frac{5}{7}$ is greater than $\frac{3}{7}$

Rule 1   *When two fractions have the same denominator, the greater is that which has the greater numerator.*

Compare the fractions $\frac{7}{21}$, $\frac{7}{18}$

Exp   In the first case a unit is divided into 21 equal parts and 7 parts are taken

In the second case a unit is divided into 15 equal parts and 7 parts are taken.

Now it is obvious that a part of the unit which is divided into 15 equal parts must be greater than the part of a unit which is divided into 21 equal parts. Therefore $\frac{7}{15}$ is greater than $\frac{7}{21}$

Rule 2 *When two fractions have the same numerator, the greater is one which has the smaller denominator*

Note A fraction, therefore increases either when its numerator increases or when its denominator diminishes and similarly it diminishes either when its numerator diminishes or when the denominator increases

§11 But if neither the numerators nor the denominators, of the given fractions are equal they must be reduced to equivalent fractions having the least common denominator and then apply Rule 1 Thus

Compare the fractions $\frac{5}{12}$, $\frac{7}{16}$

$\frac{5}{12}$, $\frac{7}{16} = \frac{20}{48}$, $\frac{21}{48}$

$\frac{21}{48}$ i.e., $\frac{7}{16}$ is greater than $\frac{20}{48}$ i.e., $\frac{5}{12}$.

Example 1 Arrange the fractions $\frac{2}{11}$, $\frac{5}{11}$, $\frac{3}{11}$, in ascending order of magnitude

Fractions after arranging in ascending order of magnitude are $\frac{2}{11}$, $\frac{3}{11}$, $\frac{5}{11}$ **Ans.** (Rule 1)

Example 2. Arrange the fractions $\frac{15}{16}$, $\frac{15}{31}$, $\frac{15}{26}$, in descending order of magnitude.

Fractions after arranging in descending order of magnitude are $\frac{15}{16}$, $\frac{15}{26}$, $\frac{15}{31}$ **Ans** (Rule 2)

Example 3 Arrange in ascending and descending order of magnitude the fractions $\frac{5}{12}$, $\frac{9}{16}$, $\frac{7}{24}$

Sol First reduce the fractions to equivalent ones having the least common denominator

$\frac{5}{12}$, $\frac{9}{16}$, $\frac{7}{24} = \frac{20}{48}$, $\frac{27}{48}$, $\frac{14}{48}$,

and of these $\frac{14}{48}$ is the least and $\frac{27}{48}$ is the greatest,

ascending order is $\frac{7}{24}$, $\frac{5}{12}$, $\frac{9}{16}$, $\Big\}$ **Ans**

and descending order is $\frac{9}{16}$, $\frac{5}{12}$, $\frac{7}{24}$

§12. The defect of a fraction from 1 is called its **complement**, thus $\frac{1}{5}$ and $\frac{2}{7}$ are respectively the *complements* of $\frac{4}{5}$ and $\frac{5}{7}$.

Rule 3   *Fractions may sometimes be very conveni-
iently compared by taking their complements, provided
that each of the complements has in its numerator the
same number*, the greatest and least fractions will be those
that have the least and the greatest complements

Example 4   Find the *greatest* and the *least* of the
fractions $\frac{5}{6}$, $\frac{24}{25}$ and $\frac{7}{8}$

Sol.   The complements of these fractions are $\frac{1}{6}$, $\frac{1}{25}$, $\frac{1}{8}$
respectively , of these $\frac{1}{25}$ is the least and $\frac{1}{6}$ is the greatest

   $\frac{24}{25}$ is the *greatest* and $\frac{5}{6}$ is the *least*.   **Ans.**

Example 5.   Arrange in order of magnitude $\frac{3}{8}$, $\frac{6}{25}$, $\frac{7}{22}$.

Sol $\frac{3}{8} = \frac{3-3}{8-3} = \frac{1}{2\frac{2}{3}}$,

          $\frac{6}{25} = \frac{6-6}{25-6} = \frac{1}{4\frac{1}{6}}$,

          $\frac{7}{22} = \frac{7-7}{22-7} = \frac{1}{3\frac{1}{7}}$

   Now of these $\dfrac{1}{2\frac{2}{3}}$ is the greatest and $\dfrac{1}{4\frac{1}{6}}$ is the least

(Rule 2) ,   the order of magnitude is $\frac{3}{8}$, $\frac{7}{22}$, $\frac{6}{25}$ **Ans.**

EXERCISE 32

Find the greatest fractions by the first method —

1 $\frac{5}{23}$, $\frac{3}{22}$, $\frac{9}{22}$ **2** $\frac{5}{12}$, $\frac{3}{16}$, $\frac{7}{24}$ **3.** $\frac{7}{8}$, $\frac{11}{12}$, $\frac{17}{20}$

Find the least fractions by the second method —

4 $\frac{3}{11}$, $\frac{3}{16}$, $\frac{3}{22}$,   **5** $\frac{3}{7}$, $\frac{5}{9}$, $\frac{4}{13}$ **6** $\frac{3}{7}$, $\frac{9}{11}$, $\frac{12}{17}$

Find the greatest and the least of the following frac-
tions by the third method —

7 $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$ **8** $\frac{4}{9}$, $\frac{5}{21}$, $\frac{9}{16}$.   **9** $\frac{9}{11}$, $\frac{7}{29}$, $\frac{5}{28}$

Arrange the following fractions in descending order of
magnitude —

10 $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ **11.** $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$.   **12** $\frac{5}{12}$, $\frac{7}{18}$, $\frac{18}{90}$

13 $\frac{6}{11}$, $\frac{10}{52}$, $\frac{16}{33}$.   **14** $\frac{7}{12}$, $\frac{8}{9}$, $\frac{7}{16}$.   **15** $\frac{5}{6}$, $\frac{9}{16}$, $\frac{11}{18}$

Arrange the following fractions in ascending order of magnitude —

16 $\frac{7}{19}, \frac{10}{27}, \frac{16}{36}, \frac{5}{72}$ **17.** $\frac{17}{21}, \frac{9}{14}, \frac{1}{7}, \frac{5}{8}$

18. $\frac{8}{7}, \frac{18}{21}, \frac{19}{35}, \frac{18}{29}$ **19.** $\frac{7}{19}, \frac{9}{38}, \frac{15}{57}, \frac{15}{76}$

20. Find a fraction lying between $\frac{5}{6}$ and $\frac{6}{7}$ whose denominator is 84, between $\frac{3}{5}$ and $\frac{2.9}{7.3}$ whose denominator is 720

ADDITION OF FRACTIONS

§13   You know that $\frac{3}{16}$ of a rupee $=3a$

$$\frac{5}{16} \text{ of a rupee} = 5a$$
$$\frac{7}{16} \text{ of a rupee} = 7a$$
$$\frac{3}{16} + \frac{5}{16} + \frac{7}{16} \text{ of a rupee} = 3 + 5 + 7 \text{ or } 15a$$
$$= \frac{15}{16} \text{ of a rupee}$$

Rule   *If the fractions have a common denominator, add their numerators for the new numerator and put the common denominator underneath.*

§14   But if the fractions have no common denominator, first reduce them to equivalent fractions having the least common denominator and then add as shown above.

Example 1   Add together $\frac{2}{3}, \frac{5}{6}, \frac{7}{8}$

Sol $\frac{2}{3}, \frac{5}{6}, \frac{7}{8} = \frac{16}{24}, \frac{20}{24}, \frac{21}{24}$ the L.C M of 3, 6, 8 = 24

$$\text{sum} = \frac{16 + 20 + 21}{24} = \frac{57}{24} = \frac{19}{8} = 2\frac{3}{8} \text{ Ans}$$

Note 1   If the answer be an improper fraction, reduce it to a mixed number

Note 2   If the fraction in the answer be not in its lowest terms, reduce it to its lowest terms   The fractions in the questions should also be reduced to their lowest terms

Example 2   Add together $\frac{3}{5}, \frac{7}{8}, \frac{9}{10}$

Sol $\frac{3}{5} + \frac{7}{8} + \frac{9}{10} = \frac{24 + 35 + 36}{40} \begin{cases} \text{The L C M. of 5, 8, 10} \\ = 40. \end{cases}$

$$= \frac{95}{40} = 2\frac{15}{40} = 2\frac{3}{8} \text{ Ans}$$

Example 3   Add together $5\frac{3}{4}, 7\frac{5}{8}, 12\frac{5}{6}$

F. 8.

Note   In adding the mixed numbers, add all the whole numbers on one side and the fractions on the other, and then combine both the sums

Sol $5\frac{3}{4} + 7\frac{5}{8} + 12\frac{5}{6} = 5 + 7 + 12 + \frac{3}{4} + \frac{5}{8} + \frac{5}{6}$

$$= 24 + \frac{18 + 15 + 20}{24} = 24 + \frac{53}{24}$$

$$= 24 + 2\frac{5}{24} = 26\frac{5}{24} \quad \textbf{Ans.}$$

EXERCISE 33.

Find the sum of the following :—

1	$\frac{3}{8} + \frac{7}{8} + \frac{5}{8}$	**2.** $\frac{5}{12} + \frac{11}{12} + \frac{7}{12}$	**3.** $\frac{5}{6} + \frac{7}{8} + \frac{9}{12}$
4	$\frac{3}{7} + \frac{5}{14} + \frac{11}{21}$	**5** $\frac{8}{25} + \frac{9}{75} + \frac{7}{15}$	**6** $\frac{7}{12} + \frac{9}{16} + \frac{11}{24}$
7	$\frac{7}{15} + \frac{11}{25} + \frac{16}{45}$	**8** $\frac{9}{14} + \frac{16}{21} + \frac{17}{28}$	**9.** $\frac{5}{11} + \frac{7}{22} + \frac{14}{33}$
10	$\frac{2}{5} + \frac{7}{15} + \frac{13}{24}$	**11** $2\frac{1}{3} + 3\frac{1}{4} + 5\frac{1}{6}$	**12** $3\frac{1}{3} + 4\frac{1}{4} + 9\frac{2}{5}$
13	$5\frac{7}{8} + 9\frac{5}{9} + 10\frac{9}{10}.$	**14** $2\frac{4}{14} + \frac{9}{25} + 7\frac{3}{35}.$	
15	$\frac{1}{9} + \frac{1}{7} + \frac{6}{63}.$	**16** $\frac{5}{16} + \frac{7}{24} + \frac{13}{48}$	
17.	$5\frac{1}{2} + \frac{3}{3} + \frac{9}{10} + 18.$	**18** $\frac{1}{3} + \frac{11.6}{9} + 1\frac{1}{27} + 12$	
19	$6\frac{3}{4} + 7\frac{2}{3} + \frac{2.9}{9} + \frac{16}{24}$	**20** $7\frac{5}{13} + 9\frac{8}{16} + 8\frac{3}{32} + 15\frac{5}{48}.$	

SUBTRACTION OF FRACTIONS

§15   The process in subtraction is similar to the process in addition   The following examples will illustrate the method :—

(i) To subtract one fraction from another when they have the same denominator.

Example 1   Subtract $\frac{4}{15}$ from $\frac{7}{15}$.

Sol $\frac{7}{15} - \frac{4}{15} = \dfrac{7-4}{15}$ $\left.\begin{array}{l}\end{array}\right\}$ For seven-fifteenths—four-fifteenths=three fifteenths.

$= \frac{3}{15}$

$= \frac{1}{5}$ **Ans.**

Rule   *Find the difference of the numerators for a new numerator and write the denominator of the given fractions as its denominator.*

(ii)   When the given fractions have different denominators

Example 2   Subtract $\frac{15}{26}$ from $\frac{38}{39}$.

Sol $\frac{38}{39} - \frac{15}{26} = \frac{76}{78} - \frac{45}{78} = \frac{76 - 45}{78} = \frac{31}{78}$. **Ans**

Rule   *Reduce the fractions to the least common denominator and then proceed as in case (i)*

(*iii*) To subtract one mixed number from another mixed number.

Example 3.   Subtract $12\frac{3}{5}$ from $19\frac{5}{6}$.

Sol $19\frac{5}{6} - 12\frac{3}{5} = 19 - 12 + \frac{5}{6} - \frac{3}{5}$

$= 7 + \frac{25 - 18}{30}$

$= 7 + \frac{7}{30} = 7\frac{7}{30}$ **Ans**

Note   In mixed numbers, first subtract the whole numbers and then the fractions as shown above   But if the fraction of the subtrahend is greater than the fraction of the minuend, then take unity from the difference of the whole numbers and add to the minuend (This is done by adding the least common denominator to the new numerator of the minuend)   The following example will show   the method clearly —

Example 4   Subtract $18\frac{7}{8}$ from $29\frac{3}{7}$

Sol $29\frac{3}{7} - 18\frac{7}{8} = 29 - 18 + \frac{3}{7} - \frac{7}{8}$

$= 11 + \frac{3}{7} - \frac{7}{8} = 11 + \frac{24 - 49}{56}$

Now we cannot subtract 49 from 24   Take one from 11 and add to $\frac{3}{7}$, $1 + \frac{3}{7} = \frac{10}{7}$.   There will be 80 in place of 24 now *i. e.*, we will add the L C. D to the minuend.

Thus $11 + \frac{24 - 49}{56} = 10 + \frac{(24 + 56) - 49}{56}$

$= 10 + \frac{31}{56} = 10\frac{31}{56}$ **Ans**

(*iv*) To subtract a fraction from another when they have a common numerator.

Example 5   Subtract $\frac{7}{13}$ from $\frac{7}{8}$.

Sol $\frac{7}{8} - \frac{7}{13} = \frac{(13 - 8) \times 7}{8 \times 13} = \frac{5 \times 7}{104} = \frac{35}{104}$ **Ans**

Rule   *Multiply the difference of the denominators by the common numerator for the new numerator and put the product of the denominators for the new denominator.*

(v)   To subtract a mixed number from an integer.

Example 6   Subtract $2\frac{2}{3}$ from 10.

Sol $10 - 2\frac{2}{3} = 8 - \frac{2}{3} = 7 + 1 - \frac{2}{3}$

$= 7 + \frac{1}{3} = 7\frac{1}{3}$. Ans

EXERCISE 34

Find the value of —

1   $\frac{7}{8} - \frac{3}{8}$

2   $\frac{12}{25} - \frac{7}{25}$

3. $\frac{12}{55} - \frac{1}{7}$.

4   $\frac{15}{56} - \frac{7}{24}$

5   $\frac{7}{24} - \frac{17}{120}$.

6   $\frac{19}{55} - \frac{9}{87}$.

7. $16\frac{7}{8} - 12\frac{5}{6}$.

8   $14\frac{3}{4} - 12\frac{5}{8}$

9   $12\frac{7}{18} - 10\frac{5}{36}$.

10. $16\frac{7}{64} - 12\frac{7}{32}$

11. $45 - 36\frac{7}{8}$

12. $37 - 25\frac{7}{18}$

13   $21\frac{17}{18} - 11\frac{19}{24}$

14. $99\frac{98}{100} - 97\frac{24}{25}$

15   $\frac{5}{6} + \frac{7}{12} - \frac{23}{24}$

16   $\frac{8}{15} + 2\frac{3}{4} - 1\frac{7}{10}$

17. $15\frac{7}{8} + 12\frac{3}{4} - 11\frac{23}{24}$.

18   $\frac{15}{28} + \frac{20}{48} - \frac{7}{30}$

19   $18\frac{3}{4} - 12\frac{7}{8} - 4\frac{5}{24}$

20. $13\frac{3}{8} - 7\frac{10}{25} - 3\frac{7}{45}$

21. $\frac{15}{80} - \frac{15}{26}$

22.   Find the diff between $\frac{5}{12} + \frac{7}{16} + \frac{11}{24}$ and $\frac{3}{4} + \frac{7}{16} + \frac{7}{24}$.

23   What number added to $\left(\frac{5}{12} - \frac{5}{18}\right)$ will give 4 ?

MULTIPLICATION OF FRACTIONS

§16.   To multiply a fraction by a whole number.

Rule.   *Multiply the numerator of the fraction by the whole number*

Example 1   Multiply $\frac{7}{24}$ by 3

Sol $\frac{7}{24} \times 3 = \dfrac{7 \times 3}{24} = \frac{21}{24} = \frac{7}{8}$. Ans.

Example 2   Multiply $8\frac{5}{6}$ by 15.

Sol. $8\frac{5}{6} \times 15 = \left(8 + \frac{5}{6}\right) \times 15 = 8 \times 15 + \frac{5}{6} \times 15$

$= 120 + \frac{25}{2}$

$= 120 + 12\frac{1}{2} = 132\frac{1}{2}$. Ans

The process in the following example should be carefully noted :—

Example 3 Multiply $555\tfrac{554}{555}$ by 100.

Sol Since $555\tfrac{554}{555}=556-\tfrac{1}{555}$

$\therefore 555\tfrac{554}{555}\times100 =(556-\tfrac{1}{555})\times100$
$=55600-\tfrac{100}{555}$
$=55599+1-\tfrac{20}{111}$
$=55599-\tfrac{91}{111}$
$=55599\tfrac{91}{111}$ **Ans.**

§17 To multiply a fraction by a fraction

Example 4 Multiply $\tfrac{3}{4}$ by $\tfrac{3}{5}$

Exp The annexed figure represents a unity AB is $\tfrac{3}{4}$ of it, and AC is one-fifth of $\tfrac{3}{4}$

AD is three-fifth of $\tfrac{3}{4}$

But AD also represents $\tfrac{9}{20}$ of the unity.

$\tfrac{3}{4}\times\tfrac{3}{5}=\tfrac{9}{20}$
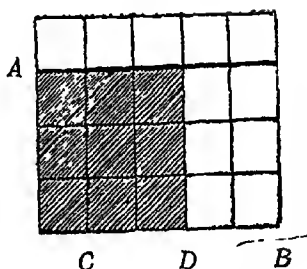
Rule *Multiply the numerators for a new numerator and denominators for a new denominator*

Example 5. Simplify $1\tfrac{1}{2}\times\tfrac{4}{9}\times3\tfrac{5}{6}\times2\tfrac{1}{4}$.

Sol. Product$=\tfrac{3}{2}\times\tfrac{4}{9}\times\tfrac{23}{6}\times\tfrac{9}{4}$
$=\dfrac{3\times2\times2\times23\times3\times3}{2\times3\times3\times3\times2\times2\times2}=\tfrac{23}{4}$
$=5\tfrac{3}{4}.$ **Ans.**

Or, we may cancel the common factors of the numerator and the denominator without splitting them into prime factors As,

$$\text{Product}=\dfrac{3\times4\times23\times9}{2\times9\times6\times4}=\tfrac{23}{4}=5\tfrac{3}{4}\quad\textbf{Ans.}$$

Note Before multiplying mixed numbers must be turned into improper fractions, and if the product is an improper fraction it must be turned into mixed number

EXERCISE 35.

Multiply '—

1	5⅓ by 9	2	7⅛ by 56	3	6⅔ by 28
4	9 15/18 by 24	5	17 5/21 by 35.	6	13 7/19 by 57.
7	111 1/111 by 10.	8	999 999/999 by 100.	9.	⅝ by 1 3/5.
10	3 6/11 by 4 2/7	11.	7 8/25 by 1 3/10.	12	16 5/16 by 2 24/27
13.	5 10/13 by 4 4/15	14.	37 81/87 by 2 7/15		

Simplify ,—

15　7½ × 2⅓ × 2⅐ × 3⅛　　　　16　2 11/21 × 5 8/29 × 11 5/7 × 5 3/7.

17.　5⅜ × 2⅐ × 3¼ × 2 1/13 × 1 3/51.　18　¾ × ⅝ × 9/10 × 1 7/9

Find the continued product of ,—

19　¾, 1 1/7, 5⅛, 1 2/13　　　　20.　5⅔, 4 2/7, 1 4/17, 7 1/15, 1⅙.

DIVISION OF FRACTIONS

§18　To divide a fraction by a whole number

Example 1.　Divide ¾ by 5

Graphical proof.　Let rectangle *ABCD* represent a unity　Then *EFCD* will represent ¾ ,

EMPD is one fifth of *EFCD*, *i e*, of ¾ which is three-twentieth of the whole figure.

$$\tfrac{3}{4} \div 5 = \tfrac{3}{20}$$
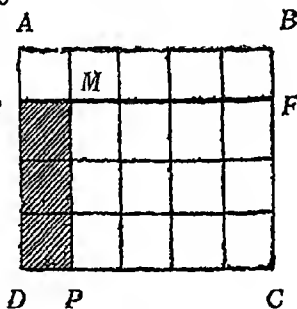
Rule. *Multiply the denominator of the fraction by the whole number leaving the numerator as it was , or in other words multiply the fraction by the reciprocal of the whole number.*

Def　When a whole number or a fraction is inverted *i.e*, the numerator is turned into denominator, and the denominator into numerator, the new fraction is called the **Reciprocal** of the original, thus the reciprocal of 5/9 is 9/5 , of

5 or $\frac{5}{1}$ is $\frac{1}{5}$ It is evident that the product of two recipro-
cal numbers is 1

Example 2   Divide $\frac{7}{8}$ by 14.

Sol $\frac{7}{8}-14=\dfrac{7\times1}{8\times1\cancel{4}}=\frac{1}{16}$ **Ans**
$\phantom{Sol \quad \frac{7}{8}-14=\frac{7\times1}{8\times1}}2$

Example 3. Divide $56\frac{7}{8}$ by 12

Sol $56\frac{7}{8}-12=\frac{455}{8}-12$
$\phantom{Sol \quad 56\frac{7}{8}-12}=\frac{455}{8}\times\frac{1}{12}=\frac{455}{96}=4\frac{71}{96}.$ **Ans.**

Aliter $56\frac{7}{8}-12=(48+8\frac{7}{8})-12$
$\phantom{Aliter \quad 56\frac{7}{8}-12}=4+\frac{71}{8}\times\frac{1}{12}=4+\frac{71}{96}=4\frac{71}{96}$ **Ans**

§19   To divide a fraction by a fraction

As division is the *reverse* of multiplication, hence the

Rule *Multiply the dividend by the reciprocal of the
divisor*

Example 4   Divide $\frac{7}{8}$ by $\frac{5}{12}$

Sol $\frac{7}{8}-\frac{5}{12}=\frac{7}{8}\times\frac{12}{5}$
$=\dfrac{7\times\cancel{4}\times3}{\cancel{4}\times2\times5}=\frac{21}{10}=2\frac{1}{10}.$ **Ans,**

EXERCISE 36

Divide —

1	$\frac{5}{12}$ by 10.	**2.** $\frac{15}{16}$ by 25	**3.**	$2\frac{1}{4}$ by 11.
4	$8\frac{8}{11}$ by 16	**5** $18\frac{1}{8}$ by 45.	**6.**	$14\frac{3}{7}$ by 37
7.	$7\frac{1}{5}$ by $2\frac{8}{9}$	**8** $30\frac{1}{4}$ by $6\frac{3}{5}$.	**9.**	$26\frac{1}{4}$ by $1\frac{7}{8}$
10.	$24\frac{5}{24}$ by $16\frac{5}{36}$	**11** $52\frac{7}{8}$ by $24\frac{15}{17}$	**12.**	$\frac{375}{8316}$ by $\frac{875}{9564}$.
13	$\frac{475}{7524}$ by $\frac{625}{4356}$	**14** $\frac{2505}{4968}$ by $\frac{3006}{2646}$	**15.**	$\frac{5744}{4500}$ by $\frac{1864}{2625}$
16	$\frac{6720}{8168}$ by $\frac{8640}{8744}$	**17** $\frac{1028}{3035}$ by $\frac{1495}{5465}$	**18**	$24\frac{287}{396}$ by $20\frac{1}{4}$.
19	Divide 130 by the sum of $7\frac{1}{2}$ and $8\frac{3}{4}$			

20. A man's annual income is Rs $2737\frac{1}{2}$, find his
income per day

21   The product of two numbers is $29\frac{1}{256}$, one of
them is $22\frac{11}{112}$, find the other

22   What number multiplied by $9-3\frac{4}{25}$ will give 1 as
product ?

———

§20. Compound fractions

A compound fraction as defined in Art 4, is a fraction of a fraction or several other fractions   Thus $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{5}{8}$ &c , &c , are compound fractions   '*Of*' indicates the sign of multiplication

For instance, $\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$ So also $\frac{2}{3}$ of $\frac{1}{2}=\frac{1}{3}$

Numbers connected by *of* are to be considered as a *single number*, i e , the word 'of' has the force of a pair of brackets   The student should, therefore, very carefully note the difference in meaning between × and *of*   Thus in $2\frac{1}{3}-1\frac{1}{2}\times\frac{3}{4}$, the sign — applies only to the next number $1\frac{1}{2}$, but in $2\frac{1}{3}-1\frac{1}{2}$ of $\frac{3}{4}$, the sign — applies to $(1\frac{1}{2}$ of $\frac{3}{4})$, as shown below —

(1) $2\frac{1}{3}-1\frac{1}{2}\times\frac{3}{4}$ $=\frac{7}{3}-\frac{3}{2}\times\frac{3}{4}$
   $=\frac{7}{3}\times\frac{2}{3}\times\frac{1}{4}=\frac{7}{6}=1\frac{1}{6}$.   **Ans**

(2) $2\frac{1}{3}-1\frac{1}{2}$ of $\frac{3}{4}$ $=\frac{7}{3}-(\frac{3}{2}\times\frac{3}{4})$
   $=\frac{7}{3}-\frac{9}{8}$
   $=\frac{7}{3}\times\frac{8}{9}=\frac{56}{27}=2\frac{2}{27}$ **Ans**

This shows that if the sign — is immediately followed by numbers connected together by the word '*of*' then the product of all such numbers following the sign — is to be considered as a single number

§21. Combined Operations.

In simplifying fractions involving various signs and brackets, we should proceed in the following order —

(*i*) all operations within brackets must be done first ,

(*ii*) then the quantities, connected together by '*of*' should be simplified ,

(*iii*) operations of division should then be done ,

(*iv*) next the operations of multiplication should be performed

(*v*) last of all the operations of addition and subtraction should be performed

In doing steps (*iii*) and (*iv*) shown above, we should remember that the sign — and × apply only to the numbers following them.

§22 Illustrations

1. $\frac{1}{7}$ of $3\frac{1}{2} + 2\frac{1}{2} = \frac{1}{7} \times \frac{7}{2} + \frac{5}{2} = \frac{1}{2} + \frac{5}{2} = 3$ **Ans**

2 $8 - 2\frac{1}{3}$ of $2\frac{1}{7} = 8 - \frac{7}{3} \times \frac{15}{7} = 8 - 5 = 3.$ **Ans.**

3 $2\frac{1}{2}$ of $\frac{4}{5} \times \frac{7}{8}$ of $2\frac{2}{3} = \frac{5}{2} \times \frac{4}{5} \times \frac{7}{8} \times \frac{8}{3} = \frac{14}{3} = 4\frac{2}{3}$ **Ans**

4 $\frac{5}{6}$ of $\frac{7}{10} - \frac{5}{8}$ of $\frac{7}{10} = (\frac{5}{6} \times \frac{7}{10}) - (\frac{5}{8} \times \frac{7}{10})$
$= \frac{7}{12} - \frac{7}{16} = \frac{7}{12} \times \frac{16}{7} = \frac{4}{3} = 1\frac{1}{3}$ **Ans.**

5 $\frac{5}{9} \times 3\frac{1}{3} - 2\frac{2}{5} = \frac{5}{9} \times \frac{10}{3} \times \frac{5}{12} = \frac{125}{162}$ **Ans**

6 $\frac{5}{9} - 3\frac{1}{3} \times 2\frac{2}{5} = \frac{5}{9} \times \frac{3}{10} \times \frac{12}{5} = \frac{2}{5}$ **Ans**

7. $\frac{5}{9} - 3\frac{1}{3}$ of $2\frac{2}{5} = \frac{5}{9} - (\frac{10}{3} \times \frac{12}{5})$
$= \frac{5}{9} - 8 = \frac{5}{9} \times \frac{1}{8} = \frac{5}{72}.$ **Ans.**

8. $\frac{5}{9} - 3\frac{1}{3} - 2\frac{2}{5} = \frac{5}{9} \times \frac{3}{10} \times \frac{5}{12} = \frac{5}{72}$ **Ans**

Note The result in 7 and 8 are the same Observe carefully the difference in solution of Examples 6 and 7 above

Example 1 Simplify —
$$13\frac{1}{3} - 6\frac{1}{2} \text{ of } \frac{10}{13} + 2\frac{2}{3} - \frac{2}{7} \text{ of } \frac{5}{6} - 1\frac{11}{14} \times 3\frac{1}{3}.$$

Sol. The exp $= \frac{40}{3} - (\frac{13}{2} \times \frac{10}{13}) + \frac{8}{3} - (\frac{2}{7} \times \frac{5}{6}) - \frac{25}{14} \times \frac{10}{3}$
$$= \frac{40}{3} - 5 + \frac{8}{3} - \frac{5}{25} - \frac{25}{14} \times \frac{10}{3}$$
$$= \frac{40}{9} \times \frac{1}{5} + \frac{8}{3} - \frac{5}{25} \times \frac{14}{25} \times \frac{10}{3}$$
$$= \frac{8}{9} + \frac{8}{9} - \frac{1}{3}$$
$$= \frac{15}{9} = 5 \quad \text{**Ans**}$$

Example 2. Simplify —
$$8\frac{5}{6} - \{\frac{1}{3} + (1 + \frac{1}{12})\} - 3(1 - \frac{3}{8} \text{ of } 2\frac{1}{2})$$

Sol The exp $= 8\frac{5}{6} - \{\frac{1}{3} + \frac{13}{12}\} - 3(1 - \frac{15}{16})$
$$= 8\frac{5}{6} - \left\{\frac{4 + 13}{12}\right\} - 3(1 - \frac{15}{16})$$
$$= 8\frac{5}{6} - \frac{17}{12} - \frac{3}{16}$$
$$= 8\frac{5}{6} - \frac{17}{12} \times \frac{16}{3}$$
$$= 8\frac{5}{6} - 7\frac{5}{9} = 1\frac{5}{18}. \quad \text{**Ans**}$$

§23 Removal of brackets.

A pair of brackets may be removed if it is preceded by the sign " + " without changing the signs of the terms inside them , thus $8 + (2 + 3 - 2) = 8 + 2 + 3 - 2 = 11$ **Ans.**

But if the pair is preceeded by "$-$," the sign of every term within the brackets is changed i e, $+$ to $-$ and $-$ to $+$, thus $8-(2+3-2)=8-2-3+2=5$. **Ans.**

In a long expression in which different kinds of brackets are used, less comprehensive brackets are removed first and then more comprehensive ones Thus in the following expression $9-\{8-[4+4-(3-2)]\}$, we remove first the least comprehensive or the innermost brackets. After removing the innermost brackets the expression stands thus .$-$

$9-\{8-[2+4-3+2]\}$.

Now remove again the innermost brackets as

$9-\{8-2-4+3-2\}$.

Now we have only a pair of brackets Remove this pair also and the expression $=9-8+2+4-3+2=6$ **Ans.**

Example 3 Remove the brackets and then simplify the expression $2+\{3-[5-(2+4)]\}$.

Sol The expression $=2+\{3-[5-2-4]\}$
$=2+\{3-5+2+4\}$
$=2+3-5+2+4=6$. **Ans.**

Example 4. Remove the brackets and then simplify the expression $1\frac{1}{2}+\{2\frac{1}{3}-[\frac{3}{4}+(\frac{5}{6}-\frac{3}{8}+\frac{1}{6})]\}$.

Sol. The exp. $=1\frac{1}{2}+\{2\frac{1}{3}-[\frac{3}{4}+(\frac{5}{6}-\frac{3}{8}-\frac{1}{6})]\}$
$=1\frac{1}{2}+\{2\frac{1}{3}-[\frac{3}{4}+\frac{5}{6}-\frac{3}{8}-\frac{1}{6})]\}$
$=1\frac{1}{2}+\{2\frac{1}{3}-\frac{3}{4}-\frac{5}{6}+\frac{3}{8}+\frac{1}{6}\}$
$=1\frac{1}{2}+2\frac{1}{3}-\frac{3}{4}-\frac{5}{6}+\frac{3}{8}+\frac{1}{6}$
$=3+\frac{1}{2}+\frac{1}{3}-\frac{3}{4}-\frac{5}{6}+\frac{3}{8}+\frac{1}{6}$
$=3+\dfrac{12+8-18-20+9+3}{24}$
$=3+\dfrac{32-38}{24}=2+\dfrac{32+24-38}{24}$
$=2\frac{18}{24}=2\frac{3}{4}$. **Ans.**

EXERCISE 37.

Simplify —

1   $\frac{1}{6}$ of $2\frac{1}{2} + 4\frac{3}{5}$.   2 $19 - 5\frac{1}{4}$ of $2\frac{2}{7}$.

3   $5\frac{1}{3}$ of $2\frac{1}{4} \times \frac{3}{8}$ of $1\frac{1}{16}$.   4 $\frac{5}{8}$ of $\frac{9}{10} - 1\frac{1}{2}$ of $\frac{5}{8}$

5   $\frac{7}{8} \times 2\frac{1}{4} - \frac{5}{6}$ of $\frac{9}{10}$.   6 $1\frac{2}{3} - 2\frac{2}{9} \times 5\frac{1}{3}$ of $4\frac{1}{2}$

7. $\frac{5}{6} - 2\frac{1}{2}$ of $2\frac{1}{2} \times 1\frac{3}{4}$ 8 $2\frac{1}{4} - 5\frac{5}{8} - 3\frac{1}{7} \times 1\frac{2}{8}$.

9   $11\frac{1}{2} - 5\frac{1}{6} \times \frac{2}{3} - \frac{3}{9} + 5\frac{1}{3}$ 10 $9\frac{1}{4} + \frac{7}{8} - \frac{1}{2}$ of $\frac{7}{8} - 5\frac{1}{2} + 1\frac{5}{6}$.

11. $2\frac{1}{3} \times 1\frac{1}{2} - 1\frac{1}{3}$ of $2\frac{4}{5}$.   12. $3\frac{7}{10} - 2\frac{1}{3} \times \frac{1}{2} + 2\frac{4}{5}$ of $\frac{9}{14}$.

13. $2\frac{1}{4}$ of $1\frac{1}{2} - 1\frac{1}{8} \times 2\frac{2}{8}$.   14 $2\frac{1}{4}$ of $1\frac{1}{2} - 1\frac{1}{8}$ of $2\frac{2}{5} + \frac{5}{2}$

15   $(\frac{1}{3} + \frac{2}{11} + \frac{5}{14} - \frac{1}{3} \times \frac{9}{11}$ of $\frac{7}{24}) \div (1 - \frac{1}{8}$ of $\frac{9}{11} - \frac{2}{11}$
                                                    of $\frac{7}{24} - \frac{7}{24}$ of $\frac{1}{3}$).

16. $(\frac{5}{8} + \frac{1}{6}\frac{1}{1} + 1\frac{7}{8} + 1\frac{11}{12} - 5) - (\frac{3}{2} + \frac{4}{3})$.

17. $\{1\frac{3}{7}$ of $26\frac{1}{3}$ of $(1 - \frac{2}{8})\} \times \{2\frac{5}{8}$ of $(4\frac{1}{5} - 3\frac{2}{8})$ of $\frac{15}{108}\}$

18   $(1\frac{2}{8}$ of $2\frac{7}{9} - 3\frac{1}{16}\frac{7}{0}) \times (5\frac{1}{3}$ of $4\frac{1}{2} - 3\frac{1}{4}$ of $3\frac{1}{5}) \times \frac{2}{8}\frac{7}{7}$ of $\frac{9}{17} \times \frac{8}{5}$.

19   $\frac{8247}{1017} - \frac{903}{1107} \times \frac{774}{613} - \frac{3928}{666}$.

20   $(2\frac{1}{3} - 1\frac{3}{4}) \div (4$ of $1\frac{1}{8} + 6 \times \frac{1}{2}) \times (5 - \frac{5}{6}) \div (\frac{1}{2} + \frac{2}{6}) + \frac{5}{6}$.

21   $(2 + \frac{1}{3} + \frac{1}{6} + \frac{1}{13} + \frac{1}{16} + \frac{1}{18}) \div (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6})$.

Remove the brackets and then simplify :—

22. $1\frac{1}{3} + \{2\frac{1}{3} - [\frac{7}{9} - (\frac{2}{3} - 1\frac{1}{2})]\}$.

23. $7 - \{\frac{3}{4} + [2\frac{1}{2} - (1\frac{1}{2} - \frac{1}{3})]\}$

24   $8 - \{2\frac{1}{3} + [3\frac{1}{2} - (5\frac{1}{6} - \overline{\frac{1}{8} + 1\frac{1}{2}})]\}$.

25   $7\frac{1}{6} + \{\frac{5}{12} - [\frac{3}{8} + (16\frac{1}{2} - \overline{4\frac{1}{3} + 5\frac{5}{6}})]\}$.

§24. Complex fractions

Complex fraction as defined in Art. 4 is one in which the numerator or the denominator or both are fractions

Thus $\frac{\frac{5}{6}}{8}$, $\frac{8}{\frac{5}{6}}$, $\frac{\frac{3}{5}}{\frac{5}{6}}$, $\frac{5\frac{1}{3}}{8\frac{1}{2}}$, $\frac{\frac{1}{6} + \frac{3}{4}}{\frac{5}{6} + \frac{7}{8}}$ are all complex fractions

The numerator is separated from the denominator by a thick line

Note   A complex fraction is read by using the word *by for divided by* between the readings of numerator and denominator

Thus, $\frac{\frac{3}{5}}{\frac{5}{6}}$ is read '$\frac{3}{5}$ by $\frac{5}{6}$'.

§25   In the sum of a whole number and a fraction, when the fraction is either complex or simple, the sign of addition is sometimes omitted, as in $7\dfrac{\frac{2}{3}}{4}$ which really means $7 + \dfrac{\frac{2}{3}}{4}$, and in a product when one of the factors is put within brackets, the sign of multiplication is often omitted as in $\frac{3}{4}(\frac{2}{3}-\frac{1}{7})$, which really means $\frac{3}{4} \times (\frac{2}{3}-\frac{1}{7})$

§26   Complex fractions are simplified by the application of the same rules as simple fractions and are reduced by regarding them as the quotient of the numerator divided by the denominator

Example 1   Simplify the complex fractions given in §24

Sol   1. $\dfrac{\frac{5}{6}}{8} = \frac{5}{6} \div 8 = \frac{5}{6} \times \frac{1}{8} = \frac{5}{48}$ Ans

2   $\dfrac{8}{\frac{5}{6}} = 8 \div \frac{5}{6} = 8 \times \frac{6}{5} = \frac{48}{5} = 9\frac{3}{5}$ Ans

Note the difference between the two solutions.

3. $\dfrac{\frac{3}{2}}{\frac{5}{6}} = \frac{3}{2} \div \frac{5}{6} = \frac{3}{2} \times \frac{6}{5} = \frac{9}{5} = 1\frac{4}{5}$ Ans

4   $\dfrac{5\frac{1}{2}}{8\frac{1}{2}} = 5\frac{1}{2} \div 8\frac{1}{2} = \frac{11}{2} \times \frac{2}{17} = 11\frac{1}{17}$ Ans.

5   $\dfrac{\frac{1}{4}+\frac{3}{4}}{\frac{5}{6}+\frac{7}{8}} = \dfrac{\frac{13}{12}}{\frac{41}{24}} = \frac{13}{12} - \frac{41}{24} = \frac{13}{12} \times \frac{24}{41} = \frac{26}{41}.$ Ans.

Or thus —Multiply the numerator and denominator by the L C M of all the denominators 3, 4, 6, 8, i e., 24.

$$\dfrac{\frac{1}{4}+\frac{3}{4}}{\frac{5}{6}+\frac{7}{8}} = \frac{8+18}{20+21} = \frac{26}{41}$$ Ans.

This step is, in many cases, very short and convenient.

Example 2 Simplify $\dfrac{\frac{1}{7}+\frac{5}{4} \text{ of } 2\frac{2}{3}}{\frac{5}{12}-\frac{2}{7}} \times \dfrac{5\frac{1}{6}-4\frac{5}{7}-\frac{1}{6}}{14\frac{4}{11}+5\frac{3}{4}-2\frac{7}{8}} - 1$

Sol.   The exp $= \dfrac{\frac{1}{7}+\frac{3}{4}\times\frac{8}{3}}{\frac{5}{12}-\frac{2}{7}} \times \dfrac{\frac{31}{6}\times\frac{7}{81}-\frac{1}{6}}{\frac{168}{11}+\frac{23}{4}\times\frac{8}{23}} - 1.$

$$= \frac{\frac{1}{7}+2}{\frac{5}{12}-\frac{2}{7}} \times \frac{\frac{7}{6}-\frac{1}{5}}{\frac{158}{11}+2}-1$$

$$= \frac{\frac{15}{7}}{\frac{11}{84}} \times \frac{1}{\frac{180}{11}}-1.$$

$$= \frac{15}{7} \times \frac{84}{11} \times 1 \times \frac{11}{180}-1$$

$$= 1-1=0 \quad \text{Ans}$$

EXERCISE 38

Simplify —

1. $\dfrac{2\frac{1}{5}}{4\frac{4}{5}}$ 2. $\dfrac{3\frac{1}{5}}{4\frac{2}{5}}$. 3. $\dfrac{5\frac{1}{5}}{2\frac{1}{5}}$ 4. $\dfrac{5\frac{1}{8}}{3\frac{4}{9}}$,

5. $\dfrac{5\frac{2}{5}}{3\frac{1}{15}+7\frac{11}{15}}$ 6. $\dfrac{3\frac{5}{8}}{2\frac{4}{15}+2\frac{1}{3}}$ 7. $\dfrac{4\frac{1}{6}-2\frac{4}{8}}{1\frac{2}{5}}$.

8. $\dfrac{5\frac{4}{7}-3\frac{6}{7}}{3\frac{1}{7}}$. 9. $\dfrac{\frac{7}{10}-\frac{2}{5}}{\frac{9}{13}+\frac{1}{5}} = \dfrac{\frac{1}{3}}{9\frac{1}{2}}$

10. $\dfrac{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}{\frac{1}{2} \text{ of } \frac{1}{3} \text{ of } \frac{1}{6}}$ 11. $\dfrac{15\frac{3}{8}+6-\frac{3}{4}}{7\frac{4}{9} \times \frac{3}{12}}$

12. $8-8 \times \dfrac{2\frac{1}{5}-1\frac{2}{7}}{2-\frac{6}{35}}$ 13. $\dfrac{1\frac{1}{2} \text{ of } 2\frac{1}{4} \text{ of } 3\frac{1}{5} \text{ of } 4\frac{1}{6}}{3\frac{2}{8} \text{ of } 2\frac{5}{12}}$

14. $\dfrac{1+\frac{1}{2}+\frac{1}{3}-\frac{1}{4}-\frac{1}{12}}{\frac{1}{3}+\frac{1}{9}} - \dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2} \times \frac{1}{9}}$

15. $\dfrac{8\frac{1}{7}-1\frac{1}{6}}{\frac{2}{3}+1\frac{1}{2}} - \dfrac{1}{5\frac{1}{4}-1\frac{1}{5}} + \dfrac{\frac{4}{5}}{\frac{5}{6}-\frac{1}{5}}$

16. $(\frac{1}{8}+\frac{\frac{3}{7}}{1\frac{3}{5}}+\frac{1}{14}+\frac{5}{9}-1)-\frac{4}{9}$ of $\frac{5}{6}$ of $2\frac{5}{7}$

17. $(4\frac{2}{3}-1\frac{1}{8}) \times (3\frac{1}{2}-\frac{3}{5})-(13\frac{1}{8}+7\frac{1}{2})$ of $\dfrac{3\frac{1}{8}}{1\frac{1}{9}}$

18. $\dfrac{2\frac{1}{2}-1\frac{3}{4}}{4 \text{ of } 1\frac{1}{8}+6 \times \frac{1}{2}} \times \dfrac{5-\frac{5}{9}}{\frac{1}{2}+\frac{2}{5}}+\frac{5}{9}$

19. $\dfrac{\frac{1}{4}+\frac{2}{9}}{4-\frac{5}{9} \text{ of } 5\frac{1}{2}} - \dfrac{\frac{2}{5}+\frac{1}{4}}{\frac{7}{10} \text{ of } 4\frac{3}{5}-2\frac{1}{4}}$.

20. $\dfrac{1\frac{1}{4}-\frac{5}{14}}{1\frac{1}{4}+\frac{5}{14}}+\frac{7}{6}$ of $\dfrac{9 \times 5}{14 \times 3}-\dfrac{14\frac{1}{2}}{15}$

21. $\dfrac{1+2\frac{1}{5}+3\frac{1}{8}}{\frac{2}{5}+\frac{6}{7}+\frac{12}{13}} \times \dfrac{55\frac{7}{9}-11}{1\frac{9}{11} \text{ of } 13\frac{1}{5}}$

22. $\dfrac{7\frac{1}{3}}{6\frac{1}{2}} + \dfrac{11\frac{1}{2} - 2\frac{2}{5}}{11\frac{1}{2} + 2\frac{2}{5}} \times 10\frac{9}{16} - 6\frac{45}{272}$.

23. (a) $\dfrac{17}{7 + 2\frac{2}{5}} \times \frac{9021}{21105} \div \left(1\frac{87}{46} - \frac{15}{16}\right)$

     (b) $\dfrac{\frac{3}{7} - \frac{2}{9}}{\frac{5}{7} + \frac{2}{9}}$ of $2\frac{11}{28} \div \dfrac{4}{13 - 3\frac{8}{9}} + 3\frac{11}{16} \times \dfrac{3}{3 - 2\frac{10}{15}}$

24. $\dfrac{2\frac{1}{4}}{2\frac{2}{8}} + \dfrac{2\frac{1}{6} + 5\frac{1}{8}}{3\frac{1}{9} + 9\frac{1}{2}} + \frac{8}{9}$ of $\frac{9}{10} + \frac{3}{8}$ of $\frac{8}{20}$.

25. $\dfrac{\frac{5}{14} - \frac{3}{7} \text{ of } \frac{1}{2}}{\frac{5}{16} + \frac{7}{13} \text{ of } 3\frac{1}{4} - \left(\frac{7}{8} \text{ of } \frac{37}{21} - \frac{1}{3}\right)} - \dfrac{\frac{1}{2} \text{ of } \frac{1}{8} + \frac{3}{8} \text{ of } 5}{9\frac{1}{3} - 1\frac{2}{5}}$

26. $\dfrac{(4\frac{2}{9} - 1\frac{1}{9}) \times (3\frac{1}{2} - \frac{3}{5})}{13\frac{1}{3} + 7\frac{1}{2}}$ of $\dfrac{3\frac{1}{4}}{1\frac{1}{5}}$.

27. $5 - 5 \times \dfrac{2 + 1\frac{1}{5}(2 + 1\frac{1}{5})}{1\frac{1}{5} + 2(2 + 1\frac{1}{5})} - \frac{3}{16}$

28. $\dfrac{3}{9} \div \left\{ \dfrac{\frac{28}{81} \text{ of } \dfrac{1}{9\frac{1}{3}}}{\frac{4}{5} - \frac{3}{4}} \right\} - \frac{4}{9}$ of $\left\{ \frac{1}{2} - \frac{3}{5} \text{ of } \dfrac{1}{3 - 1\frac{2}{7}} \right\}$.

29. $\dfrac{7}{5 - \frac{8}{3}} - \dfrac{3 - \frac{4}{9}}{4 - \frac{3}{2}} - \frac{5}{7}$ of $\left\{ \dfrac{1}{1\frac{3}{7}} + \frac{9}{5} \text{ of } \dfrac{3\frac{1}{9} - 2\frac{1}{2}}{4\frac{7}{21} - 2} \right\}$.

30. $\dfrac{5\frac{5}{6} + 4\frac{3}{6}}{8\frac{3}{5} - 5\frac{3}{4}} - \left\{ \dfrac{\frac{7}{9} - \frac{6}{7} + \frac{5}{6}}{\frac{5}{9} - \frac{7}{6} + \frac{6}{5}} \text{ of } (1\frac{3}{11} \times 3\frac{4}{5}) \right\}$.

31. $\left(\dfrac{\frac{4}{11} \text{ of } 1\frac{4}{7} - \frac{5}{6} + \frac{6}{7}}{\frac{3}{7} \text{ of } 1\frac{6}{9} + 1\frac{3}{4}} \right) \times 4\frac{3}{10} - \frac{5}{6}$ of $\frac{1}{5}$.

32. $\dfrac{\frac{3}{9}(1\frac{3}{5} - \frac{3}{6} \text{ of } 1\frac{4}{9}) + \frac{4}{17}}{\frac{3}{4} \times 1\frac{4}{7} - 1\frac{1}{2} - \frac{11}{28}} \times \dfrac{\frac{3}{4} + \frac{2}{3}}{\frac{3}{4} - \frac{2}{3}} - 20$

33. $\dfrac{6\frac{3}{5} - 4\frac{1}{10}}{5\frac{1}{2} - 4\frac{3}{5}} - \dfrac{2\frac{9}{15} - 1\frac{11}{36} + 1\frac{1}{4} - \frac{9}{14} \text{ of } 3\frac{1}{4}}{\frac{3}{2} \times 3\frac{4}{7} \times 5\frac{5}{6} \div 3\frac{4}{15}} \times 33\frac{3}{7}$.

34. $\dfrac{(\frac{1}{3} + \frac{1}{7}) - (\frac{1}{7} - \frac{1}{9})}{(\frac{1}{3} + \frac{1}{7}) - \frac{1}{7} - \frac{1}{9}} - \dfrac{1\frac{7}{8} + \frac{3}{28}}{1\frac{7}{12} - \frac{1}{7}} - \dfrac{\frac{1}{2} \text{ of } \frac{1}{4}}{\frac{1}{3} - \frac{1}{8}}$

35. $\dfrac{\frac{3}{5} - \frac{1}{6} \text{ of } \frac{7}{8} + \frac{8}{9}}{\frac{4}{9} - \frac{3}{7} - \frac{1}{5} + \frac{5}{8}} - \dfrac{\frac{5}{11} - \frac{4}{6}}{1 - \frac{5}{6} - \frac{3}{8}}$.

36. $\dfrac{\frac{5}{7} \text{ of } 1\frac{3}{5} - \frac{5}{8} \text{ of } \frac{8}{7}}{1 - \frac{1}{7} \times (\frac{5}{13} + \frac{1}{5})} \times \dfrac{\frac{1}{4} + \frac{1}{7} - (\frac{1}{7} - \frac{1}{9})}{(\frac{1}{5} + \frac{1}{7}) - \frac{1}{7} - \frac{1}{6}}$.

37. $\dfrac{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}}{\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}}\times 3\frac{1}{2} \ \dot{-}\ \dfrac{\frac{1}{15}+\frac{1}{20}+\frac{1}{24}}{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}-\frac{1}{6}}.$

38. $2\frac{5}{7}$ of $\dfrac{13\frac{1}{2}-9\frac{2}{3}}{15\frac{1}{2}-11\frac{7}{30}}\ -3\frac{2}{7}+\dfrac{1\frac{7}{9}}{9\frac{2}{9}-8\frac{4}{9}}.$

39. $\dfrac{\frac{5}{11}\left(\frac{2}{15}\text{ of }2\frac{5}{7}+\frac{1}{3}\text{ of }1\frac{2}{7}\right)}{\frac{2}{15}\times 1\frac{5}{8}\times 1\frac{1}{7}-\frac{1}{11}}-\dfrac{\frac{1}{3}}{\frac{1}{6}\text{ of }3-\frac{1}{17}\text{ of }5\frac{2}{8}}.$

40. $\left\{3+\dfrac{1\frac{5}{8}+\frac{11}{12}}{2\frac{11}{15}-1\frac{9}{20}}\text{ of }1\frac{170}{84}\right\}-\left\{\left(3\frac{1}{2}\times 5\frac{4}{7}\right)-\left(\frac{16}{5}+3\frac{3}{4}\right)+\frac{1}{2}\right\}$

§27.   Continued fractions

The fraction of the form of $\cfrac{1}{1+\cfrac{1}{1+\frac{2}{3}}}$ is called a

continued fraction The method of solving such fractions will be best illustrated by the following solved example .—

Example 1.   Simplify the continued fraction

$$\cfrac{1}{4+\cfrac{1}{2-\cfrac{1}{1-\frac{5}{12}}}}$$

Sol   The fraction $=\cfrac{1}{4+\cfrac{1}{2-\cfrac{1}{\frac{7}{12}}}}$

$$=\cfrac{1}{4+\cfrac{1}{2-1\frac{5}{7}}}$$

$$=\cfrac{1}{4+\cfrac{1}{\frac{2}{7}}}$$

$$=\cfrac{1}{4+\frac{7}{2}}$$

$$=\cfrac{1}{\frac{15}{2}}=\frac{2}{15}.\quad\text{Ans.}$$

Rule.   *Begin at the bottom and work upwards*

§28. To convert a given fraction to a continued fraction.

The following example shows how we convert a fraction to a continued fraction with unit numerators and with all signs positive

Example 2. Convert $\frac{13}{59}$ to a continued fraction

Sol The fraction $= \dfrac{1}{\frac{59}{13}}$

$$= \dfrac{1}{4 + \frac{7}{13}}$$

$$= \dfrac{1}{4 + \dfrac{1}{\frac{13}{7}}}$$

$$= \dfrac{1}{4 + \dfrac{1}{1 + \frac{6}{7}}}$$

$$= \dfrac{1}{4 + \dfrac{1}{1 + \dfrac{1}{\frac{7}{6}}}}$$

$$= \dfrac{1}{4 + \dfrac{1}{1 + \dfrac{1}{1 + \frac{1}{6}}}}$$

which is often written in the form

$$\frac{1}{4+} \quad \frac{1}{1+} \quad \frac{1}{1+} \quad \frac{1}{6}.$$

EXERCISE 39

Simplify (1 to 12) —

1. $2 + \dfrac{1}{5 + \dfrac{1}{1 + \frac{1}{3}}}$

2. $\dfrac{1}{10 + \dfrac{1}{2 + \frac{1}{50}}}$

3. $\cfrac{1}{4-\cfrac{1}{2-\cfrac{1}{1-\frac{5}{13}}}}$

4. $\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{5}}}}$

5 $1+\cfrac{2}{3+\cfrac{1}{4+\cfrac{1}{5\frac{2}{3}}}}$

6. $9+\cfrac{1}{1+\cfrac{1}{7+\frac{1}{6}}}$

7 $\cfrac{2}{2+\cfrac{2}{2-\cfrac{2}{2-\frac{2}{3}}}}$

8. $1-\cfrac{3}{7-\cfrac{3}{6-\cfrac{3}{5-\frac{3}{4}}}}$

9. $\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\frac{1}{2}}}}}$

10 $9+\cfrac{1}{1-\cfrac{3}{2+\cfrac{4}{2+\frac{5}{6}}}}$

11 $\cfrac{5}{4+\cfrac{3}{3+\cfrac{2}{2+\cfrac{1}{1+\frac{1}{6}}}}}$

12 $\cfrac{1}{2+\cfrac{3}{4+\cfrac{1}{5-\cfrac{1}{7-\frac{2}{3}}}}}$

13 Divide

$$2\tfrac{1}{3}+8\tfrac{6}{11}-\tfrac{1}{5}\ \text{of}\ (7\tfrac{1}{2}-3\tfrac{1}{3})\ \text{by}\ 11+\cfrac{1}{1-\cfrac{1}{1+\cfrac{1}{8\frac{1}{11}}}}$$

Simplify —

14 $\dfrac{10\frac{3}{4}-(5\frac{2}{5}+4\frac{9}{20})}{10\frac{1}{3}-(2\frac{4}{5}-\frac{4}{15})-7}-\cfrac{1}{6+\cfrac{1}{1+\frac{7}{8}}}+\cfrac{1}{4-\cfrac{1}{2-\cfrac{1}{1-\frac{5}{15}}}}$

F. 9

$$6 + \cfrac{1}{6 + \cfrac{1}{6 - \frac{1}{6}}}$$

15 $\dfrac{6\frac{41}{68}}{6-4\frac{4}{11}} + \frac{7}{9} \times 1\frac{22}{27}$ of $\frac{9}{56} - \dfrac{}{6}$

16   (i) $4 + \dfrac{1}{1 + } \dfrac{1}{1 + } \frac{1}{3}.$ (ii) $2 + \dfrac{1}{1 + } \dfrac{1}{1 + } \dfrac{1}{5 + } \frac{1}{6}$

17   Convert $\frac{40}{73}$ to a continued fraction.

§29.   Application of fractions to compound quantities

Example 1   Add together Rs 15. 8a 7$\frac{1}{5}$p , Rs 18 9a 5$\frac{1}{8}$p , Rs 2 3a 6$\frac{7}{9}$p

	Rs	a	p
Sol.	15	8	7$\frac{1}{5}$
	18	9	5$\frac{1}{8}$
	2	3	6$\frac{7}{9}$
	36	5	7$\frac{14}{45}$ Ans

$\frac{1}{5} + \frac{1}{8} + \frac{7}{9} = \frac{59}{45} = 1\frac{14}{45}.$
Put down $\frac{14}{45}p$ and carry on
1p Now proceed further.

Example 2   Subtract £17 8s 5$\frac{3}{4}$d. from £19 7s 3$\frac{5}{8}$d

	£	s	d
Sol.	19	7	3$\frac{5}{8}$
	17	8	5$\frac{3}{4}$
	1	18	9$\frac{7}{8}$

Here $\frac{5}{8}$ is less than $\frac{3}{4}$, therefore add 1 to $\frac{5}{8}$ making it $1\frac{5}{8}$
Now $\frac{13}{8} - \frac{3}{4} = \frac{7}{8}$ Put down $\frac{7}{8}d.$
Add 1d to 5d and proceed further.

Example 3   Multiply Rs 5 7a 6$\frac{5}{8}$p by 29.

	Rs.	a	p.
Sol.	5	7	6$\frac{5}{8}$
		7	
	38	4	11$\frac{5}{8}$
		4	
	153	3	11$\frac{1}{4}$
	5	7	6$\frac{5}{8}$
	158	11	6$\frac{1}{8}$

$29 = 7 \times 4 + 1$
$\frac{5}{8} \times 7 = \frac{35}{8} = 5\frac{5}{8}$, put down $\frac{5}{8}$ and carry on 5.   Now proceed further
$\frac{5}{8} \times 4 = \frac{10}{3} = 3\frac{1}{3}$, put down $\frac{1}{3}$ and carry on 3 and proceed further.
$\frac{3}{8} + \frac{5}{8} = \frac{7}{8} = 1\frac{1}{8}$, put down $\frac{1}{8}$, carry 1 and proceed further

Example 4   Find the value of $\frac{7}{8}$ of Rs 3, 5a. 4p

Sol.	Rs	a	p.
	3	5	4
			7
8)	23	5	4
	2	14	8 Ans

Or

	Rs	a.	p
8)	3	5	4
	0	6	8
			7
	2	14	8 Ans

Example 5   Simplify Rs 7. 8a. 9p $\times 5\frac{3}{4}$

Sol

$$
\begin{array}{ccc}
\text{Rs} & a & p. \\
7 & 8 & 9 \times 5\frac{3}{4} = 7 \ \ 8 \ \ 9 \\
& & 5
\end{array}
$$

$$
\begin{array}{rcl}
\text{Rs} & a. & p. \\
7 & 8 & 9 \\
& & 3 \\
\hline
4)22 & 10 & 3 \\
\hline
5 & 10 & 6\frac{3}{4} = \frac{1}{4} \text{ of Rs } 7 \ \ 8a. \ 9p.
\end{array}
$$

$$
\left.
\begin{array}{rcl}
37 & 11 & 9 \\
5 & 10 & 6\frac{3}{4}
\end{array}
\right\} \text{Add.}
$$

$$
\begin{array}{ccc}
43 & 6 & 3\frac{3}{4}. \text{ Ans}
\end{array}
$$

Example 6   Divide Rs 13 10a 8p. by $2\frac{1}{3}$ of $3\frac{3}{5}$.

Sol $2\frac{1}{3}$ of $3\frac{3}{5} = \frac{7}{3} \times \frac{18}{5} = \frac{42}{5}$.

Rs. 13 10a 8p.$-\frac{42}{5} = $Rs. 13. 10$a$. 8$p$. $\times \frac{5}{42}$.

$$
\begin{array}{rcl}
\text{Rs.} & a. & p. \\
= 13 & 10 & 8 \\
& & 5 \\
\hline
6)68 & 5 & 4 \\
\hline
7)11 & 6 & 2\frac{5}{3} \\
\hline
1 & 10 & \frac{8}{-1}. \text{ Ans.}
\end{array}
$$

$42 = 6 \times 7$

Example 7   Divide £56. 7s. 5$\frac{5}{8}$d. by 24.

Sol

$$
24 \left\{
\begin{array}{r|ccc}
4 & \pounds 56 & 7 & 5\frac{5}{8} \\
6 & 14 & 1 & 10\frac{13}{32} \\
\hline
& 2 & 6 & 11\frac{141}{192} \text{ Ans}
\end{array}
\right.
$$

Rem $= 1\frac{5}{8} = \frac{13}{8}$

$\frac{13}{8} - 4 = \frac{13}{8} \times \frac{1}{4} = \frac{13}{32}$

Rem. $= 4\frac{13}{32} = \frac{141}{32}$.

$\frac{141}{32} - 6 = \frac{141}{192}.$

EXERCISE 40.

Add together :—

£.	s.	d.
1. 15	7	3$\frac{5}{6}$
16	3	4$\frac{3}{4}$
18	2	3$\frac{7}{9}$

£.	s.	d.
2 21	9	3$\frac{7}{5}$
15	4	6$\frac{5}{4}$
15	3	3$\frac{1}{13}$

Rs	a	p
3. 8	7	4$\frac{1}{2}$
12	13	3$\frac{5}{16}$
16	4	7$\frac{3}{32}$

mds.	sr.	ch
4. 21	15	9$\frac{1}{13}$
56	18	3$\frac{2}{9}$
18	25	7$\frac{5}{18}$
16	21	6$\frac{5}{24}$

ton	cwt.	qr
5 78	16	3$\frac{7}{8}$
15	12	2$\frac{1}{13}$
16	13	1$\frac{4}{5}$
12	7	1$\frac{5}{18}$

mi.	fur.	po.
6. 17	2	35$\frac{1}{9}$
18	3	16$\frac{2}{3}$
16	4	15$\frac{5}{18}$
23	3	18$\frac{14}{27}$

Perform the following subtractions —

	£.	s.	d		Rs	a	p.		yd.	ft	in
7.	48	5	$7\frac{1}{4}$	8.	21	7	$4\frac{1}{3}$	9.	36	3	$7\frac{2}{9}$
	19	6	$8\frac{5}{8}$		15	9	$8\frac{7}{18}$		18	1	$9\frac{3}{4}$

10   Subtract 18 mds 24 srs $12\frac{3}{4}$ ch from 29 mds. 36 srs. $7\frac{9}{18}$ chk

Simplify —

11   Rs. 5. 12a 9p $\times \frac{2}{3}$ 12   Rs 9. 13a. 4p. $\times \frac{7}{8}$

13   Rs. 21 11a. 9p $\times 1\frac{3}{13}$ 14   Rs 24 13a. 6p $\times 1\frac{4}{15}$.

15.   20 mds 15 sr. 8 chk $\times \frac{7}{8}$

16   16 mds 21 sr 8 chk. $\times 1\frac{5}{16}$

17   5 tons 10 cwt. 3 qr. $\times 1\frac{4}{9}$

18.   12 tons 10 cwt 2 qr. $\times 1\frac{7}{9}$

19.   Rs. 9 10a $8\frac{1}{3}p$ $\times 1\frac{5}{9}$ 20   Rs 16. 11a $7\frac{3}{5}p$. $\times 5\frac{1}{4}$.

21   28 mds 21 sr $7\frac{1}{2}$ chk $\times 5\frac{1}{8}$.

22.   16 tons 12 cwt $3\frac{1}{2}$ qr $\times 9\frac{1}{3}$

23   21 tons 13 cwt $1\frac{5}{6}$ qr $\times 7\frac{1}{2}$

24.   15 yards 2 ft. $3\frac{1}{8}$ in $\times 3\frac{1}{8}$

25   24 yards 1 ft $7\frac{1}{8}$ in $\times 2\frac{1}{4}$

26   Rs 307 4a 4p $-\frac{4}{25}$ 27.   Rs. 45 13a. 4p. $-1\frac{7}{8}$.

28.   Rs 173 5a 4p $-20\frac{4}{5}$

29   12 mds 27 sr 8 chk $-1\frac{7}{9}$.

30   13 mds 21 sr. 7 chk $-5\frac{5}{6}$

31.   18 tons 11 cwt $3\frac{1}{2}$ qr $-5\frac{1}{4}$.

32   11 tons 18 cwt 2 qr 21 lb. $-9\frac{1}{3}$

33. $\frac{5}{6}$ of Rs. 16 12a 8p $+1\frac{3}{5}$ of Rs 15 9a. 7p $+1\frac{5}{16}$ of Rs 21 13a 4p.

34    $\frac{3}{5}$ of Rs 3 5a. 4p $+\frac{5}{7}$ of Rs 21. 14a $+\frac{3}{11}$ of Rs 47 3a 4p.

35    $\dfrac{15\frac{3}{5}}{7\frac{4}{5}}$ of £1 $+\frac{1}{3}$ of £140. 10s 6d. $+2\frac{13}{5}$ of half-a-guinea

36 $\frac{3}{4}$ of 3 mds. 34 srs. $+\frac{3}{7}$ of 8 mds. 9 srs $+\frac{2}{3}$ of 3 srs 12 chk

37. $\frac{480}{869}$ of Rs 2 9a $+\frac{117}{480}$ of Rs $7\frac{1}{2}+\frac{29}{869}$ of Rs 9. 4a

38 $£\frac{11}{12}+\frac{3}{4}$ of 2 guineas$-\frac{5}{9}$ of 3s. 9d $+\frac{7}{8}$ of 1s

39 $(2\frac{1}{2}-3\frac{1}{3})$ of $£3$. 6s $+(\frac{2}{3})^2$ of 27s $-3\frac{2\frac{1}{2}}{6}$ of 5s.

40. $\dfrac{1\frac{4}{5}-1\frac{1}{5}}{\frac{4}{5}\text{ of }\frac{5}{9}-10\frac{1}{3}}$ of $\dfrac{1\frac{1}{2}\text{ of }4\frac{1}{6}}{6\frac{15}{16}\times5\frac{4}{5}}$ of Rs 50.

§30. To express one concrete quantity as a fraction of another quantity of the same kind

Rule *By dividing one concrete number by another concrete number of the same kind we get an abstract number which may be an integer or a fraction. To find this number reduce both the quantities to the same denomination and divide the first by the second.*

Example 1 Express Rs 2 4a as a fraction of Rs 9

Sol The fraction$=\dfrac{\text{Rs 2 4}a}{\text{Rs. 9}}=\dfrac{36a}{144a.}$
$$=\tfrac{1}{4}\text{ Ans}$$

Note A beginner makes a mistake by dividing Rs 9 by Rs 2 4a

Example 2 Express 2 mds 27 sr 8 chk as a fraction of 8 mds. 2 sr 8 chk.

Sol. The fraction$=\dfrac{\text{2 mds 27 sr 8 chk}}{\text{8 mds 2 sr. 8 chk}}$
$$=\dfrac{1720\text{ chk.}}{5160\text{ chk.}}=\tfrac{1}{3}\text{ Ans}$$

Note We can solve the above example by reducing each compound quantity to the fraction of its highest denomination thus

2 mds 27 srs 8 chks.$=$2 mds. $27\frac{1}{2}$ srs

$$=2\frac{55}{2\times40}\text{ or }2\frac{11}{16}\text{ mds }=\tfrac{43}{16}\text{ mds.}$$

8 mds. 2 srs. 8 chks $=$8 mds $2\frac{1}{4}$ srs.

$$=8\frac{5}{2\times40}\text{ or }8\frac{1}{16}\text{ mds.}=\tfrac{129}{16}\text{ mds}$$

The required fraction $=\tfrac{43}{16}-\tfrac{129}{16}=\tfrac{43}{16}\times\tfrac{16}{129}=\tfrac{1}{3}$. Ans

Note. We can make the statement of such questions in a number of different ways For instance we can state the 1st example in the following ways also —

1 Express Rs. 2 4a as a fraction of Rs 9
2 Reduce Rs 2 4a to the fraction of Rs 9
3. What fraction is Rs 2. 4a of Rs 9 ?
4. What fraction of Rs 9 is Rs 2 4a
5 What part is Rs 2 4a. of Rs 9 ?
6. Express Rs 2 4a. in terms of Rs 9
7 Find the measure of Rs 2 4a in terms of Rs 9
8 Find the quotient when Rs. 2 4a. is divided by Rs 9
9 Find the ratio of Rs 2 4a to Rs 9
10 How many times is Rs 9 contained in Rs 2 4a ?
11 How often is Rs 9 contained in Rs 2 4a

Example 3 Simplify $\dfrac{\text{Rs}\ 2\ 5a\ 6p}{\text{Rs. } 3\ 12a} - \dfrac{1 \text{ hr } 16 \text{ m. } 45 \text{ sec}}{2 \text{ hrs } 7 \text{ m, } 55 \text{ sec.}}$

Sol The exp $= \dfrac{450p}{720p} - \dfrac{4605 \text{ sec}}{7675 \text{ sec}} = \frac{5}{8} - \frac{3}{5} = \frac{1}{40}$. **Ans**

Example 4 Simplify $\dfrac{\text{Rs}\ 9.\ 1a,}{\text{Rs } 17\ 13a.} - \dfrac{1 \text{ sr. } 9 \text{ chk}}{5 \text{ sr } 15 \text{ chk}}$.

Sol. The quantity $= \dfrac{145a}{285a} - \dfrac{25 \text{ chk.}}{95 \text{ chk.}}$
$= \frac{145}{285} - \frac{25}{95}$
$= \frac{29}{57} - \frac{5}{19} = \frac{14}{57}$. **Ans**

EXERCISE 41

1. Express 5a 4p as a fraction of Re 1
2. Express 13a. 4p as a fraction of Rs. 5
3. Express Rs. 3. 10a. 8p as a fraction of Rs 11.
4. Reduce Rs 5. 1a 4p to the fraction of Rs. 7 10a.
5 What fraction of Rs. 21 is Rs. 5. 4a. ?
6 Find the quotient when Rs 5. 8a 9p is divided by Rs. 15. 8a. 6p
7 What fraction is £6. 13s. 4d. of £10 ?
8. Find the measure of £25. 16s. 9d. in terms of 41. 6s 8d
9. Express 15 mds. 27 srs 8 chks. in terms of 25 mds. 4 srs

10   How many times is 8 hours 40 minutes 20 seconds contained in 2 hours 10 minutes 5 seconds ?

11.   What part of 8 miles is 1 mile 586 yards 2 feet ?

12   Reduce 3 miles 110 yards to the fraction of 8 miles 176 yards

13   Express 4 cwt. 2 qrs 7 lbs. in terms of 14 cwt. 7 lbs

14   What is the ratio of Rs 12. $13a$ $4p$. to Rs. 19. $10a$ $8p$?

15.   What is the ratio of £15 16s 8d to £213. 15s ?

16.   Reduce $\frac{1}{8}$ of Rs. 2 $10a$ $8p. + \frac{2}{3}$ of Rs. 3. $5a$ $4p$ to the fraction of Rs. 10 $13a$ $4p$

17   Express $\frac{1}{9}$ of a guinea $+ \frac{3}{8}$ of £1 $+ \frac{5}{24}$ of 1s $+ \frac{1}{4}$ of 1d. as the fraction of £24 3s.

18.   Express $\frac{2}{5}$ of Rs 3. $8a. + \frac{3}{7}$ of Rs. 5 $4a - \frac{2}{9}$ of Rs 10 $8a$. as the fraction of Rs 39 $8a$

19   Express $\frac{2}{5}$ of $8\frac{1}{5}$ mds $+ \frac{1}{11}$ of $5\frac{1}{2}$ mds $- \frac{2}{9}$ of $4\frac{1}{2}$ mds. as a fraction of $\frac{1}{5}$ of $23\frac{1}{3}$ mds

20   Express $8\frac{3}{4}$ days of $\frac{2}{7} - 5\frac{1}{4}$ days of $\frac{2}{8} + 6\frac{3}{4}$ days of $\frac{2}{9}$ as a fraction of $8\frac{1}{4}$ days of $\frac{1}{11}$.

In the following examples express the former of the two quantities as a fraction of the latter —

21 $(8\frac{4}{7} - 3\frac{3}{8})$ of £5. 9s $11\frac{1}{2}d$, $\frac{7\frac{4}{5}}{4\frac{2}{8}}$ of £30 13s $2\frac{3}{4}d$.

22 $\frac{4}{15}$ of £$1\frac{1}{2} + \frac{5}{8}$ of 5s $4d - 8\frac{1}{2}$ of $\frac{1}{4\frac{1}{4}}$ of 5s. $3\frac{3}{4}d$;

2s $1\frac{1}{4}d$.

23 $\frac{3\frac{1}{2}}{4\frac{1}{2}}$ of $\left\{£\frac{3}{4} - \frac{3}{7}$ of 5s.$\right\}$, $\frac{2}{3}$ of 27s

24.   Simplify —

$\left(5\frac{5}{6} - \frac{5}{12}\right)$ of $\left(\frac{\frac{5}{6}}{3\frac{1}{3}} - \frac{7}{8}$ of $\frac{4}{3}\right) \div \frac{5}{7}$ of $\frac{3 \text{ tons 3 cwt.}}{9 \text{ cwt}}$.

25   Find the value of

$12 \times (\frac{2}{9} - \frac{1}{24} - \frac{1}{68} - \frac{1}{174} - \frac{1}{392}) + \frac{329}{861} - \frac{2\frac{5}{8}}{2\frac{5}{7}}$ of $\frac{11s. 4d.}{12s. 3d.}$.

26.   Simplify

$$\frac{\frac{3}{7} + \frac{7}{8}}{\frac{8}{9} + \frac{9}{10}} \text{ of } \frac{13s \quad 5d}{9s \quad 10d} - \frac{2}{8}\left(\frac{3}{7} + \frac{4}{7}\right) \text{ of } \frac{3 \text{ tons } 3 \text{ cwt.}}{4 \text{ tons } 3 \text{ cwt.}}.$$

27   What fraction of a ton added to $\frac{11}{28}$ of 2 cwt. will make it equal to 1 cwt 2 qr. 11 lbs.?

28   What fraction of Rs 29. 12a must be added to $\frac{3\frac{1}{7}}{4\frac{1}{9}}$ of $\left(3\frac{3}{7} + 1\frac{9}{5}\right)$ of Rs 6. 9a. to make the sum equal to Rs 32 8a ?

29   Find the weight which is the same fraction of 15 cwt 2 qrs 13 lb as £1 11s. $10\frac{1}{2}d$ is of £3. 10s $1\frac{1}{2}d$

30   What fraction of Re 1 13a 7p must be added to $\frac{3\frac{5}{3}}{2\frac{1}{4}}$ of $\left(\frac{3}{4} + \frac{4}{9}\right)$ of 1a. 4p to make the sum equal to Re. 1 ?

31.   Express $\frac{3}{14}$ of 3s. 6d $+\frac{2}{7}$ of £11. 7s 6d. $-\frac{5}{8}$ of £4 17s 4d $+1d$. to the fraction of Rs 12 when one rupee $=1s$ 6d

32   Reduce $\frac{1}{8}$ of Rs. 30. 6a. 6p $+\frac{2}{5}$ of 13a 9p. $-\frac{3}{4}$ of Rs 2 9a 4p. to the fraction of £5 when one rupee $=1s$ 4d

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§24   G C. M and L C M of fractions

The theory of the G. C. M and L. C M of whole numbers is also applicable when the given numbers are fractions

Rule   *If the fractions are of the same denominator, find the G C M or L C M of the numerators and put it above the denominator. But if the fractions are not of the same denominator reduce them to their least common denominator and then find the G. C. M. or L C. M of the new numerators and write it above the common denominator*   Thus—

Example 1   Find the G C M and L C.M. of $\frac{5}{13}, \frac{10}{13}, \frac{15}{13}$

Sol.   The fractions have the same denominator.   The

G. C M of the numerators $= 5$.

and L C M of the numerators $= 30$.

the required G C M. $= \frac{5}{13}$ Ans

and   „   L C. M $= \frac{30}{13}$ or $1\frac{7}{13}$. Ans

Example 2   Find the G C M and L C M of $\frac{5}{12}$, $\frac{5}{18}$ and $\frac{5}{8n}$

Sol   The fractions when reduced to their L. C. D.
$$=\frac{15}{46}, \frac{10}{88} \text{ and } \frac{5}{86}$$

G C M.$=\frac{5}{96}$ and L C M $=\frac{80}{86}$ or $\frac{5}{6}$ Ans

From the examples solved above we can deduce the following rules also —

(*i*) the G C M. of any number of fractions
$$=\frac{\text{G C M of numerators}}{\text{L C M of denominators}}$$

(*ii*) the L C. M. of any number of fractions
$$=\frac{\text{L C M of numerators}}{\text{G C M of denominators}}$$

Note   When the given fractions are not in their lowest terms they must be reduced before applying the rule

EXERCISE 42.

Find the G. C. M and L C M. of —

1 $\frac{5}{12}, \frac{7}{12}, \frac{15}{13}$ 2 $\frac{7}{40}, \frac{14}{40}, \frac{21}{40}$ 3 $\frac{9}{38}, \frac{15}{34}, \frac{21}{36}$.

4. $\frac{5}{12}, \frac{3}{24}, \frac{5}{18}$. 5 $\frac{7}{15}, \frac{10}{25}, \frac{21}{40}$ 6 $2\frac{1}{2}, \frac{3}{7}, \frac{5}{9}, \frac{5}{12}$.

7 $3\frac{1}{4}, \frac{5}{8}, \frac{13}{4}, \frac{11}{36}$ 8 $\frac{13}{17}, \frac{21}{34}, \frac{35}{51}, \frac{3}{68}$ 9. $3, \frac{5}{11}, \frac{4}{11}, \frac{5}{55}$.

10   One pendulum ticks 57 times in 58 seconds, another 608 times in 609 seconds   If they are started together, how often will they have ticked together in the first hour ?

11   What English sum of money can buy an exact number of rupees or dollars? Re 1=1s 4d., and 1 dollar=3s 8d.

12   One-sixth of the number of books in a library consists of Mathematics, $\frac{2}{5}$ of the remainder of Fiction, $\frac{5}{9}$ of what still remains of History and the remaining books are on Science   What should be the least number of books in the library to satisfy these conditions ?

13   5 competitors start together and continue running round a circular course $1\frac{1}{2}$ miles in circumference   They run at the rate of 2, 3, 4, 5 and 6 miles an hour respectively. How soon will they all be at the starting place ?

14   One Imperial maund $=82\frac{2}{7}$ lbs. avoirdupois, one Madras maund$=26\frac{14}{25}$ lbs. avoirdupois.   Find the least number of pounds that can be expressed as an exact number of Imperial and Madras maunds.

Some Important Typical Examples in Fractions.

Example 1. A man owns $\frac{3}{5}$ of the house and sold $\frac{1}{4}$ of it for Rs 250 , find the value of the house

Sol   The portion he sold $=\frac{3}{5}$ of $\frac{1}{4}=\frac{1}{6}$

$\cdot$ he sold $\frac{1}{5}$ for Rs 250,

the value of the whole $=$Rs $250\times 5=$Rs 1250 **Ans.**

Example 2. A person bequeathed his property to his three sons thus to the first $\frac{2}{5}$ and to the second $\frac{1}{4}$ of the remainder and the remaining property worth Rs 1500 to the third , find the value of the property.

Sol   First boy's share $=\frac{2}{5}$.

Remainder $=1-\frac{2}{5}=\frac{3}{5}$

$\cdot\cdot$ second boy's share $=\frac{3}{5}$ of $\frac{1}{3}=\frac{1}{5}$

Now remaining property $=1-(\frac{2}{5}+\frac{1}{5})=\frac{2}{5}$.

$\quad\frac{2}{5}$ of the property is worth Rs. 1500

$\cdot\cdot$ whole property is worth Rs $1500\times\frac{5}{2}$

$\qquad\qquad\qquad\qquad\qquad =$Rs 3750   Ans

Example 3. One-seventh of a sum exceeds its one-ninth by Rs 192 , find the sum.

Sol. $\frac{1}{7}-\frac{1}{9}=\frac{2}{63}$

$\cdot$ $\frac{2}{63}$ of the sum$=$Rs 192

$\cdot$ the sum $=$Rs $192\times\frac{63}{2}=$Rs 6048   **Ans.**

Example 4   A man was given $1\frac{1}{12}$ of a sum and was asked what the sum was. He multiplied the given sum by $1\frac{1}{12}$ and so got the answer too little by Rs. 115 , find the sum.

Sol   Suppose the sum $=$Re 1

$\cdot$ the sum given $=$Re $1\frac{1}{12}$

$\cdot$ the answer found $=1\frac{1}{12}\times 1\frac{1}{12}=$Re $\frac{121}{144}$

$\cdot$ the diff $=1-\frac{121}{144}=$Re $\frac{23}{144}$

$\cdot\cdot$ the actual sum $=$Rs. $115\times\frac{144}{23}=$Rs 720 **Ans.**

EXERCISE 43

1. $\frac{8}{18}$ of an estate is worth Rs. 501 , find the value of the whole estate

2 $\frac{3}{8}$ of an estate is worth Rs 125. 5a. 3p. , find the value of $\frac{5}{12}$ of the estate.

3. A man owns $\frac{9}{16}$ of a building and sells $\frac{2}{3}$ of it for Rs 948 , find the value of the whole building.

4 A had some money , he spent $\frac{2}{5}$ of it and found that $\frac{2}{3}$ of the remainder is equal to Rs 13 7a. 4p , find the amount he had

5 Peter spent $\frac{2}{5}$ of his money and found that $\frac{3}{5}$ of the remainder is equal to Re 1. 8a. What amount of money had he ?

6. A has Rs 14 7a 11p. with him which is $3\frac{3}{4}$ times of B's money , find the amount of B

7. A man divided some apples among three boys ; to the first he gave $\frac{1}{4}$ of them and to the second $\frac{2}{3}$ of the remainder and to the third, the remaining 10. How many apples had he ?

8. A person distributed some money among 3 beggars , the first got $\frac{1}{5}$ of the money and the second $\frac{4}{5}$ of what was then left and the third got 4a. only. What did the first and second beggars get ?

9 If a person gets a bequest of $\frac{4}{7}$ of an estate of 2000 acres, and sells $\frac{2}{5}$ of his share, how many acres does he retain ?

10. A person willed that the elder son should get $\frac{1}{2}$ of his property, the younger $\frac{1}{4}$ and wife $\frac{1}{6}$ The remaining property worth Rs 2000 is utilized in opening a public library , find the value of the property.

11 A person divided a piece of land among his three sons thus he gave 16$\frac{2}{3}$ acres to the first, $\frac{1}{3}$ of the whole to the second and $\frac{1}{2}$ to the third How many acres did he divide ?

12 A father divided a piece of land among his three sons thus he gave 35 acres to the first, $\frac{6}{20}$ of the whole to the second and to the third as much as to the other two together how many acres did the third get ?

13. A person bequeathed his property to his three sons thus , $\frac{3}{5}$ of the property to the first, $\frac{2}{5}$ of the remainder to the second and to the third Rs 2000 more than to the second What is the value of the property ?

14 A person sold $\frac{3}{5}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of an estate for Rs 501 1a 4p , find the value of $\frac{2}{3}$ of it

15. What is the least number which when divided by $\frac{8}{5}$, $\frac{9}{10}$, $\frac{11}{15}$ gives a whole number as quotient in each case ?

16 What is the greatest amount which is contained an exact number of times in both Rs $15\frac{3}{4}$ and Rs $18\frac{3}{4}$?

17 Find the greatest length that is contained an exact number of times in $5\frac{1}{4}$ ft and $9\frac{1}{8}$ ft

18. Four bells begin to toll together and toll respectively at intervals of $1\frac{1}{2}$, $1\frac{3}{4}$, $3\frac{3}{8}$ and $4\frac{1}{2}$ seconds What time will elapse before they all toll together again ?

19 A man owns $\frac{1}{8}$ of an estate, he sold $\frac{11}{25}$ of $\frac{9\frac{7}{11}}{8\frac{7}{10}}$ of it for Rs 960, find the value of $\frac{5}{13}$ of $\frac{2\frac{1}{2}}{3\frac{3}{8}}$ of the estate.

20 There is a stick, $\frac{1}{10}$ of which is red, $\frac{1}{20}$ black, $\frac{1}{30}$ yellow, $\frac{1}{40}$ orange, $\frac{1}{50}$ blue, $\frac{1}{60}$ white and the remaining 302 inches of violet colour. Find the length of the stick.

21. What sum is that, $\frac{2}{3}$ of $\frac{1}{5}$ of which is $\frac{3}{7}$ of $\frac{7}{9}$ of Rs 5 10a ?

22 Find the length, $\frac{25}{80}$ of which is $\frac{3}{8}$ of $7\frac{1}{2}$ of $16\frac{1}{2}$ yards

23. Find the sum $\frac{18}{1380}$ of which is $(4\frac{14}{21} - 10\frac{44}{45} + 9\frac{9}{25} - \frac{52}{117})$ of 8p. Find also what fraction it is of $\frac{8}{78}$ of Rs 6 8a

24 In a book on Arithmetic an example was printed thus, "Add together $\frac{1}{14\frac{2}{3}}$, $\frac{1}{19\frac{1}{4}}$, $\frac{1}{\_}$, $\frac{1}{13\frac{3}{4}}$" the denominator of one fraction being accidentally omitted The answer given at the end of the book was $\frac{11}{28}$ Find the missing denominator.

25 For two parts of an examination there are 572 candidates, $\frac{8}{11}$ of the number entered for part 1, and $\frac{9}{13}$ for part 2, how many entered for both parts ?

26 One-fifth of a number exceeds its one-seventh by 54 , find the number.

27 A boy was given $\frac{18}{19}$ of a sum and was asked what the sum was He multiplied the given sum by $\frac{18}{19}$ and so got the answer too little by Rs. 83, 5a. 4p. , find the sum

CHAPTER VIII.

DECIMAL FRACTIONS

§1   In a number like 3333, you know that the local
value of 3 decreases ten fold at each step from left to right.
First 3 on the left indicates   3000

    Second 3 indicates   300
    Third 3   „   30
    Fourth 3   „   3

If we place a dot (called the decimal point) to the right
of the unit's figure and then write another 3 as 3333'3, the
value of this last 3 will also decrease ten fold i e , it will
represent three-tenths and another 3 will represent three-
hundredths and so on   The following diagram will make
the method of **notation** clear —

Etc.	thousands.	hundreds	tens	units,	tenths	hundredths.	thousandths	etc
	2		5	4	7	3	5	
	2000	300	50	4	$\frac{7}{10}$	$\frac{3}{100}$	$\frac{5}{1000}$	

The number indicated above is " two-thousand three
hundred fifty-four and seven-tenths, three-hundredths, five-
thousandths" and is read as two thousand, three hundred,
fifty-four decimal seven, three, five   Thus the above
number is written as 2354 735

Note   In reading the figures after the decimal point they are
read in order   Thus 735 is read as   decimal seven, three and five"
and not as 'decimal seven hundred and thirty-five

The number expressed in the above notation is called
a **decimal fraction** ; because each figure to the right of the
decimal point indicates a fraction having 10 or some power

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of ten for its denominator   Hence the

Definition   A decimal fraction is a fraction which has *ten* or some power of *ten* for its denominator

$$5\ 24 = 5 + \tfrac{2}{10} + \tfrac{4}{100}$$

Note 1   The decimal point is always placed towards the top of the figures and not in the middle   Thus 6 9 is six decimal nine and 6 9 is 6×9   The boys should note the difference very carefully

Note 2   Number of figures after the decimal point is called the number of decimal places   In 5 24 there are two decimal places   and in 5 024 there are three decimal places and so on

Note 3   The part to the left of the decimal point is the integral part and to the right decimal part   Thus in 5 24, 5 is the integral part and 24 is the decimal part

EXERCISE 44

Read the following —

1	'1, '01, 001	2	5 2, 5 02, 5 5502.
3	16 789, 16 0789	4	13 7089, 3'0025
5.	30 0025, 30 2005		

Express into vulgar fractions as $15'02 = 15 + \tfrac{2}{100}$ —

6.	7 125, 8 034.	7	12 003, 16 203
8	20 0025, 15'1354	9	7'0105, 21 1205.

Express as decimals —

10.	Six tenths	11.	Six hundredths
12	Six thousandths	13.	Six ten thousandths.
14	Five and three tenths		

15.   Fifteen and nine tenths and six thousandths

16	$\tfrac{3}{10} + \tfrac{5}{100}$	17.	$\tfrac{8}{10} + \tfrac{5}{100} + \tfrac{7}{1000}$
18	$\tfrac{9}{100} + \tfrac{5}{1000} + \tfrac{9}{10000}$	19	$\tfrac{8}{10} + \tfrac{7}{1000} + \tfrac{9}{10000}$.
20	$5 + \tfrac{3}{10} + \tfrac{8}{1000} + \tfrac{7}{100000}$	21	$8 + \tfrac{9}{10000} + \tfrac{7}{1000000}$.
22.	$15 + \tfrac{3}{10} + \tfrac{8}{10} + \tfrac{5}{100000}$.	23	$21 + \tfrac{4}{1000} + \tfrac{8}{100000}$
24	$15 + \tfrac{7}{100} + \tfrac{9}{100000}$	25	$15 + \tfrac{8}{10} + \tfrac{7}{100000}$

§2   To convert a decimal into a vulgar fraction.

Observe the following examples —

Example   Convert '2053 and 15 0255 into vulgar fractions.

Sol. (*i*) $\cdot 2053 = \frac{2}{10} + \frac{5}{1000} + \frac{3}{10000}$

$$= \frac{2000 + 50 + 3}{10000} = \frac{2053}{10000}. \text{ Ans.}$$

(*ii*) $15\ 0256 = 15 + \frac{2}{100} + \frac{5}{1000} + \frac{6}{10000}$

$$= 15 + \frac{200 + 50 + 6}{10000}$$

$$= 15\frac{256}{10000} = \frac{150256}{10000}   \text{ Ans.}$$

Hence the following

Rule. *Write down the given number omitting the decimal point for the numerator and one followed by as many zeroes as there are decimal places in the given number for its denominator   Simplify the fraction thus obtained, if possible*

Thus $5\cdot 002 = \frac{5002}{1000} = \frac{2501}{500}$

Note   The student should commit to memory the following results —

$5 = \frac{1}{2}$, $25 = \frac{1}{4}$, $\cdot 75 = \frac{3}{4}$, $125 = \frac{1}{8}$, $375 = \frac{3}{8}$, $625 = \frac{5}{8}$, $\cdot 875 = \frac{7}{8}$.

§3   Conversely we can convert every vulgar fraction having 10 or any power of 10 as denominator into a decimal fraction.   Observe the following examples —

$\frac{3}{10}$ means 3

$\frac{3}{100}$,,   03

$\frac{3}{1000}$,,   003

Hence the following

Rule. *Put the decimal point leaving as many figures in the numerator on the right hand as there are zeroes after 1 in the denominator   If the number of figures in the numerator be less than the number of zeroes in the denominator, prefix necessary zeroes in the numerator.*

Thus $\frac{1256}{100} = 12\ 56$ and $\frac{1256}{1000} = 1\ 256$

and $\frac{1256}{10000} = 1256$ and $\frac{1256}{100000} = 01256$

Note   This is really a division by 10 or any power of 10

EXERCISE 45.

Express as vulgar fractions in their lowest terms —

1. ·5, 25, ·75
2. ·125, 375, 625
3. 05, ·025, 0125
4. 124, 576, 728.
5. ·504, 054, 0504.
6. 5 1, 5 029, 5 625.
7. 16 25, 18 75, 21·375
8. 90 5, 90 05, 900 55

Express as decimals —

9. $\frac{7}{10}$, $\frac{17}{100}$, $\frac{177}{100}$
10. $\frac{75}{100}$, $\frac{186}{1000}$, $\frac{182\cdot5}{10000}$
11. $\frac{9}{10}$, $\frac{9}{100}$, $\frac{9}{1000}$
12. $\frac{875}{1000}$, $\frac{75}{1000}$, $\frac{5}{1000}$
13. $5\frac{7}{100}$, $8\frac{9}{1000}$, $15\frac{175}{10000}$
14. $\frac{500}{10000}$, $\frac{40}{10000}$, $\frac{540}{10000}$
15. $\frac{78936}{100}$, $\frac{764856}{1000}$, $\frac{75001}{10000}$
16. $\frac{15}{1000}$, $\frac{7}{10000}$, $\frac{101}{100000}$

§4 The following facts should be noted very carefully —

(ı) *The value of a decimal fraction is not changed by affixing zeroes to the right of the last figure*

Thus $175 = 1750 = ·17500$ etc

Since 175 $= \frac{175}{1000}$
and ·1750 $= \frac{1750}{10000} = \frac{175}{1000}$
also ·17500 $= \frac{17500}{100000} = \frac{175}{1000}$
175       $= ·1750 = 17500$, etc , etc

(ıı) *Zeroes affixed just after the decimal point decrease its value ten fold.*

Thus $1 = \frac{1}{10}$, $01 = \frac{1}{100}$, $001 = \frac{1}{1000}$ etc

§5 To multiply or divide a decimal by 10 or any power of 10

It has been explained that the local value of figures increases ten fold at each step from right to left and decreases from left to right   Hence to multiply a decimal by 10, 100, 1000, etc etc , remove the decimal point 1, 2, 3, etc. etc , places respectively to the right, and to divide the decimal by 10, 100, 1000, etc. etc. remove the decimal point 1, 2, 3, etc etc places to the left.   Thus

$$
\begin{array}{ll}
\cdot537 \times 10 &= 5\,37 \\
537 \times 100 &= 53\,7 \\
537 \times 1000 &= 537 \\
537 \times 10000 &= 5370
\end{array}
\qquad
\begin{array}{ll}
153\cdot7 - 10 &= 15\cdot37 \\
153\,7 - 100 &= 1\cdot537 \\
153\,7 - 1000 &= \cdot1537 \\
153\cdot7 \div 10000 &= \cdot01537.
\end{array}
$$

Note   Deficiency of figures to the right or to the left may be made up by adding zeroes as shown above

EXERCISE 46   (Oral.)

Multiply the following by 10 —

1   1, 01, 001, 0001   2   1·01, 1 001, 1·0001

3.   2 5, 2 55, 25·25, 25·002   4.   ·15, ·115, ·1015

Divide the following by 10 —

5.   6·1, 8·25, 10 123   6.   15 3, 15 03, 12 156,

7.   153, ·1057, 1425   8   01, ·001, ·0001

Multiply the following by 100 —

9   1 001, 1001, 01001   10   16 023, 15 125, 21 1307

11.   154 25, 123 10, 158 02   12   103 002, 103·0002

Divide the following by 100 —

13.   728 13, 1226 02   14   1202 09, 12020 9

15   ·1, 01, ·001, 0001   16   2 001, 20 001, 200 01

Multiply the following by 1000 —

17.   ·02, 01, 001   18.   003, 004, ·504.

19   4 03, 15 003, 21·007   20.   ·1002, 0012, 00025

Divide the following by 1000 —

21.   ·003, 516, 4·123   22   16 123, 1880 116

23   56210 02, 70231·05   24   212 31, 723 45, 5560 5

§6   Addition of Decimals

Rule   *Write the numbers under one another so that the decimal points may fall in a vertical straight line and the digits of the same local value exactly under each other, then proceed as in simple addition*

F 10

Example   Add together 2 05, 15'912, 002 and 21.

Sol.   (1)   2 050   (2)   2 05
                     15 912   15 912
                      · 002   002
                     21'000   21
                     ───────   ──────────
                     38'964   38'964 **Ans**

Proof :—

$$2\cdot05 + 15\cdot912 + 002 + 21 = \tfrac{205}{100} + \tfrac{15912}{1000} + \tfrac{2}{1000} + 21$$

$$= \frac{2050 + 15912 + 2 + 21000}{1000}$$

$$= \tfrac{38964}{1000} = 38\;964 \quad [\text{Art } 3]$$

Note   The student may in the beginning, fill in with zeroes the missing powers of 10 as shown in solution (1)   It may also be noted that an integer is expressed as a decimal by affixing zeroes in the decimal part   Thus 21 = 21 0 = 21 00 = 21 000, etc.

EXERCISE 47

Add together —

1.   '002, '9002, 5'1, 12.

2.   '0005, 8, 3 125, 15

3   15'1, 15 001, 150 01, 18.

4.   1, '001, 101 1, 10 1, 25

5.   '02, 25 023, 100 2, 117

6.   124 1235, 1 02, 18 075, 11

7   '052, '5, 351, 54 501, 31.

8   501'5607, 5 02, 5003 24, '8.

9   72801, '01, 612, 556 1.

10.   137 023, 124'135, 00235, 56

§7   Subtraction of Decimals

Rule.   *Write the smaller number underneath the greater as explained in Art. 6 and proceed as in simple subtraction.*

Example   Subtract 5'00235 from 11'1.

Sol $11\overset{\cdot}{\cdot}1\overset{.}{0}000$
               $\underline{5\ 00235}$
               $\overline{6\overset{\cdot}{\cdot}09765}$ Ans

Proof $11\overset{\cdot}{\cdot}1 - 5\ 00235 = \frac{111}{10} - \frac{500235}{100000}$

$$= \frac{1110000 - 500235}{100000}$$

$$= \frac{609765}{100000} = 6\ 09765.$$

EXERCISE 48

Subtract —

1. $\overset{\cdot}{\cdot}25$ from $\overset{\cdot}{\cdot}5$.   2 $\overset{\cdot}{\cdot}45$ from $\overset{\cdot}{\cdot}51,$

3. $\overset{\cdot}{\cdot}025$ from $\overset{\cdot}{\cdot}1.$ 4. $\overset{\cdot}{\cdot}078$ from 201

5 $12\overset{\cdot}{\cdot}1$ from $121.$ 6. $\overset{\cdot}{\cdot}8736$ from $\overset{\cdot}{\cdot}985$

Find the difference of —

7. $2\overset{\cdot}{\cdot}00035$ and $15\ 87.$ 8 $7\ 12302$ and $8\ 01$

Simplify —

9. $56 - 45\ 0235$ 10. $41 - 40\overset{\cdot}{\cdot}9356$

11. $43 - 42\overset{\cdot}{\cdot}00235$ 12 $121 - 111\overset{\cdot}{\cdot}980765.$

13. $105\ 34 - 45\ 43 - 15\ 012 - 30\ 012.$

14 $17\ 143 - 14\overset{\cdot}{\cdot}01235 + 21\ 0123 - 2\overset{\cdot}{\cdot}123.$

15 $162\overset{\cdot}{\cdot}013 + 00235 - 156\overset{\cdot}{\cdot}9135 - 2\ 0023$

§8. Multiplication of Decimals.

(i) To multiply a decimal by a whole number

Example 1 Multiply $2\ 35$ by $21.$

Sol. $2\overset{\cdot}{\cdot}35 \times 21 = \frac{235}{100} \times 21$
             $= \frac{4935}{100} = 49\ 35.$ Ans

There are two places of decimals in the multiplicand and so in the product too   Hence the following

Rule. *Multiply the numbers as in simple multiplication and point off in the product as many places of decimals as there are decimal places in the multiplicand Prefix zeroes if necessary*

Example 2.   Multiply ˙0026 by 15

Sol.   0026
                        15
                      ─────
                       390

The number of decimal places should be 4, because there are 4 decimal places in the multiplicand

∴ the product=˙0390= 039 **Ans.**

(*ii*) **To multiply a decimal by a decimal.**

Example 3   Multiply 5 0023 by ˙00025.

Sol.   5 0023 × 00025 $= \frac{50023}{10000} \times \frac{25}{100000}$

$= \frac{1250575}{1000000000} =$ 001250575 **Ans.**

There are 9 places of decimals in the multiplicand and multiplier and so in the product too   Hence the following

Rule. *Multiply the numbers as in simple multiplication and point off in the product as many places of decimals as there are decimal places in the multiplicand and the multiplier together.   Prefix zeroes if necessary.*

Example 4   Multiply 121˙50987 by 5˙25 correct to 3 decimal places

Sol.   121 5,0987   five decimal places.
             5 25   two   „   „
           ─────────
           60754935
           24301974
           60754935
           ─────────
           637 9268175   seven   „   „

∴ the required product=637˙927   **Ans.**

EXERCIE 49.

Multiply :—

1. 4 2   by 5   2   15 25   by 16
3   18˙026 by 25   4   21 023   by 20
5   51˙908 by 35   6. 31˙0235 by 46
7. 12˙25 by 2 5   8   13 21,02 by 3˙25.
9. 18 023 by 16 124   10. 21˙0005 by 1 0025.
11 8 001 by 8001   12   75 005 by 5 0016

Multiply the following correct to 4 decimal places :—

13   16 5803 by 8 5302.   **14**   31 8035 by 165 02

15   156 7835 by 9 1356.   **16**   121 7802 by 35 125.

Find the continued product of —

17.   1 × 01 × ·001 × 0001   **18**   002 × 0005 × 16.

19.   ·4 × 04 × 05 × 005.   **20**   1 6 × 16 × 1600 × ·016

---

§10   Division of decimals

(i)   To divide a decimal by a whole number.

Example 1.   Divide 5 025 by 25

Sol $5\ 025 \div 25 = \frac{5025}{1000} \times \frac{1}{25}$

$= \frac{201}{1000} = ·201.$ **Ans.**

There are three places of decimals in the dividend and so in the quotient too   Hence the following

Rule   *Proceed as in simple division and then point off in the quotient as many places of decimals as there are decimal places in the dividend   Deficiency of figures, if any, may be supplied by zeroes.*

Example 2.   Divide 2565 by 15. .

Sol $15)\underline{\!\cdot\!2565}$
                  $\overline{171}$

There are four decimal places in the dividend and so there should be in the quotient too ,

the reqd. quotient = 0171. **Ans.**

(ii)   To divide a decimal by a decimal

Example 3.   Divide 5 125 by 25.

Sol. $5\ 125 \div ·25 = \frac{5\ 125}{25}.$

To make the divisor a whole number, multiply the dividend and the divisor by 100.

$$\frac{5\ 125}{25} = \frac{5·125}{25} \times \frac{100}{100} = \frac{512·5}{25}.$$

Now proceed as in example 2

The required quotient=20 5 **Ans.**

From the above example we deduce the following

Rule   *Remove the decimal point from the divisor and move the decimal point as many places to the right in the dividend as there are decimal places in the divisor, and then proceed as in Art 10 (1)*

§11   In dividing if there is any remainder left after the last digit from the dividend has been brought down, we should add zeroes to the right of the dividend and proceed till there is no remainder left

Example 4   Divide 1 365 by 1 25.

Sol   1'25)1 365=   125)136 500(1 092 **Ans**

                            125

                            ────

                            1150

                            1125

                            ────

                             250

                             250

                             ────

EXERCISE 50.

Divide —

1   25'6 by 16, 17 28 by 32 and 897'2 by 40

2.   773 682 separately by 13, 78, 169 and 1014

3.   '00750116 separately by 677, 1354 and 10832

4   35 9424 by 7 02 and 11 444495 by 4 735

5.   89 725 by 005 and 4'2225 by '000015

6   020872522 by 08635 and '39538 by 5300

7   1 by 01, 01001 by 001 and 99 by 0009

8.   9864 1698175 by 35 0645, 124'59993 by 3 194 87.

9   1 365 separately by 1 25, 12 5, 00125 and 12500

10   0064096 by 2 003 and 614'50824 by 0010201.

Divide and find the quotient to 3 places of decimals —

11   210 8972 by 128'25   **12** 156'8935 by 115 89.

13.   51352 by 892 7893.   **14**   117 003 by 21'00235.

15.   59 78312 by 21'563   **16.**   12 354 by 7256 04.

Simplify —

17. $\dfrac{500 \times \cdot 00025}{625}$.

18. $\dfrac{001 \times \cdot 1421}{2\ 03}$.

19. $\dfrac{4\ 25 \times 5\ 135}{102\ 7 \times\ 0085}$.

20. $\dfrac{15\ 001 \times \cdot 004}{8 \times 300\ 02}$.

21. $\dfrac{\cdot 1 \times \cdot 01 \times\ 0001 \times 16}{2 \times\ 02 \times \cdot 002 \times 4}$.

22. $\dfrac{5\cdot 24 \times 7\cdot 28}{104\cdot 8 \times\ 091}$.

§12   To reduce a vulgar fraction to a decimal.

Rule   *Reduce the vulgar fraction to its lowest terms and divide the numerator by the denominator as in division of decimals.*

Example 1   Convert $\frac{5}{32}$ into a decimal.

```
            15625
    32)5 00000
         3 2
        ────
         1·80
         1·60                    ·15625   Ans
         ────
          200
          192
          ───
           80
           64
          ───
          160
          160
```

Example 2.   Arrange $\frac{9}{155}$, $\frac{15}{186}$, $\frac{25}{372}$, $\frac{27}{62}$, $\frac{9}{124}$, in order of magnitude by reducing to decimals

$\frac{9}{155}$ = 058. . .   or $=\dfrac{1}{17\cdot 2}$

$\frac{15}{186}$ = ·080 . .   or $=\dfrac{1}{12\ 4}$

$\frac{25}{372}$ = 067. ..   or $=\dfrac{1}{14\cdot 8}$

$\frac{27}{62}$ = ·435.. ..   or $=\dfrac{1}{2\ 2}$

$\frac{9}{124}$ = ·072......   or $=\dfrac{1}{13\cdot 7}$

∴ order of magnitude is $\frac{27}{62}$, $\frac{15}{186}$, $\frac{9}{124}$, $\frac{25}{372}$, $\frac{9}{155}$.

EXERCISE 51

Convert into decimals —

1. $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{25}$ 2 $\frac{5}{8}$, $\frac{7}{20}$, $\frac{11}{40}$

3 $\frac{4}{5}$, $\frac{3}{8}$, $\frac{7}{25}$ 4 $\frac{32}{125}$, $\frac{25}{128}$, $\frac{16}{125}$.

5 $\frac{99}{625}$, $\frac{91}{250}$, $\frac{41}{420}$ 6 $\frac{124}{250}$, $\frac{199}{192}$, $\frac{15}{128}$.

Reduce the following to 5 decimal places —

7 $\frac{513}{140}$, $\frac{729}{140}$, $\frac{353}{160}$ 8. $56\frac{11}{79}$, $59\frac{64}{75}$, $56\frac{121}{528}$

Arrange in the descending order of magnitude, by reducing to decimals —

9 $\frac{5}{12}$, $\frac{7}{16}$, $\frac{3}{4}$ 10 $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$

Convert the following into decimals —

11. $3\frac{1}{2}$ of $2\frac{4}{5}$ and $\frac{3}{5}+\frac{7}{8}+\frac{9}{10}+\frac{7}{32}$

12   $\frac{6\frac{3}{4}}{11\frac{1}{4}}$ and $\frac{17\frac{1}{8}}{12\frac{1}{2}}$ 13 $\frac{5\frac{1}{4}}{4\frac{1}{6}} \times \frac{\frac{3}{8}}{\frac{5}{16}}$.

14. $3\frac{5}{8}$ of $\frac{1}{8\frac{1}{17}}$ and $3\frac{5}{8}$ of ·003 — ·0011 of $7\frac{1}{4}$

Find the greatest and the least fractions by reducing to decimals —

15   $\frac{13}{144}$, $\frac{15}{150}$, $\frac{17}{168}$, $\frac{5}{48}$ 16 $\frac{7}{16}$, $\frac{9}{24}$, $\frac{18}{56}$, $\frac{39}{72}$

---

§13. G. C. M and L. C. M. of decimals

Example 1. Find the G. C. M and L. C. M of 5, ·15, ·25.

Sol.   5, ·15, ·25$=\frac{5}{10}$, $\frac{15}{100}$, $\frac{25}{100}$

$=\frac{50}{100}$, $\frac{15}{100}$, $\frac{25}{100}$

G. C M. $=\frac{5}{100}$ or ·05 }
L. C M $=\frac{150}{100}$ or 1·5. } Ans.

From the above we conclude the following

Rule *Make the same number of decimal places in all the numbers by affixing zeroes and then find the G. C. M. or L. C. M. of the new numbers as if they were integers and then mark off in the result the said number of decimal places prefixing zeroes if necessary.*

We shall apply the rule in the first example thus —
$$5, \text{·}15, \text{·}25 = \text{·}50, \ 15, \ 25$$
G C M of 50, 15 25,   = 5 and L C M.=150
. the required G. C. M. = 05   } Ans.
and the required L C M =1·50 or 1 5 }

Example 2 Find the G C M. and L C M of 05, 7 and ·002

Sol   05, 7 and 002= 050, ·700, ·002
G C M of 50, 700, 2=2
L C M of 50, 700, 2=700
   the required G C M =·002   } Ans
and   „   L. C M = 700 or ·7. }

EXERCISE 52

Find the G. C. M. and L C M of —

1. ·14, 21, ·28, 56	2 11, ·22, 33, 44		
3 15, ·25, 45, ·75	4 13, 26, ·39, 52		
5. 14, 42, 56, 8+	6 5, 25, 025, 005		
7 7, 1·4, 007, 0014	8 ·45, 09, ·009, 8·1		
9 1·2, 24, 3 6, 108.	10 00121, 1 21, ·0011, 11.		

§14   Complex fractions involving decimals.

Example 1   Simplify $\dfrac{3\,5+2\frac{1}{3} \text{ of } \frac{9}{7}}{2\text{·}5+5\frac{1}{6}-1\frac{7}{2\frac{1}{4}}} + \dfrac{1\,05+\,95}{5}$

Sol   The expression $= \dfrac{3\,5+\frac{7}{3}\times\frac{9}{7}}{2\,5+\frac{31}{6}\times\frac{24}{81}} + \dfrac{\frac{2}{5}}{5} = \dfrac{3\,5+3}{2\,5+4}+4$

$$= \dfrac{6\,5}{6\,5}+4 = 1+4 = 5 \quad \text{Ans}$$

Example 2.   Simplify $\dfrac{1\,75}{\frac{5}{4}} + \dfrac{2\,35-1\,05}{-4\text{·}7-\frac{21}{10}} + \dfrac{575}{92}$

Sol   The expression $= \dfrac{1\,75}{1\,25} + \dfrac{1\text{·}3}{4\,7-2\,1} + \dfrac{575}{920}$

$$= \dfrac{175}{125} + \dfrac{1\text{·}3}{2\,6} + \dfrac{575}{\text{·}920}$$
$$= \tfrac{175}{125} + \tfrac{13}{26} + \tfrac{575}{920}$$
$$= \tfrac{7}{5} + \tfrac{1}{2} + \tfrac{5}{8}$$
$$= 1\,4 + 5 + \text{·}625 = 2\text{·}525 \quad \text{Ans}$$

Note 1 In such questions, sometimes vulgar fractions are converted to decimal fractions and sometimes decimal fractions are converted to vulgar. A careful study in decimals and constant practice will enable the student to understand which method is simple in an example.

Note 2 The result in such questions should be expressed in its decimal form

EXERCISE 53.

Simplify the following —

1. $\dfrac{1\frac{1}{9}\text{ of }10\ 26}{2\frac{2}{7}\text{ of }1\ 05}$.

2. $\dfrac{1\frac{1}{4}\times 6+\text{'}25}{2\frac{3}{5}\text{ of }8\frac{1}{4}-13\ 25}$.

3. $\dfrac{2\ 37\text{ of }(5\ 21+6\text{'}84)}{\text{'}1185}$.

4. $\text{'}05\text{ of }\dfrac{3\ 4}{2\frac{1}{5}\text{ of }1\ 2}+\frac{1}{14}$.

5. $399\times 007-000019+\dfrac{\text{'}003125\times 48}{0000125}$.

6. $\dfrac{3\frac{4}{11}\times (1\frac{8}{9}\text{ of }1\ 08)}{1\frac{11}{13}\times (0\ 6+\frac{3}{9})}$

7. $\dfrac{4}{7\text{'}5}\left\{\dfrac{\frac{1}{8}+\dfrac{\text{'}75}{4.5}}{\dfrac{1-25}{2-5}}+\frac{7}{8}\right\}$

8. $\dfrac{1\ 25}{\frac{5}{4}}+\dfrac{4\text{'}85-2\text{'}25}{\text{'}85-81}-55\ 0025$

9. $\dfrac{1}{1\frac{1}{20}}\times\dfrac{1+\text{'}0025\times 05}{1\ 0025-05}+\dfrac{45\times\frac{10}{45}}{8}$.

10. $\dfrac{\text{'}67\times 67\times\text{'}67-\text{'}001}{\text{'}67\times 67+067+01}+\dfrac{\text{'}57}{1+\dfrac{1}{3\frac{1}{14}}}$

11. $\dfrac{1\text{'}125}{\frac{9}{8}}+\dfrac{5\text{'}6\text{ of }(6\frac{1}{4}-1\text{'}25)}{1\ 4\text{ of }\text{'}2+10}$.

12. $\dfrac{\frac{2}{15}\times\text{'}39+\frac{7}{26}}{3\frac{1}{4}+(1\ 375\text{ of }2\frac{1}{5})}\times\dfrac{14\text{'}27}{\text{'}0102}$.

13. $\dfrac{3\frac{1}{8}\text{ of }3\frac{3}{4}+8}{3\text{'}2+2\frac{4}{5}}+\dfrac{\text{'}527\times 527-\text{'}223\times\text{'}223}{527-\text{'}223}$.

14. $\dfrac{7\;5}{6\;5} + \dfrac{11\dot{5} - 2\;4}{11\dot{5} + 2 +} \times 10\frac{0}{1\,8} - 6\frac{4\,2}{2\,7\,8}.$

15. $\dfrac{8\frac{1\,1}{6\,5}}{\frac{7}{8}\text{ of }2\;25}\text{ of }\dfrac{1625}{\frac{1}{1\,5}\text{ of }5\frac{4}{7}} + \left(\frac{2}{3\,1} + \frac{7}{8\,1}\right) + \frac{3}{5\,6\,7}$

16 $\dfrac{3\frac{3}{4} + 4\frac{4}{5}}{5\frac{1}{1\,0} - 4\frac{3}{1\,0}} - \dfrac{4\;75 + 3\;8}{6\;1 - 5\;15} \times \dfrac{\cdot021 \times \cdot0021 \times 210}{\cdot007 \times 14}.$

RECURRING DECIMALS

§15   In reduction of vulgar fractions to decimals the division in some cases does not terminate   For the sake of illustration we solve a few examples of this type here.

Examples   Express $\frac{1}{3}$, $\frac{5}{11}$ and $\frac{17}{90}$ into decimals.

     (*i*) $\frac{1}{3} = 3)\underline{1\ 000}$
                333

     (*ii*) $\frac{5}{11} = 11)\underline{5\ 0000}$
                 $\cdot45, 45$

     (*iii*) $\frac{17}{90} = 90)\underline{17\ 000}$
                  188

We see that the division in each of these cases does not terminate and can be extended to unlimited length. Such decimals are called **Repeating** or **Circulating** or **Recurring** decimals

Definition. *A decimal in which a figure or a set of figures repeat continually is called a Recurring decimal.*

The first two examples are examples of **Pure recurring** decimals and the third is an example of **mixed recurring** decimals.

Definition. *A decimal in which all the figures recur is called a pure recurring decimal, and in which some figures do not recur is called a mixed recurring decimal.*

Notation. The result is expressed by placing a dot ( ) over the repeated figure or figures   Thus the result in the first example will be expressed as $\dot{3}$ where the dot indicates that the figure 3 is continually repeated   Similarly the results in the second and third examples will be written as $\cdot\dot{4}\dot{5}$ and $\cdot1\dot{8}$.

$\overset{.}{3}$ will be read as decimal 3, three recurring and $\overset{..}{4}5$ will be read as decimal 4, 5 four five recurring

Note The figure or figures which recur form what is called the **Period**

§16. Terminating or Non-terminating.

If we were to know that a certain vulgar fraction will produce a *terminating* decimal or not, we should reduce the given vulgar fraction to its lowest terms, if its denominator be wholly made up of the factors 2 and 5. It will be expressed as a terminating decimal, otherwise not

Example Find whether $\frac{3}{80}$ and $\frac{5}{36}$ are terminating or non-terminating.

Sol. (i) $\frac{3}{80} = \left(\dfrac{3}{2 \times 2 \times 2 \times 2 \times 5}\right)$, since its denominator is wholly made up of the factors 2 and 5, therefore it can be expressed as a terminating decimal

(ii) $\frac{5}{36} = \left(\dfrac{5}{2 \times 2 \times 9}\right)$, since its denominator is not wholly made up of the factors 2 and 5, therefore it cannot be expressed as a terminating decimal

EXERCISE 54.

State in each case, whether the equivalent decimal is terminating or non-terminating —

1. $\frac{1}{9}$ 2 $\frac{5}{21}$ 3. $\frac{9}{64}$ 4. $\frac{11}{84}$ 5 $\frac{96}{125}$

6 $\frac{81}{125}$ 7 $\frac{91}{165}$ 8 $\frac{128}{925}$ 9 $\frac{101}{728}$ 10. $\frac{101}{875}$

Express the following as a recurring decimal —

11. $\frac{5}{6}$ 12 $\frac{4}{9}$ 13. $\frac{10}{11}$ 14 $\frac{6}{7}$ 15 $\frac{5}{13}$

16 $\frac{7}{30}$ 17. $\frac{7}{30}$ 18 $\frac{8}{27}$ 19. $\frac{9}{88}$ 20. $\frac{3}{37}$

21 $5\frac{4}{7}$ 22. $3\frac{3}{13}$ 23 $9\frac{3}{11}$ 24. $5\frac{8}{37}$ 25. $14\frac{3}{29}$

26 $\dfrac{5}{12}$ 27 $\dfrac{8}{27}$ 28 $\dfrac{11}{13}$ 29. $\dfrac{12}{21}$ 30. $\dfrac{3\frac{3}{8}}{5\frac{1}{2}}$

§17. To convert a pure recurring decimal into an equivalent vulgar fraction

Example 1. Convert ˙5 into its equivalent vulgar fraction

Sol   Let $x = ˙5 = ˙5555$
Multiplying both sides by 10, $10x = 5˙555$ (1)
But $x = ˙555$. (2)
∴ Subtracting (2) from (1), we have $9x = 5$,
$$x, i\ e\ \ 5 = \tfrac{5}{9}\ \ \text{Ans.}$$

Example 2 Convert ˙45 into its equivalent vulgar fraction.

Sol.   Let $x = ˙45 = ˙454545$.
Multiplying by 100, we get $100x = 45˙454545$ (1)
                        but $x = ˙454545$ (2)
Subtracting (2) from (1), we get $99x = 45$
$$∴\ x, i\ e, ˙45 = \tfrac{45}{99}\ \ \text{Ans.}$$

From the above solutions we deduce the following
Rule   *Make the period the numerator of the fraction and put a number of as many nine as there are figures in the period in the denominator, then reduce the fraction to its lowest terms, if possible.*

§18 To convert a mixed recurring decimal into its equivalent vulgar fraction.

Example 3 Express ˙45 to an equivalent vulgar fraction

Sol.   Let $x = ˙45 = ˙4555$.
                             $10x = 4˙555$ (1)
                        and $100x = 45˙555$ . . (2)
Subtracting (1) from (2), $90x = 45 - 4$
$$∴\ x, i\ e,\ ˙45 = \frac{45 - 4^{*}}{90} = \frac{41}{90}\ \ \text{Ans.}$$

Example 4. Express ˙4569 to its equivalent vulgar fraction.

Sol   Let $x = ˙4569$ $= ˙456969$.   .
           $100x$ $= 45˙6969$ (1)
        and $10000x$ $= 4569˙6969$ . . (2)

Subtracting (1) from (2), $9900x = 4569 - 45$.

$$x, \; i \; e, \quad \cdot 45\dot{6}\dot{9} = \frac{4569 - 45^*}{9900} = \tfrac{4524}{9900} = \tfrac{377}{825} \quad \text{Ans.}$$

From asterisked figures in the above solutions we have the following

Rule   *Subtract the figures which do not recur from the given expression and make the difference the numerator of the fraction and underneath it write a number consisting of as many nines as there are figures that recur followed by as many zeroes as there are figures that do not recur*

Note 1   If a pure or mixed recurring decimal be preceded by a whole number, the whole number may be kept separate

Note 2   From the above rules we conclude that

$9 = \tfrac{9}{9} = 1, \quad 0\dot{9} = \cdot 1$ and $0\,09 = 01$

$4\,9 = 5, \; 4\,59 = 4\cdot 6, \; 4\,02349 = 4\,0235$, etc.

and $99 = 1, \; 999 = 1, \; 9999 = 1, \; 5\,399 = 5\,4$, etc

That is to say, in simplifying recurring decimals containing the figure 9 only, the recurring part may be omitted and the preceding figure be increased by one   But you must not be tempted to apply the same rule if the recurring portion contain 9 along with other digits in any position

EXERCISE 55.

Express as vulgar fractions in their lowest terms (Questions asterisked to be done mentally)

1	$\cdot\dot{4}.$	2	$\cdot\dot{2}\dot{5}.$	3	$\cdot\dot{2}\dot{7}.$	4	$\dot{4}5$
5.	$\cdot\dot{0}\dot{8}1.$	6	$117.$	7	$333.$	8	576
9.	$2\,\dot{0}18.$	10	$5\,444$	11	$7\,148$	12.	$10\cdot 296$
^13.	$7\,9\dot{9}9$	~14	$12\,999\dot{9}$	15	$18\,2727$	16.	$15\,7272$
17	615384	18	857142	19.	923076		
20.	714285	21.	$\cdot 81.$	22	$\cdot 127.$		
23	256	24	26	*25.	49		
*26.	$5\,9\dot{9}$	$\cdot$	*27.	099	$\cdot 28.$	$419.$	
29	$25\,9045$	30	$21\cdot 5638$	31	$2\,257345.$		
32	Prove that						

$$\frac{1}{9} = \frac{\cdot\dot{1}}{1} = \frac{\cdot\dot{2}}{2} = \frac{\cdot\dot{3}}{3} = \frac{\cdot\dot{4}}{4} = \frac{5}{5} = \frac{6}{6} = \frac{7}{7} = \frac{\cdot\dot{8}}{8} = \frac{9}{9}.$$

33 Prove that $\dfrac{1}{11}=\dfrac{\dot{0}\dot{9}}{1}=\dfrac{\cdot\dot{1}\dot{8}}{2}=\dfrac{\cdot\dot{2}\dot{7}}{3}=\dfrac{\dot{3}\dot{6}}{4}=\dfrac{\cdot\dot{4}\dot{5}}{5}$ etc.

34 Prove that $\dfrac{1}{7}=\dfrac{\cdot\dot{1}42857}{1}=\dfrac{\cdot 28571\dot{4}}{2}=\dfrac{42857\dot{1}}{3}$ etc.

35. Prove that $25\ 85\dot{6}28571\dot{4}=50\ 856+\frac{2}{1000}$.

36 Write at sight the following as recurring decimals $\frac{95}{990},\frac{526}{990},\frac{96}{990},\frac{96}{9990},\frac{141}{9990}$

---

§19 Cyclic order of digits.

The following forms should be verified and committed to memory —

$$\tfrac{1}{7}=\cdot142857,\ \tfrac{2}{7}=\cdot28571\dot{4}$$
$$\tfrac{3}{7}=\cdot428571,\ \tfrac{4}{7}=\cdot571428$$
$$\tfrac{5}{7}=\cdot714285,\ \tfrac{6}{7}=\cdot857142$$

If the digits of the decimal of $\frac{1}{7}$ be placed round the circle we can obtain the results of $\frac{2}{7}$, $\frac{3}{7}$ etc., etc., by taking the digits in order in the direction of the arrow heads. It will also be noted that the decimals equivalent to fractions with denominator 7 are all pure recurring decimals
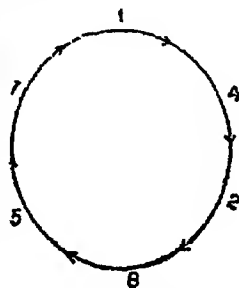
The decimals equivalent to fractions with denominator 13 are pure recurring decimals and can be arranged in cyclic order in two sets. Verify and commit these results also to memory, if possible.

First Set	Second Set
$\tfrac{1}{13}=\cdot\dot{0}7692\dot{3}$	$\tfrac{2}{13}=\cdot15384\dot{6}$
$\tfrac{3}{13}=\cdot230769$	$\tfrac{5}{13}=\cdot384615$
$\tfrac{4}{13}=\cdot\dot{3}07692$	$\tfrac{6}{13}=\cdot\dot{4}61538$
$\tfrac{9}{13}=\cdot692307$	$\tfrac{7}{13}=\cdot53846\dot{1}$
$\tfrac{10}{13}=\cdot76923\dot{0}$	$\tfrac{8}{13}=\cdot615384$
$\tfrac{12}{13}=\cdot923076$	$\tfrac{11}{13}=\cdot846153$

§20   To begin the period in a given recurring decimal

In a given recurring decimal the period may be supposed to begin at any point we please after the first repeating figure   Thus

$$253636 = ·2536 = 253636 = \text{etc. etc.}$$

Note   Number of figures in the period of a recurring decimal may be any multiple of that number without changing the value of the decimal   Thus

$$125 = ·12525 = 1252525 = \text{etc}$$

§21.   Similar recurring decimals.

When recurring decimals have the same number of non-recurring figures, and also the same number of recurring figures, they are said to be Similar   Thus

42023, ·15897 and 8·29055 are similar recurring decimals

§22   To make given recurring decimals similar

Example.   Make 4 023, 2563 and 564 similar

Exp.   To make similar recurring decimals, there should be the same number of non-recurring figures and also the same number of recurring figures in each case   As the highest number of non recurring figures in the given numbers is 2, extend each decimal to two non-recurring figures. Also the numbers of figures in the periods are 1, 3 and 2 respectively, so to make the same number of recurring figures in each case, extend each decimal to 6 recurring figures which is the L C M. of 1, 3 and 2.

$$
\begin{aligned}
\text{Thus } 4\,023 &= 4\ 02\ \big|\ 333333 \\
·2563 &= ·25\ \big|\ 635635 \\
564 &= 56\ \big|\ 464646
\end{aligned}
$$

EXERCISE 56.

In each of the following recurring decimals begin the period at the fourth decimal place —

1.   5234   2   5468   3.   ·567   4.   2156
5.   23563   6   25438.   7   2564.   8   123567.
9   Extend 24, 576 and 57 so that they may have the same number of figures in the period

Make the following sets of recurring decimals similar

10. ˙25, ˙56.   11.   35$\dot{6}$, 78
12.   3456, ˙378   13   ˙02$\dot{3}$, 56$\dot{7}$8.
14   ˙5, 0$\dot{0}$5, 0$\dot{0}$0$\dot{5}$.   15   ˙065, ˙085, ˙0$\dot{8}$5
16. ˙225, ˙022$\dot{5}$, 0022$\dot{5}$ 17.   576, 00356, 0004325
18 3 1̇256, 5 012356, 2˙1235

19   Extend 23$\dot{4}$, ˙2345 and 234$\dot{5}$ so that they may
have the same number of recurring figures.

§23. Addition of Recurring decimals

Rule *Make the decimals similar and then to be
accurate in answer retain two more figures of each than
the required number and then add them as usual*

Example 1. Add together 23$\dot{5}$, 5 002$\dot{4}$ and 6 71$\dot{4}$.

Sol. ˙235 =   23 | 555555 | 55
      5 0024 = 5 00 | 242424 | 24
      6 714 = 6 71 | 471471 | 47
      sum = 11 95269451   Ans

§24   Subtraction of Recurring decimals

Rule. *Proceed as in the case of addition*

Example 2. Subtract 15˙00345 from 18 00235

Sol.   18 00235 = 18 002 | 352352 | 35
       15 00345 = 15 003 | 454545 | 45
       Difference = 2 998897806   Ans

Note   If the answer in addition or subtraction is required
correct to certain decimal places, the extra figures are neglected and
the retained portion is increased by unity if the first figure of the
neglected portion is either 5 or greater than 5

EXERCISE 57.

Add together accurately ·—

1   32, 48, 58, 4$\dot{5}$. 2   2 02, 5 043, 8˙56425, 9 8935
3   4 25, 0023, 3 78$\dot{4}$, 526$\dot{3}$2
4.   15 235, 02356, 9 8023, 15 1̇2.
5.   2 5769, 7 20123, 8 2356, 5˙7$\dot{1}$.
6.   21˙2$\dot{3}$56, 5 432, 8 0̇23, 6˙235$\dot{4}$
7.   12 5326, 18˙2563, 7 563402, 2 1

8.   13˙235$\dot{6}$, 21˙02356, 12 02$\dot{3}$, 8˙24$\dot{6}$.

9   7˙12356, 11 2356, 2˙0235, 8 9356

10   16 23, 21 56$\dot{7}$, 45˙2356, 9 8235

Subtract —

11   15 23$\dot{6}$ from 21˙562$\dot{3}$.   **12.**   19 503$\dot{4}$ from 35 983$\dot{5}$

13   12 3$\dot{7}$56 from 18 98356   **14.**   32˙7432 from 48˙7495.

15   28 2356$\dot{7}$8 from 35 235678

---

§25   **To multiply a recurring decimal by a whole number or by a terminating decimal**

Rule   *Extend the decimal 2 or 3 places beyond the end of the period in order to ensure the correctness of the last digit retained and proceed in the usual way. In the product, point off as many decimal places as there are decimal places in both the multiplicand and multiplier together The product will also be a recurring decimal of the same kind as the multiplicand, i e, with a period containing the same number of digits.*

Example 1   Multiply 5 246$\dot{5}$ by 215

Sol.   (1)

5 2465	65
215	
26 2328	
52 465	
1049 31	

     (2)

5 2465	65
215	
26 2328	28
52 4656	56
1049 3131	31
1128 0116	**Ans**

Extend each line by repeating the digits of the period and then add as shown in (2)

Example 2   Multiply 3 5426$\dot{8}$ by 144.

Sol

3 54268	268
˙144	
1417073	
1417073	
354268	

1417073	073
14170730	730
35426826	826
51014630	629

$= 5101463\dot{0}$.   Ans.

§26   **To divide a recurring decimal by a whole number.**

Example 3   Divide 15˙$\dot{1}$23$\dot{5}$ by 6.   ..

Sol.   6)15 1235   =6)   15˙1235123512351235

2˙5205853918725205

2˙520585391872.   Ans.

Example 4.   Divide 15 9356 by 124.

Sol   124)15 9356356356356356356(1285131906099648035

124

353

248

1055

992

636

620

163

124

395

372

236

124

1123

1116

756

744

1235

1116

1196

1116

803

744

595

496

996

992

435

372

636

620

˙1285131906099964803˙. Ans   16

§27 To multiply or divide a recurring decimal by a recurring decimal.

Rule *Reduce the recurring decimals to vulgar fractions, then multiply or divide as the case may be and reduce the resulting fraction back to the equivalent decimal*

Example 5. Multiply ·10̇5̇ by ·3̇.

Sol $·10̇5̇ = \frac{105}{999} = \frac{35}{333}.$

$·3̇ = \frac{3}{9} = \frac{1}{3}$

∴ the product $= \frac{35}{333} \times \frac{1}{3} = \frac{35}{999} = ·03̇5̇$ **Ans.**

Example 6. Divide 1·2̇8̇ by ·06̇4̇

Sol. $1·2̇8̇ = \frac{128-12}{90} = \frac{116}{90} = \frac{58}{45}$

$·06̇4̇ = \frac{64-6}{900} = \frac{58}{900}$

∴ the quotient $= \frac{58}{45} \div \frac{58}{900} = \frac{58}{45} \times \frac{900}{58} = 20$ **Ans**

EXERCISE 58

Simplify —

1. ·5̇12̇×·6   **2** ·33̇5̇×1·5   **3.** ·025×·36

4. 1·516×·24   **5** 2·0315×·45   **6** ·63025×·216.

7. ·4125÷·7   **8** 3·156÷·12   **9** ·6·5601÷·16

10 1·240125÷·124.   **11** 15·0286÷·136.

12 ·872÷·288.   **13** ·81 × ·011.

14 ·3̇7̇ × ·29̇7̇   **15.** ·033̇ × ·6̇6̇.

16. ·2̇85714̇ × ·4̇6̇   **17** ·6̇428571̇ × 2·0̇74̇,

18 ·2̇142857̇ × 1·3̇   **19.** 49·3̇ × ·29954

20 2·3̇571428̇ ÷ 10·2̇142857̇.   **21** 1·9̇0̇ ÷ ·583̇.

22 11·83̇ ÷ ·249   **23** ·1̇42857̇ ÷ 1·857142̇

24. ·891̇ ÷ 1·29   **25** ·125̇ ÷ ·251̇

26 ·153846̇ ÷ 1·0̇76923̇.   **27** 411·3519 ÷ 16·5881.

28. ·307692̇ ÷ ·538461̇   **29.** 7·39̇ ÷ ·079̇.

30 ·857142̇ × ·538461̇ ÷ 1·153846̇.

31 2·1̇428571̇ ÷ ·07692307 × 2·3̇

DECIMAL MEASURES

§28   Reduction of simple decimal quantity from one unit to another

Example 1   Reduce Rs. 4·16625 to pies

  Sol   4·16625 rupees
  _____16_____
  66 66000 annas
  ____12____
  799 92 pies Ans

Example 2   Reduce 95445 lbs. to tons

  Sol   (4)95445 lbs
28=4×7   (7)23861 25
  4) 3408 7500 qr.
  20)   852 187500cwt.
  42 609375 tons.
  Ans.

EXERCISE 59.

Reduce (*based on Ex 1*) —

1   Rs. ·375 to pies.   2   Rs. 15 1875 to pies.
3   ·01125 of Rs 120 to pies
4.   £ 15625 to pence   5   ·035375 of £60 to pence.
6   ·15875 days to sec.   7   8·125 of 20 days to min.
8.   2·1625 mds. to chks   9   21 1325 of 20 mds to srs.
10.   4 1225 tons to lbs.   11   2 625 of 125 tons to qrs.
12   1 1625 of 20 tolas to ratties.

Reduce (*based on Ex 2*) —

13.   5 chks to maunds   14   117 pies to rupees.
15   1645 lbs to tons   16   12321 pence to £.
17   7875 seconds to hrs   18   121 yards to miles
19.   15620 4 chks to mds
20.   825 of 1600 pies to rupees

---

§29   Reduction of compound quantities to simple quantities and *vice versa*

The method will be best illustrated by the following solved examples —

Example 1   Reduce Rs. 12 109375 to Rs $a. p.$

Sol.   Rs.   12·109375⎞   The Rs. 12 ıs not reduced
                        16⎟   to annas
    $a.$ ‾1·750000‾⎟
                    12⎟   The anna 1 ıs not reduced
        p ‾9 ∅∅‾⎠   to pıes

    Rs 12 109375＝Rs. 12 1a. 9p. Ans

Example 2.   Reduce Rs 15 7a 9p. to Rupees.

Sol.   12)9 00p
                        ‾·75a.‾
                        7·
                    ‾‾‾‾‾‾‾‾‾‾
                16) 7·750000a
                    ‾484375‾
                    15
                    ‾‾‾‾‾‾‾‾
            Rs.   15 484375   Ans

Example 3   Fınd the value of 3·725 of 12 tons 12 cwt. 2 qr.
    Sol.   4)2 0 qr.
                        ‾5‾ $c_{wt.}$
                    12·
                    ‾‾‾‾‾‾
                20)12 500 cwt.
                    ‾625‾ tons
                12
                ‾‾‾‾‾‾‾
                12 625 tons

∴ 3·725 of 12 tons 12 cwt 2 qr ＝12·625 × 3·725 tons
                                    ＝12·625
                                        3 725
                                    ‾‾‾‾‾‾‾
                                    63125
                                    25250
                                    88375
                                    37875
                                    ‾‾‾‾‾‾‾
                                ·47·028125 Tons
                                        20
                                ‾‾‾‾‾‾‾‾‾‾‾
                                0·562500 cwt.
                                        4
                                ‾‾‾‾‾‾‾‾‾‾‾
                                2·2500   qr.

∴ 47 tons 0 cwt. 2·25 qr   **Ans.**

Note   Sometimes the compound quantity is not reducible to decimals   In such cases we proceed thus —

12 tons 12 cwt. 2qr. $=1010$ qr.

3 725 of 12 tons 12 cwt 2 qr.$=1010 \times 3\ 725$ qr

                          $=3762\,25$ qr

         4) 3762 25   qr.

        20)   940 cwt. 2˙25 qr.

              47 tons 0 cwt. 2˙25 qr.   **Ans.**

Aliter. $3˙725 = \frac{3725}{1000} = \frac{149}{40}$

   3 725 of 12 tons 12 cwt. 2 qr.

$= \dfrac{(12 \text{ tons } 12 \text{ cwt. } 2 \text{ qr.}) \times 149}{40}$ and so on,

Example 4.   Find the value of double of £ 31 2583̇.

Sol   31 2583/33

                    2

           £62 5166̇/66

               20

        s.10˙3333/20

              12

       d. 3 9998/40

    the required value $= £62.\ 10s.\ 4d.$ **Ans**

EXERCISE 60.

Reduce to compound quantity .—

1   Rs 12 129375   2   Rs. 16˙22875

3.   16 29625 mds.   4   24˙9125 mds.

5.   £16 1625.   6   £21 4875

7.   12 18725 yards   8   12˙16925 tons

Express the following as a decimal of its highest denomination .—

9.   Rs 15. 8a. 6p.   10   Rs. 21 13a. 9p

11   £18 12s. 9d.   12.   £12 17s. 6d

13.   5 tons 15 cwt. 2 qr. 7 lb.

14   16 tons 16 cwt. 1 qr 14 lb

15.   12 yards 2 ft 3 in   16   12 mds 28 sr 4 chk

Find the value of (by use of 1st method Ex. 3 if possible).

17   ·25 of Rs 12 13a 9p   18.   ·5 of Rs 2. 6a. 4·8p,
19.   125 of Rs 21 8a. 6p   20.   3 0̇6̇ of Rs. 2. 1a.
21   625 of 2 yards 1 ft 6 in.
22   375 of £12 13s 9d   23   ·428 of £3. 8s.
24   ·4135 of £1   25   4583 of 1s.
26   ·358⅓ of £+ 18s. 6d   27.   01 × 01 of Rs. 7+9. 4a.
28   8 71875 of 5a. 4p. + 1 146875 of Rs. 3 5a   4p —
        0625 of Rs. 10 8a.
29.   325 of £1. 1s. + 54 of 8s 3d. + ·027 of £14 6s.
30.   ·025 of 7 mds. + 2 12 of 15 mds −3 15 of 4 mds.
31.   ·7 of 7s 6d —·84 of 16s 6d. + ·927 of £2. 10s. 5d
32   5 of Rs 7 8a — 05 of Rs 4 6a. + 02 of Rs. 21 9a.
33   7365 of £3   6s   8d + 504 of £15. 12s. 6d. +
        2·102083 of £2
34   38⅞ of £8. 16s. 3d + 6½ of ⅜ of 7s. 8½d. + ₁⁷₁ of 1d.

§30   To express one concrete quantity as the decimal of another concrete quantity of the same kind.

Example 1   Reduce Rs. 2 1a. 6p. to the decimal of Rs. 8. 6a.

Sol   The reqd. decimal $= \dfrac{\text{Rs } 2 \ 1a \ 6p}{\text{Rs } 8 \ 6a} = \dfrac{2\cdot09375}{8\cdot375}$

$$= 8375) 2093\cdot 75 (25. \quad \textbf{Ans}$$
$$\underline{16750}$$
$$\overline{41875}$$
$$\underline{41875}$$

Aliter.   The reqd. decimal $= \dfrac{\text{Rs } 2 \ 1a. \ 6p}{\text{Rs } 8 \ 6a} = \dfrac{\frac{67}{32}}{\frac{67}{8}}$

$$= \tfrac{67}{32} \times \tfrac{8}{67} = \tfrac{1}{4} = 25. \quad \textbf{Ans.}$$

Rule   *Express both quantities in fraction of the same denomination and then proceed as in Art. 30, chapter VII*

EXERCISE 61.

Reduce the first of the two given quantities to the decimal of the second :—

1   Rs 2 8a , Rs 5.

2   Rs 5 4a , Rs 21

3. Rs 5 8a., Rs 27 8a

4   Rs 7 8a 9p , Rs 30 3a

5   Rs 21. 2a 6p , Rs. 169 4a.

6   £2. 8s 9d , £9. 7s. 6d.

7. £25 1s 6d., £50. 3s

8. 15 mds. 4 sr. 8 chk , 75 mds 22 sr. 8 chk

9   17 mds 8 sr 6 chk , 68 mds 33 sr. 8 chk.

10 12 tons 16 cwt 2 qr., 51 tons 6 cwt

11 Rs. 21 2a 4p , Rs. 169 2a 8p.

12 Rs 16 8a 8p , Rs 66. 2a. 8p

13. £5 8s 4d , £43 6s 8d

14. £12. 13s 8d , £50 14s. 8d.

15. 7 ft. 4 in , 9 yards 2 ft. 4 in

16 Reduce $5\frac{1}{2}$ sq. yds. to the decimal of an acre.

17. What decimal of £2. 13s. 4d. is ·0625 of 2'6 of £1. 6s 8d ?

18 Simplify

$$1 - \cfrac{2}{3 + \cfrac{4}{5 - \cfrac{6}{7 + \frac{8}{9}}}} - 2'08\frac{2}{3} \text{ of } \frac{3 \text{ tons } 2 \text{ cwt } 2. \text{ qr } 20 \text{ lb.}}{10 \text{ cwt } 2 \text{ qr } 11 \text{ lb}}$$

and reduce the result to the decimal of 1 1.

19 Find the value of $1\frac{2}{17}$ of 01236 of Rs. 5. 11a. 8p. and taking the rupee as worth 1s $4\frac{1}{2}d$ express the result as the decimal of one shilling

20. Express $\frac{3}{8}$ of 7s. 6d + 1'25 of 5s — ·545 of 9s. 2d. as a decimal of £10

§31   We now close this chapter with the following miscellaneous exercise :—

EXERCISE 62

Simplify and give answer in decimals if possible :—

1. $\dfrac{81 \times \cdot 005}{45}$.

2. $\dfrac{\cdot 003 \times \cdot 15}{\cdot 0035}$.

3. $\dfrac{2 \cdot 46 - 2 \cdot 30}{\cdot 3 + 1 \cdot 27} + \dfrac{4\frac{1}{3}}{19}$.

4. $\dfrac{125 \times (\cdot 1750 \text{ of } \cdot 285714)}{\cdot 00025}$

5. $\dfrac{\cdot 0025 + 1 \cdot 6}{3 \cdot 6 - 2 \cdot 3} - \dfrac{713 + 2 \cdot 625}{12 \cdot 7 - 10 \cdot 2}$

6. $\dfrac{\cdot 12 \text{ of } (\cdot 0104 - \cdot 002) + 36 \times \cdot 002}{\cdot 12 \times 12}$.

7. $\left\{ 37 + \dfrac{3 \cdot 7037}{100} \right\} \times \cdot 54$.

8. $\left\{ 41 + \dfrac{4 \cdot 1041}{100} \right\} - 82 - \frac{61}{111}$.

9. $\dfrac{4 \cdot 2 - 3 \cdot 14}{1 \cdot 3 + 2 \cdot 102} \text{ of } \dfrac{1 \cdot 3 \text{ of } 4}{\cdot 37 \text{ of } 8 \cdot 81}$.

10. $\dfrac{\cdot 285714 \times 4\frac{2}{3}}{1 \cdot 3} + \dfrac{81}{1\frac{2}{6}} + \cdot 3$.

11. $\dfrac{2\frac{1}{2} - 1\frac{1}{3}}{2\frac{1}{4} - \frac{1}{2} \text{ of } 3\frac{1}{3}} \text{ of } \dfrac{4 \cdot 3}{25} \text{ of Rs. } 15$.

12. $\dfrac{\frac{1}{2} + \frac{3}{4}}{\frac{5}{6} - \frac{7}{8} + \frac{9}{10}} - \cdot 142857 \text{ of } 10\frac{20}{109}$.

13. $\dfrac{3 \cdot 125}{2 \cdot 16} \text{ of } \dfrac{24}{\cdot 125} - \dfrac{2 \cdot 2}{1 \cdot 5} \text{ of } \dfrac{187 \cdot 5}{3 \cdot 42}$.

14. $\left(\dfrac{\cdot 0019}{3 \cdot 16} \text{ of } \dfrac{4 \cdot 4}{\cdot 0005} \right) - \left(\dfrac{8 \cdot 8}{7} \text{ of } \dfrac{4}{5 \cdot 625} \right)$.

15. $\dfrac{2 \cdot 8 \text{ of } 2 \cdot 27}{1 \cdot 136} + \dfrac{4 \cdot 4 - 2 \cdot 83}{1 \cdot 6 + 2 \cdot 629} \text{ of } \dfrac{6 \cdot 8 \text{ of } 3}{2 \cdot 25}$.

16. $\dfrac{\cdot 142857 \times \cdot 076923}{\cdot 010989} + \dfrac{2 \cdot 75 \times 11 \cdot 25}{6 \cdot 2} + \frac{1}{8}$.

17 $\dfrac{1\cdot83+2\ 0416+\cdot3-3\frac{1}{5}}{1\ 0025+\ 0625-1\frac{1}{16}}+\dfrac{66\frac{2}{3}}{100}$

18. $\cdot426$ of $\dfrac{3\ 3}{08}$ of $\dfrac{3}{735}$ of $\dfrac{\cdot147\times4\cdot\dot{4}}{11\cdot1}$ of $\pounds1.\ 17s\ \ 6d.$

19 $\dfrac{2\frac{1}{4}+\frac{3}{4}\ \text{of}\ 2\frac{2}{3}-1\frac{1}{4}}{3\cdot6+\ 15\times4-\cdot24-2\ 1}$ of $\dfrac{20\ 1\ \text{of}\ 1\text{md.}\ 30\frac{1}{2}\ \text{sr}}{17\cdot5625\ \text{of}\ 2\ \text{md.}\ 20\frac{1}{2}\ \text{sr.}}$,

of Rs. 2 8a

20. $\dfrac{5\frac{1}{2}\ \text{of}\ \cdot2\ \text{of}\ 2\ 571428-1-(\frac{1}{5}+\cdot5)}{1-\frac{3}{14}\ \text{of}\ \left\{\ \cdot5+\frac{1}{2}\ \text{of}\ \dfrac{05}{142857\ \text{of}\ 1\frac{1}{20}}\right\}}$

21. $(2\cdot36\dot{4}-1\cdot6\dot{9}7)+1\ \dot{3}\times(2\ 4+7\cdot5)\ \div\dfrac{2}{2+\dfrac{1}{2+\frac{1}{3}}}$.

22. $\dfrac{0016\times\cdot025}{325\times\cdot05}-\dfrac{\cdot1216\times\ 105\times\cdot002}{08512\times\cdot625\times\cdot039}\times\dfrac{1\ \dot{6}}{2\ 3}$

23. $\left[\dfrac{(3\frac{2}{3}\times5\frac{1}{4})-(3\frac{1}{2}\times4\frac{5}{6})}{3\frac{2}{3}-(5\frac{1}{4}-3\frac{1}{2})+4\frac{5}{6}.}\ \text{of Rs. 10}\ \right]+\left[\dfrac{1\cdot405}{2\ 1}\times\dfrac{6\cdot3}{28\cdot1}\right.$

    $\left.\text{of Rs}\ 7\frac{1}{2}\right]+\cdot02\dot{7}$ of Rs 125 $7a$ $6p.$

24. $\dfrac{2\cdot5-6\cdot08+4\ 7}{(2\times8\ 3)-(2\times\cdot55)}+\dfrac{(3\frac{2}{3}\ \text{of}\ \frac{2}{5})-\frac{5}{12}}{(\cdot\dot{3}\ \text{of}\ \frac{1}{16})+(\frac{5}{16}-\frac{1}{6})}$.

25. $\dfrac{2\frac{1}{6}-1\frac{2}{3}}{3\ 3\times1\frac{2}{5}}-\dfrac{\frac{2}{7}-\frac{1}{6}}{\frac{2}{7}+\frac{1}{6}}+\dfrac{\cdot05\times\ 7}{\cdot071}$ of $\dfrac{\text{Rs. 2.}\ \ 7a.}{\text{Re}\ 1.\ 11a}$

26. $\dfrac{5\ 75-\ 428571\ \text{of}\ 15\frac{3}{4}+2\frac{2}{85}-1\ 44}{2\frac{11}{55}\times5\ 6-3\ 36}\times\dfrac{2\ 8\times1\cdot13\dot{6}}{31\ 8\dot{1}}$

27. How many times can 34 be subtracted from 27 679, and what will be the magnitude of the remainder ?

CHAPTER IX.

APPROXIMATION.

§1. The need for approximation.

Suppose we have to divide a distance of 10 yards 2 ft.
3 6 in into 100 equal parts, then each part is, by calculation,
3 876 in If we are to mark these divisions, we should draw
a straight line and mark off along it a length of 3'87 in To
do this, we first take 3 inches, then along the next inch,
which is divided into 10 equal parts, we take 8 tenths or 8
small sub divisions.   To get the remaining 7 hundredths,
we must sub-divide the next tenth of an inch into 10
equal parts and take 7 of these This is very difficult to do,
for a tenth of an inch is too small to admit of further equal
sub divisions   If, moreover, we wish to get a straight line
3 876 inches long, a hundredth of an inch is to be sub-divi-
ded into 10 equal parts and 6 of these are to be taken. This
is a very small line which cannot be judged by the eye or
measured with any ordinary instrument   Thus owing to
either imperfection of our senses or want of very delicate
instruments, we are obliged to impose certain limitations on
the extent of our measurements   In the calculation of
lengths expressed in inches, we should be satisfied with two
decimal places and if there is a third decimal place as in
3 876 in., the figure in the third place, *viz*, six thousandths
may be roughly taken to be one hundredth of an inch Thus
3'876 in is *approximately* equal to 3'88 inches.

§2 Error.

Approximation always involves some error, *i. e.*, there
must be some difference between the actual value and that
roughly taken for our purpose In the above example, when
3 876 in. is changed to 3'88 in., the error is 3 88 − 3 876 or
'004 in , but if 3 876 in. is changed to 3 87 in , the error
is '006 in   Since the error '004 is less than the error   006
in., the former approximation, *viz* , 3 88 is nearer the truth,
i e , the actual value of 3 876, 3'88 is said to be correct to
the nearest hundredth or *correct to 2 decimal places*

Again $\frac{5}{17} = \cdot 29411764$　　If we wish to *approximate*
to the result by terminating the operation at the 5th place,
we write $\frac{5}{17} = \cdot 29412$, but if at the fourth place, then we
write $\frac{5}{17} = \cdot 2941$, and so on. From this we evidently have
the following

Rule. *Increase the last figure in the result by 1, if
the next figure is 5 or greater than 5*

The reason for this is evident from the illustration
given above

§3　Significant figures.

The following examples explain clearly the meaning of
the expression 'significant figures' —

(a)　(i) The distance between two places is 1700 miles
correct to the *nearest hundred*　Here the unit of measure-
ment is one *hundred miles* and the distance is stated to be
17 such units *correct to the nearest unit*　The figures 17,
which give the number of units, are said to be significant;
while the two zeroes, which express the *magnitude* of the
unit, are said to be non-significant.

(ii)　The distance between two places is 1700 miles
correct to the *nearest mile*, here the unit is one mile and
hence all the figures are significant.

(b) The length of a line is $\cdot 07$ inch, correct to the
2nd decimal place.

This means that the length is 7 hundredths of an inch
correct to the nearest *hundredth*, here the unit is *hundreth
of an inch*,

. the *significant* figure is 7 and the zero at the
beginning is non-significant.

Note　Thus zeroes at the very beginning of a decimal are always
non-significant　From the examples given above it is clear that
significant figures are those which in any approximate result express
the *number* of units, correct to the nearest such unit

§4.   Absolute error, relative error and percentage error

Errors are of two kinds   (1)   Absolute   error   and   (2) Relative error

(1)   Absolute error = Actual value − approximate value

For example if 1   metre = 39 3707 in. and if we take 40 in to be approximately equivalent to 1 metre, then the absolute error = 40 − 39˙3707 = 6293 in

(2)   Relative error = $\dfrac{\text{Absolute error}}{\text{True value}}$.

In the above example, the relative error = $\dfrac{6293 \text{ in.}}{39˙3707 \text{ in.}}$

$$= \dfrac{6293}{39\ 3707}$$

$$= 0159.$$

The percentage error = the relative error × 100.

the percentage error in the above example

$$= 0159 \times 100 = 1\ 59$$

The relative error is   more important than the absolute error *e. g.*, if 1008 is taken as 1000 and 68 is taken as 60, the absolute error 8 is the same in both cases, but the relative error in the first case is $\frac{8}{1000} = \frac{1}{125}$ and in the last case   it is $\frac{8}{60} = \frac{2}{15}$ Hence the relative error is   much greater in the latter than in the former case

§5   Contracted Addition and Subtraction

In all approximate additions   and   subtractions where a certain degree of accuracy is required, it is enough to retain in each quantity two more places of decimals than the number specified

Example 1   Find, correct to three decimal places, the sum of 16 81984 ,   016 , ˙142857 and $\frac{5}{6}$

Sol.   *Explanation.*

	16 819	84
	˙016	66
	˙142	85
$\frac{5}{6} =$	˙833	33
	17˙812	68

A   vertical   line   is   drawn after the third decimal   figure to show that the figures follow-ing are unnecessary except for determining the figure to be carried to the third decimal place

∴ the sum is 17˙813 correct to the third decimal place   **Ans**

Example 2   Subtract 361⁵⁄₇ from 1082·835, showing the remainder correct to four decimal places

Sol.   1082 835=1082·8358 | 35
                          361⁵⁄₇= 361 7142 | 85
                          ────────────────────
                                 721·1215 | 50

·   the remainder correct up to four decimal places is 721·1216. **Ans**

§6.   Contracted Multiplication

In multiplying one long decimal by another, it is generally required to get the product *approximately* correct, *i e*, as far as a certain number of decimal places   The following Rule enables us to shorten the work —

Rule.   *Mark off in the decimal parts of the multiplicand as many figures as is one more than the number of decimal places we are required to retain in the product, under the last of these figures place the unit's figure of the multiplier writing the figures in the reverse order. Omit decimal points of both the multiplicand and multiplier and add 0's (if necessary) in the multiplicand, so that every figure of the multiplier shall have a figure above it. Begin the multiplication with the last right hand figure of the multiplier and multiply in succession by each of the other figures, in each case beginning the multiplication from the figure above the one we are multiplying by, taking care to carry to it the nearest ten from its product with the next figure on the right in the multiplicand   Place the unit's figure of all these partial products in the same vertical line, add as usual and mark off the required number of decimal places in the result striking out the last figure.*

Note   In carrying the *nearest ten*, if the product is a number from 5 to 14 carry 1 , if from 15 to 24 carry 2 and so on   If the product is 4 or less than 4, carry 0, *i e*, reject it (Art 2)

Example 3   Multiply 459 63524 by 25·4637 giving the product correct to 3 decimal places.

Sol

```
4596352   4
  73645   2
919270ɨ   8
2298176   2
 183854   1
  27578   1
   1378   9
    321   7
1170ɨ013  8        1170ɨ'014  Ans
```

Example 4 Multiply 00040635 by 241 6358 retaining 6 places of decimals.

Sol.

```
4063   50
85361  42
 8127  00
 1625  40
   40  64
   24  38
    1  22
       20
        3
09818  87        098189  Ans.
```

Example 5. Multiply ˙453 by 01694 correct to 4 decimal places.

Sol.

```
 4530  0
49610  0
  453
  272
   41
    2
00768          ˙. '0076  Ans.
```

Note 01694 is written as 0 01694

§7 Contracted Division

In dividing one decimal by another where the quotient is required correct up to certain number of decimal places we observe the following

Rule   *Make the divisor a whole number, try to find out—by mere inspection by taking one single step in the ordinary way,—the nature of the required quotient, i e, how many integral figures are expected in the quotient and also the whole number of figures in the quotient, from the left of the divisor cut off this number of figures and one more for approximation and strike out the rest. Proceed one step with this new divisor but in multiplying its first figure by the quotient figure, carry the nearest ten (see Note Art. 4) from its product with the next figure on the right   Instead of bringing down a figure to the remainder, strike off another figure from the divisor and proceed as before, till all the figures in the divisor are exhausted. If the number of figures in the divisor is less than the number of figures to be cut off in the very beginning, then proceed in the ordinary way until the number of figures still to be found in the quotient is one less than the number of figures in the divisor and then apply the Rule*

Example 6.   Divide 2508 928065051 by 92 410357 correct to four decimal places.

Sol   Evidently the integral part of the quotient will contain 2 figures and since 4 places of decimals are to be, retained, therefore $2 + 4 = 6$ figures are retained in the divisor and 1 more for *approximation* makes the divisor have 7 figures, viz, it is written as 924103,5

```
X X X X X X
9 2 4 1 0 3,  ⁵)2508928  (271498
              1848207
               -------
               660721      here the divisor is 924103
               646872      and carry 2 from 7 × 3
               -------
                13849         .              .   .
                 9241
                -------
                 4608      here the divisor is 924 and
                 3696      carry nothing from 4 × 1
                 ----
                  912         .       .
                  832  --        -
                  ---
                   80      here the divisor is 9 and
                   74   -   carry 2 from 8 × 2       .
                   ---
```

F 12

With each new figure in the quotient a × mark is put upon that figure in the divisor which is not to be taken in the next step and so on until all the figures in the divisor are exhausted.

the quotient is 27 1498   Ans.

§8   Multiplication and Division Combined

Example 7   Find the value of $\dfrac{346\ 25 \times 32\ 164}{732\ 96}$

correct to 3 places of decimals

Sol   The exp $= \dfrac{34\ 625 \times 3\ 2164}{7\ 3296}$, whence we esti-

mate the nature of the answer roughly thus $\left(\dfrac{35 \times 3}{7} = 15\right)$,

i e., the integral part in the result shall contain 2 figures. But as the result is required to be correct to 3 decimal places, we require $(2 + 3)$ or 5 *significant* figures in the result, so that $5 + 1 = 6$ figures must be retained in the divisor, but as there are only 5, we shall proceed in the division one step by the ordinary method , (or we could put one zero after 6 in the divisor, making it 7 32960)

Again the rough estimate of the numerator shows that to make the first step of the division, 6 figures will be required in the numerator, but as the numerator (34×3) has 3 figures in its integral part, the product is to be correct to 3 decimal places.

The work, therefore, is as follows —

```
                         × × × × ×
   34625  0        7  3  2  9  6)111367(15194
    4612  3                      73296
  103875  0                      38071
    6925  0                      36648
     346  3                       1423
     207  7                        733
      13  8                        690
  ───────                          659
  111367                            31
                                    29
```

i e., the product is 111 367 upto 3 decimal places

.   the required answer is 15'194. **Ans**

Example 8. Divide ˙257917 by 2˙03458 approximately correct to 7 places of decimals

Sol In this case, evidently the quotient will have no integral part   Since **7** places of decimals are to be retained, the divisor must consist of **8** figures , but as there are only 6 figures in the divisor, proceed in the ordinary way for 8−6=2 figures in the quotient, when the number of figures still to be obtained will be one less than the number of figures in the divisor.   Then apply the rule given above

```
  x x x x
2 0  3  4  5, 8)25791˙70( 1267667
            20345 8
            5445 90
            4069 16
            1376 74  here begin to apply the Rule
            1220 75
             155 99
             142 42
              13 57
              12 20
               1 37
               1 22
                 15
                 14
```

the quotient is 1267667   Ans

Example 9   Divide 549533676 by 931˙2167 correct up to 7 places of decimals

Sol   By inspection, we find that there will be 3 zeroes after the decimal point in the quotient, hence 7−3=4 figures are wanted in the quotient,   we retain 4+1=5 figures in the divisor and proceed thus—

```
   x x x
9 3 1 2, 1)54953  (5901
          46561
           8392
           8381
             11
              9          the quotient is ˙0005901. Ans
```

§9　Series.

The value of a *series* is often required to be correct to a certain number of decimal places　In such cases, we observe the following

Rule　*Work out each term in decimals, taking help in each step, of the preceding term and continue this method till a term becomes so small that it does not affect the required result.*

Example 10　Find the value of

$$1 + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \quad , \text{ correct}$$

to 6 decimal places.

Sol				
$1 = 1$			$= 1\,000000$	0
$\dfrac{1}{1.2}$	$= \frac{1}{2}$		$=\quad 500000$	0
$\dfrac{1}{1.2.3}$	$= \frac{1}{3}$ of previous result	$=\quad 166666$	6
$\dfrac{1}{1.2.3.4}$	$= \frac{1}{4}$ of $\quad$,, $\quad$,,	$=\quad 041666$	6
$\dfrac{1}{1.2.3.4.5}$	$= \frac{1}{5}$ of $\quad$,, $\quad$,,	$= \cdot 008333$	3
$\dfrac{1}{1.2.3.4.5.6}$	$= \frac{1}{6}$ of $\quad$,, $\quad$,,	$=\quad 001388$	8
Similarly next term		$=\quad 000198$	4
,, $\quad$,, $\quad$,,		$=\quad 000024$	8
,, $\quad$,, $\quad$,,		$=\quad 000002$	7
,, $\quad$,, $\quad$,,		$=\quad 000000$	2
		$= 1\,718281$	4

the required value is 1 718281.　**Ans**

Note　We stopped at the term which being converted to decimals, produced six zeroes in the first six places and as such this term as well as the term following it could not evidently affect the result

EXERCISE 63 _

Write down the following numbers correct to 3 *significant* figures —

1	640834	2	487634.	3	9 06784.
4	·0008346	5	8·482	6.	·008092

7 Given that 1 cwt = 2465 grammes, find the equivalent of 1 cwt, in grammes correct to two *significant* figures

Find the result correct to 2 *significant* figures in the following —

8 150˙3806 + 64˙0098 + 1009 + 0 8345

9 1008 053 − 678 349

10. $\frac{3}{7} + \frac{5}{8} - \frac{2}{9}$ **11** 818˙6025 + 41˙3892 − 008765

12. Find the value of 51 142857 − 5 323 correct to 7 places of decimals

13 Find the sum of 5˙23, 162 4302, 12˙0789 and 14 3096 to within one thousandth

14. Multiply 0008347 by 1 0856 correct to 3 significant figures.

Give the following products correct to 2 decimal places —

15 0 6208 × 3˙453 **16** 8 4376 × 153˙2986

17 708˙00083 × 0004684 **18** ˙005683 × 101278˙3.

Find the products in the following correct to the third decimal place —

19 0 0007895 × 108˙394 **20** (1 3825)².

21. 62 843 × 008345. **22.** 6 0008375 × 000483

23. Multiply 86858896 by 1˙0986123 retaining 5 places of decimals

Divide —

24 865˙345 by 12˙438 to 3 significant figures

25 056789 by 139 28 „ „ „

26. ˙834952 by 156 24 „ „ „

27 Divide 12˙384698 by 168 0876 correct to the second decimal place

28. Divide 1 0896543 by ˙689763 correct to the second decimal place.

29. Divide 0 38465 by 0 48796 correct to the second decimal place.

30. Divide 1 by 3 1415926 to four places of decimals

31 Find the reciprocal of 6 8438 to 3 places of decimals

32. Simplify $\frac{14641}{63479}$ correct to 3 places of decimals

33 Divide 2 34721 by 2'27924 retaining 7 places of decimals

Find the value of —

34 $1 + \frac{1}{1\ 3} + \frac{1}{1\ 3\ 5} +$ ... to 7 places of decimals

35 $\frac{1}{5} + \frac{1}{3} \times \frac{1}{5^3} + \frac{1}{5} \times \frac{1}{5^5} + \frac{1}{7} \times \frac{1}{5^7} +$

to 6 places of decimals

36 $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} +$ to 3 places of decimals.

37 $1 + \frac{1}{4} + \frac{1}{4\ 6} + \frac{1}{4\ 6\ 8} +$ 3   „   „

38 $\frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} +$. 4   „   „

39 $\frac{3}{16} + \left(\frac{3}{16}\right)^2 + \left(\frac{3}{16}\right)^3 +$ ... 5   „   „

40 $\frac{1}{3\ 5} + \frac{2}{3^2\ 5^3} + \frac{2}{3^3.5^5} +$ 7   „   „

Evaluate the following correct to two decimal places —

41. $\dfrac{8\ 785 \times 16\ 403}{48\ 349}$.   **42** $\dfrac{000567 - 26'893}{4183}$.

43 Find the value of $(1\ 07)^{16}$ correct to four decimal places.

44 Find correct to 5 places of decimals the value of

$\frac{1}{9} + \frac{1}{3\ 9^3} + \frac{1}{5\ 9^5} + \frac{1}{7.9^7} +$.

45. Find the value of $\frac{1}{1\ 4} + \frac{1}{3\ 4^3} + \frac{1}{5\ 4^5} +$

correct to 4 places of decimals.

CHAPTER X.

PRACTICE.

§1 Aliquot part. An *aliquot part* of a quantity is a fraction of it, having unity for its numerator, *i e.*, when one quantity is an exact part of another quantity it is called an *aliquot part* Thus $8a = \frac{1}{2}$ of a rupee and is therefore an aliquot part of a rupee. For the sake of convenience we give below a table of aliquot parts which are generally used in practice.

§2. Table of Aliquot Parts

Of a Rupee		Of a Seer	
$8a$	$=\frac{1}{2}$ Rupee	8 chk.	$=\frac{1}{2}$ Seer
$5a.\ 4p.$	$=\frac{1}{3}$,,	4 ,,	$=\frac{1}{4}$,,
$4a.$	$=\frac{1}{4}$,,	2 ,,	$=\frac{1}{8}$,,
$2a\ 8p.$	$=\frac{1}{6}$,,	1 ,,	$=\frac{1}{16}$,,
$2a.$	$=\frac{1}{8}$,,	**Of a Pound**	
$1a\ 4p$	$=\frac{1}{12}$,,	10s.	$=\frac{1}{2}$ Pound.
$1a.$	$=\frac{1}{16}$,,	6s. 8d.	$=\frac{1}{3}$,,
Of an anna		5s.	$=\frac{1}{4}$,,
$6p$	$=\frac{1}{2}$ Anna	4s.	$=\frac{1}{5}$,,
$4p$	$=\frac{1}{3}$,,	3s 4d	$=\frac{1}{6}$,,
$3p.$	$=\frac{1}{4}$,,	2s. 6d.	$=\frac{1}{8}$,,
$2p$	$=\frac{1}{6}$,,	2s	$=\frac{1}{10}$,,
$1\frac{1}{2}p$	$=\frac{1}{8}$,,	1s. 8d.	$=\frac{1}{12}$,,
$1p.$	$=\frac{1}{12}$,,	1s 4d	$=\frac{1}{15}$,,
Of a Maund		1s 3d	$=\frac{1}{16}$,,
20 sr	$=\frac{1}{2}$ Maund	1s	$=\frac{1}{20}$,,
10 ,,	$=\frac{1}{4}$,,	**Of a Shilling.**	
8 ,,	$=\frac{1}{5}$,,	6d.	$=\frac{1}{2}$ Shilling
5 ,,	$=\frac{1}{8}$,,	4d	$=\frac{1}{3}$,,
2 sr 8 chk.	$=\frac{1}{16}$,,	3d.	$=\frac{1}{4}$,,
1 ,, 4 ,,	$=\frac{1}{32}$,,	2d.	$=\frac{1}{6}$,,
1 sr.	$=\frac{1}{40}$,,	$1\frac{1}{2}d.$	$=\frac{1}{8}$,,
		1d.	$=\frac{1}{12}$,,

Of a Ton.			Of a quarter	
10 cwt		$=\frac{1}{2}$ Ton	14 lbs	$=\frac{1}{2}$ quarter
5 ,,		$=\frac{1}{4}$,,	7 ,,	$=\frac{1}{4}$,,
4 ,,		$=\frac{1}{5}$,,	4 ,,	$=\frac{1}{7}$,,
2 ,,	2 qr	$=\frac{1}{8}$,,	$3\frac{1}{2}$,,	$=\frac{1}{8}$,,
2 ,,		$=\frac{1}{10}$,,	2 ,,	$=\frac{1}{14}$,,
1 ,,	1 qr	$=\frac{1}{16}$,,	$1\frac{3}{4}$,,	$=\frac{1}{16}$,,
1 ,,		$=\frac{1}{20}$,,	1 lb.	$=\frac{1}{28}$,,

Practice is a short method of calculating the cost of a given quantity by means of aliquot parts when the cost of a unit is given

There are two kinds of *Practice*—Simple and Compound

I. SIMPLE PRACTICE.

§3 The following examples will explain the rule for Simple Practice —

Example 1 Find the price of 25 mds. of rice at Rs 5 13a. 4p. per md

Sol.

	Rs	a	p	
	25	0	0	=price at Re. 1 per md.
		5		
	125	0	0=	,, ,, Rs. 5 ,,
8a. $=\frac{1}{2}$ of Re. 1	12	8	0=	,, ,, 8a ,,
5a 4p. $=\frac{1}{3}$ of Re 1	8	5	4=	,, ,, 5a 4p. ,,
	145	13	4	Ans.

Note It is sometimes very convenient to subtract an aliquot part (see Ex 2)

Example 2. Find the value of $121\frac{3}{4}$ articles at Rs 5. 14a. 8p. per article.

Sol Re. $\frac{3}{4} = 12a.$ and Rs. 5. 14a. 8p. = Rs 6. — 1a. 4p.

The price of $121\frac{3}{4}$ articles at Re. 1 = Rs. 121. 12a

	Rs.	a	p	
	121	12	0	= Price at Re 1 per article.
		6		
	730	8	0	= ,, ,, Rs 6 ,, ,,
1a. 4p. $=\frac{1}{12}$ of Re. 1.	10	2	4	= ,, ,, 1a. 4p. ,, ,,
by subtraction	720	5	8	= ,, ,, Rs 5. 14a. 8p

Example 3   Find the price of 2155 tables at £22
13s 4½d. per 100

Sol | £
            | 21 55   =Price at £1 per 1C0 tables
            | 22
            | ————————
            | 4310
            | 4310
            | ————————
            | 474 10   =   ,,   ,, £22   ,,   ,,   ,,
10s =½ cf a £| 10 775   =   ,,   ,, 10s   ,,   ,,   ,,
 2s =⅕ of 10s) 2 155   =   ,,   ,, 2s.   ,,   ,,   ,,
 1s =½ of 2s| 1 0775   =   ,,   ,, 1s.   ,,   ,,   ,,
 3d =¼ of 1s| ·269375   =   ,,   ,, 3d   ,,   ,,   ,,
1½d =½ of 3d| ·1346875   =   ,,   ,, 1½d. ,,   ,,   ,,
            ————————————————
            488 5115625
                     20
            ————————————
            10 2312500
                 12
            ————————
             2 77500

.   the required price is £488 10s 2·775d   **Ans**

Example 4   Find the price of 124⅓ articles at Rs 21,
7a. 7p per article

Sol | Rs ¹· a. p
            | 124   5. 4 =price at Re. 1 per article.
            | 7
            | ————————————
            | 870   5 4 =   ,,   Rs. 7 ,,   ,,
            | 3
            | ————————————
            | 2611 0 0 =   ,,   Rs 21 ,,   ,,
7a =¹⁄₁₆ of Rs 7| 54   6 4 =   ,,   7a.   ,,   ,,
7p.=¹⁄₁₂ of 7a | 4   8 6⅓=   ,,   7p.   ,,   ,,
            ————————————————
            2669 14 10⅓   Ans

Note   The student should very carefully note the aliquot parts
in such examples

EXERCISE 64.

Find, by Practice, the price of the following articles ·—

1. 32 at 2a. each   2   40 at 5a 4p each
3. 50 at 6a each   4   65 at 10a each.

5.   75 at 13*a*, 4*p*, each   6   85 at 12*a*. each

7   121 at Rs 2 6*a*. each   8.   124 at Rs 4 10*a*. each

Use one aliquot party only (9—20) :—

9   125 at Rs 3 3*a* each   10.   127 at Rs 5 5*a* each.

11   145 at Rs 7 7*a*. each 12   123 at Rs. 9 9*a* each.

13   127 at Rs. 11 11*a* ,, 14.   128 at Rs 13 13*a*   ,,

15   125 at Rs 21. 7*a*.   ,, 16   137 at Rs 27 9*a*   ,,

17. 150 at Rs 26 13*a* ,, 18   145 at Rs 45. 15*a*. ,,

19   156 at Rs 12 14*a* 8*p* ,,20   147 at Rs. 7 5*a* 4*p*   ,,

21   147 at Rs 9 13*a* 4*p* ,,22   121 at Rs 3 10*a* 8*p* ,,

23   165 at Rs 8 9*a* 6*p* ,,24   172 at Rs 15 11*a*. 4*p* ,,

25   160 at Rs 9 9*a* 9*p*   each.⎫
26   180 at Rs. 11 11*a* 11*p*. each.⎪ Use two aliquot
27. 192 at Rs 28. 7*a* 7*p* each ⎬ parts only.
28   156 at Rs 45. 9*a*. 9*p*. each ⎭

29   232$\frac{1}{2}$ at Rs. 25 7*a*, 8*p* each

30   221$\frac{1}{2}$ at Rs. 45. 12*a* 8*p* each.

31   321$\frac{1}{8}$ at Rs. 16 14*a*. 8*p* each

32   521$\frac{1}{4}$ at Rs. 32 11*a* 8*p* each

33   416$\frac{1}{2}$ at Rs 11. 10*a* 8*p*. each.

34   512$\frac{1}{3}$ at Rs. 24 13*a* 8*p*. each.

35   320$\frac{1}{2}$ at Rs 27 9*a*. 8*p*. each.

36. 728$\frac{1}{8}$ at Rs 15. 7*a* 8$\frac{3}{8}$*p* each

37   728$\frac{1}{4}$ at £53. 3*s*. 4*d* each   [53=10×5+3]

38. 325$\frac{1}{8}$ at £59   7*s*. 6*d*. each. [59=10×6—1]

39   751$\frac{3}{8}$ at £87 16*s*   8*d* each   [87=17×5+2]

40   975$\frac{5}{8}$ at £107 17*s*. 6*d* each. ⌈107=12×9—1]

41. 1625 at £25 16*s*   6*d*. per hundred

42   4524 at £40 12*s*. 7$\frac{1}{2}$*d* per hundred

43   175 at Rs 62 8*a* per score.

44. Find the price of 127 pen-holders at 10$\frac{1}{2}$*p* each.

45   Find the price of 155 pencils at 4$\frac{1}{2}$*p*. each

46. A bankrupt's debts are Rs 15926 10*a* 8*p* and he can pay the creditors 7*a* 4*p* in a rupee Find his assets

47 A bankrupt's debts are Rs. 96728 5*a* 4*p* and he can pay the creditors 13*a* 4*p* in a rupee Find his assets.

II COMPOUND PRACTICE

§4 The following examples will illustrate the rule for Compound Practice —

Example 1 Find the cost of 12 m is 27 sr 8 chk. at Rs. 5 10*a*. 6*b* per md

Sol

	Rs	*a*	*p*.				
	5	10	6	= price of 1	md		
		12					
	67	14	0	=	,,	, 12	,,
20 sr = ½ of 1 md	2	13	3	=	,,	,, 20 sr	
5 sr = ¼ of 20 sr	0	11	3¾	=	,,	,, 5	,,
2½ sr = ½ of 5 sr	0	5	7⅞	=	,,	,, 2½	,,
	71	12	2⅝	Ans.			

Example 2 Find the cost of 12 tons 15 cwt 2 qr. 21 lb at £5. 16s. 8d per ton

Sol

	£	*s*	*d*				
	5	16	8	= price of 1 ton			
		12					
	70	0	0	= price of 12 tons.			
10 cwt. = ½ of 1 ton	2	18	4	=	,,	,, 10 cwt.	
5 cwt = ½ of 10 cwt	1	9	2	=	,,	,, 5 cwt.	
2 qr = 1/10 of 5 cwt	0	2	11	=	,.	,, 2 qr	
14 lb = ¼ of 2 qr.	0	0	8¾	=	,,	,, 14 lb	
7 lb = ½ of 14 lb	0	0	4⅜	=	,,	,, 7 lb	
	74	11	6⅛	Ans.			

Example 3 Find the value of 41 bales of cotton each weighing 4 mds. 37 srs. 8 ch at Rs 13. 7*a*. 6*b* per maund

Sol   4 mds 37 sr. 8 ch $= 5$ mds. $- 2\frac{1}{2}$ srs., $41 = 8 \times 5 + 1$

		Rs.	a	p				
Hence		13	7	6	$=$ value of 1 md.			
				5				
		67	5	6 $=$	,,	,,	5 mds	
$2\frac{1}{2}$ sr $= \frac{1}{16}$ of a md		0	13	$5\frac{5}{8} =$	,,	,,	$2\frac{1}{2}$ srs	
by subtraction		66	8	$\frac{3}{8} =$	,,	,,	1 bale	
				8				
		532	0	3 $=$	,,	,,	8 bales.	
				5				
		2660	1	3 $=$	,,	,,	40 ,,	
		66	8	$\frac{3}{8} =$	,,	,,	1 bale.	
		2726	1	$3\frac{3}{8} =$	,,	,,	41 bales.	

Example 4   Find the value of 880 articles at £3 16s. 8d per hundred

Sol

	£	s	d					
	3	16	8	$=$	Value of 100 articles			
			8					
	30	13	4 $=$	,,	,,	800	,,	
$50 = \frac{1}{2}$ of 100	1	18	4 $=$	,,	,,	50	,,	
$25 = \frac{1}{2}$ of 50	0	19	2 $=$	,,	,,	25	,,	
$5 = \frac{1}{5}$ of 25	0	3	10 $=$	,,	,,	5	,,	
	33	14	8 $=$	,,	,,	880	,, **Ans**	

EXERCISE 65

Find, by Practice, the value of —

1   3 mds. 10 sr.   at Rs 3. 5a. 4p per md.
2   4 mds 15 sr.   at Rs 5 10a 8p per md
3. 6 mds. 25 sr   at Rs. 4 13a 4p per md.
4   8 mds 27 sr. 8 chk   at Rs 5 12a. 6p per md
5   10 mds 37 sr 8 chk   at Rs. 8   6a. 8p per md
6   7 mds. 28 sr 8 chk.   at Rs. 6. 14a. 8p. per md
7. 8 mds 11 sr. 7 chk.   at Rs. 6 10a 8p per md
8   12 mds. 25 sr 12 chk   at Rs 2 3a 4p. per sr
9. 8 mds. 17 sr. 8 chk.   at Rs. 3 9a. 6p per sr.
10. 7 mds 14 sr 14 chk.   at Rs. 5. 6a 8p. per md

11.   5 mds 10 sr 10 chk at Rs 9 10*a* 8*p* per md

12   27 mds 18 sr. 9 chk at Rs 12 13*a*. 4*p* per md

13   5 cwt. 2 qr. 14 lb   at Rs 16 10*a* 8*p* per cwt.

14   7 cwt 1 qr. 21 lb   at Rs 12 12*a* 6*p* per cwt.

15   23 tons 15 cwt 2 qr 7 lb at £3 13*s* 4*d* per ton

16,   5 tons 12 cwt 2 qr 16 lb at £5 16*s* 8*d* per ton.

17   16 yds 2 ft. 3 ın at Rs 6 9*a*. 8*p*. per yd

18   43 yds 1 ft 4 ın at Rs 5 13*a*. 4*p* per yd.

19   17 tolas 8 mashas 4 ratıs at Rupees 22. 10*a* 4*p*. per tola

20.   27 tolas 9 mashas 5 ratıs at Rupees 26 13*a*. 4*p* per tola

21   24 tolas 10 mashas 4 ratıs at Rupees 25. 4*a*. 6*p*. per tola.

22   Fınd the rent of 586 acres 1 rood 31 sq poles at £4 1*s*. 10$\frac{1}{3}$*d* per acre

23   Fınd the value of 1 ton 5 cwt. 2 qr. 14 lb, at £ 3 15*s* 7*d* per cwt

24   Fınd the value of 5 acres 3 roods 7 sq poles 5$\frac{1}{2}$ sɑ yards of land at £161 6*s* 8*d*. per acre

25   Fınd the coɛt of 7 mıles 5 fur. 165 yds. at Rs. 682 7*a*. 4*p* per mıle.

26.   Fınd the rent of 156 acres 3 roods 24 sq. poles 11 sq yards at Rs 25. 3*a* 4*p* per acre.

27   Fınd the prıce of 15 bags of wheat weıghıng 2 mds 27 sr 8 chk. at Rs 6 10*a* 8*p* per md.

28.   Calculate the prıce of 25 bags of sugar weıghıng 3 mds 35 sr 8 chk each at Rs. 16. 13*a*. 4*p* per md

29.   What ıs the cost of 40 bales of cotton weıghıng 4 mds 27 sr 8 chk each at Rs. 25 8*a* per md. ?

30   Fınd the prıce of 35 boxes of tea at Rs 9. 2*a*. 8*p* per md , the weıght of each box beıng 2 mds. 15 sr 8 chk

31   Fınd the prıce, ın Englısh coın, of 45 bags of sugar weıghıng 2 mds. 15 sr 8 chk. each at Rs. 20 10*a* 8*p* per md (Re 1=2*s*. 2*d*)

32   If a man's debts amount to Rs 15789 4*a* , and he can pay only 12*a* 3⅜*p* for each rupee, how much do his creditors get ?

33. Find the price of 10 lbs. 11 oz 16 dwts 16 grs of gold at £3 17*s* 10½*d* per oz

34   Find the value of 2 tons 15 cwt 1 qr 7 lb. at £13 6*s* 8*d* per ton

Find the value of —

35   5 tons 5 cwt 2 qr 17½ lb. at £3. 6*s* 8*d* per ton.

36. 2 tons 7 cwt. 3 qr 11 lb. at £21 12*s* 6*d* per cwt

37   1347 cwt 3 qr. 21 lb at £3 17*s*. 10¼*d*. per cwt

38   1565 cwt of coal at Rs 125 12*a* 6*p*. per 100 cwt

39   7 tons 2 cwt. 2 qr at Rs 3. 2*a* per maund, assuming that one ton is equal to 27¼ maunds

40   Find the cost of a fence of length 5 miles 1104 yds 2 ft at £13 15*s* per mile

41   Find the rent of 24 acres 3 roods 26 sq. poles at £3 18*s* 4*d* per acre

42. Find the cost of 5 tons 3 cwt 3 qr 27 lb. 12 oz at £14 per cwt

43   Find the rent for 3 months 3 weeks 4 days from January 1 at Rs 106 12*a* per month

44   Find to the nearest penny the value of 11 tons 17 cwt 3 qr 21 lbs at £4. 17*s* 6*d* a ton   (*Most easily done by the decimal system*)

INVOICE OR BILL.

§5   An **invoice** is a detailed description of the goods together with their price, quality and quantity supplied by a seller to the buyer.

If the payment is made then and there, the invoice is receipted *i e* , the seller writes the words "Received pay. ment" puts his signature and date   If the total amount of the bill is over Rs 20, or one-anna stamp must be affixed. [See page 191]

Note   Each separate amount in an invoice is called an item

§6.   Specimen of an invoice or bill

LAHORE.

No. 231   APRIL 13, 1936

The Headmaster, Jain High School, Indore.

Bought of The Mathematical House

Quantity	Description	Rate	Rs	A	P	Rs	A	P.
50	"Algebra Made Simple" by Ghosh and Dhawn	2/-/-	100					
50	"Arithmetic Made Easy" by Ghosh and Dhawn	2/-/-	100					
50	Standard Geometry by M. K. K. Pillay	1/12/-	87	8				
			287	8				
	Deduct discount at 15 p c		43	2		244	6	0
50	Selected Exercises in Geometry with solutions	-/12/-	37	8				
40	20 model papers in Matriculation Mathematics A with solutions by K M Ghosh, M A	-/12/-	30					
50	Ataliq Hisab by L P Dhawn	1/-/-	50					
			117	8				
	Deduct discount at 20 p c		23	8	0	94	0	0
		Total Rs				338	6	0

E & O E.   (Sd)

For The Mathematical House

Received Payment

R K Dhawn

April 13, 1936

§7   Specimen of an account

LAHORE
June 4, 1936.

The Headmaster, Jain High School, Indore   *Dr*
To the Mathematical House, Lahore

Date	Description	Rs	A	P
April 13, 1935	To goods as per invoice No. 231	338	6	0
May 15, 1936	To goods as per invoice No. 352	56	7	3
May 29, 1936.	To goods as per invoice No. 402	36	3	6
	Total Rs	431	0	9

EXERCISE 66

Make the following invoices, giving name, date, etc —

1   20 slates at $7a$ per slate , 50 boxes of Geometrical instruments at Re 1 $2a$ per box , 2 gross of pencils at $3a$ $6p.$ a dozen , 15 colour boxes at Re. 1. $7a$ a box

2   40 copies of "Ataliq-i-Hisab" by L R Dhawn at Re 1 per copy , 30 copies of Domestic Arithmetic and Household Accounts Part I by L R Dhawn at Re 1. $4a$ per copy , 20 copies of "Algebra Made Simple" by Ghosh and Dhawn at Rs 2 per copy   (Deduct discount at 15 p c)

3   10 yards of linen at Re 1. $3a$ per yard , 20 yards flannel at Rs 5 $8a.$ per yard , $28\frac{1}{2}$ yards of velvet at Rs 3 $12a,$ per yard , 20 pairs of sarees at Rs 7. $8a$ per pair , two dozen of towels at $9a$ per towel.

4.   20 seers of sugar at $8a$ $9p$ per seer , $1\frac{3}{4}$ maunds of rice at Rs 5. $8a$ per maund , 15 seers of ghee at Re 2 $3a$ per seer , 21 lbs of butter at $8\frac{3}{4}a$ per lb. , $3\frac{3}{4}$ maunds of flour at Rs 5 $7a$ per maund

5.　20 pairs of stockings at 8*a* 9*p*. per stocking , 30 handkerchiefs at 2*a* 9*p* per handkerchief , 40 umbrellas at Re 1 6*a* 6*p* per umbrella , 20 boxes of soap at 11*a*. 3*p*. per box , 1½ dozen of gloves at Re 1 3*a* 9*p* per pair

6.　10 lbs of tea at 1*s* 1¼*d* per lb , 15 lbs of coffee at 1*s* 2¾*d* per lb , 30 lbs of chicory at 1*s* 9½*d* per lb 20 lbs of sugar at 9¾*d* per lb

7　One gross of pen-holders at 4*a* 6*p* per dozen , 300 envelopes at 12*a*. 3*p* per hundred , 23 exercise books at 2*a* 9*p* per book , 5 dozen quill pens at 2*a* 6*p* per dozen, 4½ reams of foolscap paper at 6*a* per quire , ½ gross inkpots at 7*a*. 6*p* per dozen

8　15½ yds of linen at 14*a*. per yd. 21½ yds of flannel at Rs 2 4*a* per yd , 45 yds of carpet at 5*a* 4*p* per yd , 3 pairs of socks at 3*a* 6*p* per pair , 10½ yds of sheeting at Re. 1 10*a*. 8*p* per yd

9　50 mds of wheat at Rs 5 6*a* per md. 40 mds of gram at Rs 6 3*a* per md , 75 mds of rice at Rs 4 2*a*. per md , 21½ mds of sugar at Rs 15 10*a* per md , 30½ mds of coal at Rs 3 6*a* 6*p* per md

10　21 lbs of tea at 1*s* 3*d* per lb , 13 lbs of coffee at 1*s* 5*d* per lb , 24⅓ lbs of chicory at 1*s* 2*d* per lb　54 lbs of sugar at 9*d*. per lb , 17¾ lbs. of green tea at 1*s* 1*d* per lb.

11　20 copies of Arithmetic Made Easy by Ghosh and Dhawn at Rs 2 per copy , 24 copies of Algebra Made Simple by Ghosh and Dhawn at Rs 2 per copy , 32 copies of 20 Model Papers in Matriculation Mathematics A by Prof. Ghosh and Atma Ram at 12*a*. per copy , 26 copies of Typical Questions in Arithmetic by L R Dhawn at 2*a* per copy , 27 copies of Typical Questions in Algebra by L R Dhawn at 2½*a* per copy , 25 copies of Key to Arithmetic Made Easy at Rs. 2 , 24 copies of Standard Geometry revised by M K K Pillay at Rs 2 per copy Deduct discount at 12½ %

F. 13

CHAPTER XI.

INVOLUTION AND EVOLUTION.

I INVOLUTION.

§1 A **power** of a number is the product which is obtained from successive multiplication by itself, the operation by which it is obtained is called **Involution.** Thus the 1st power of 2 is 2, the second power is $2 \times 2 = 4$, the third power is $2 \times 2 \times 2 = 8$ and so on For the sake of convenience we denote these operations by the help of indices or small figures placed above the number a little to the right, thus, we may write as

$$2^1 = 2, \ 2^2 = 4, \ 2^3 = 8, \text{ etc , etc.}$$

§2 The second and third powers of numbers are called their squares and cubes respectively

§3 Study the following table of the numbers and their squares

Numbers	1	2	3	4	5	6	7	8	9	10
Squares	1	4	9	16	25	36	49	64	81	100.

Notice carefully that no square number ends in 2, 3, 7 or 8 , hence it follows that *no number which ends in 2 3, 7 or 8 is a perfect square.*

§4 **Abbreviated methods of finding the square of any number.**

Rule 1 *Split up the number in two parts and apply the algebraical formula* $(a + b)^2 = a^2 + b^2 + 2ab$

Example 1. Find the square of 64

Sol, $64^2 = (60 + 4)^2$
$= 3600 + 16 + 2 \times 60 \times 4$
$= 4096.$ **Ans.**

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Rule 2 *Add and subtract the unit figure of the given number, to and from the number itself, Multiply the sum and difference thus obtained and then add the square of the unit's figure to the product*

Example 2 Find the square of 64 by this rule

Sol. $64^2 = (64 + 4) \, (64 - 4) + (4)^2$
                $= 68 \times 60 + 16$
                $= 4080 + 16 = 4096.$ **Ans**

This method may also be proved by the algebraical formula

$$a^2 = a^2 - b^2 + b^2$$
$$a^2 = (a + b) \, (a - b) + b^2$$

Rule 3. *If the number contains more than 2 digits, then more than 1 digit on the right of the number may be taken instead of unit's figure* Thus

Example 3 Find the square of 526

Sol $526^2 = (526 + 26) \, (526 - 26) + (26)^2$
                $= 552 \times 500 + 676$
                $= 276000 + 676 = 276676.$ **Ans**

Rule 4 *If the number ends in figure 5, then neglect the five and multiply the remaining figure by the next higher figure and affix 25 to the right* Thus

Example 4 Find the squares of 65 and 135

Sol. (i) $65^2 = 6 \times 7$ with 25 affixed $= 4225$ **Ans**
        (ii) $135^2 = 13 \times 14$ with 25 affixed $= 18225$ **Ans**

Rule 5 *If the number ends in 25, then neglect the twenty-five and multiply the remaining figure by the remaining figure with a 5 to its right, and affix 625 to the right* Thus

Example 5 Find the squares of 525 and 1625

Sol (i) $525^2 = 5 \times 55$ with 625 affixed $= 275625$ **Ans**
        (ii) $1625^2 = 16 \times 165$ with 625 affixed $= 2640625$ **Ans**

EXERCISE 67.

Write down the squares of the following numbers —

1. 44	**2** 57.	**3** 36.	**4.** 48
5. 49	**6** 46	**7** 59.	**8.** 88.
9 53.	**10.** 47	**11.** 99	**12.** 56
13 128	**14.** 156.	**15** 166	**16** 106.
17. 65	**18** 75	**19** 145	**20** 155
21 165	**22** 205	**23** 245	**24** 305
25. 625	**26.** 929	**27** 1325	**28** 2125

§5   A Solid Square and a Hollow Square

A Solid Square is one in which the number of rows is equal to the number of men in each row. In diagram (1) there are 8 rows, each containing 8 men   The total number of men is 8^2 or 64
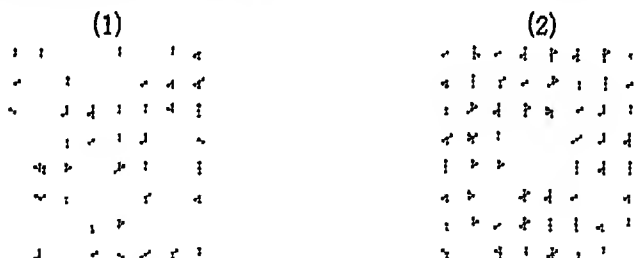
Diagram (2) is a hollow square three deep having 8 men in the front rank   It is three deep because the number of complete rows counted from any front is three. The number of men in this hollow square $= 8^2 - (8 - 2 \times 3)^2 = 60$.

Example   Find the least number of soldiers which can be drawn up in a hollow square 6, 8, 12 and 15 deep and also in a solid square

Sol   We have to find the least number which is exactly divisible by 6×4, 8×4, 12×4, 15×4

The L C.M of 24, 32, 48, 60 $= 480 = 16 \times 30$

least square number $= 16 \times 30 \times 30 = 14400$ **Ans**

II. EVOLUTION

§6. Evolution is the inverse process of involution. Whereas involution is the process of raising a number to any power so evolution is the process of extracting a required root of a given number   Thus

$5^2 = 5 \times 5 = 25$ Here 25 is the square of 5 and

5 is the second root or square root of 25, also

$5^3 = 5 \times 5 \times 5 = 125$.   Here 125 is the cube of 5 and

5 is the third root of 125   Similarly

5 is the fourth root of $5 \times 5 \times 5 \times 5$ or 625 and so on.

§7. Square root

The Square Root of a given number is that number which when multiplied by itself will produce the given number. Thus 5 is the square root of 25 because 5 multiplied by 5 produces the given number 25.

§8   The square root of a given number is indicated by the symbol $\sqrt{}$ placed before it.   It is sometimes denoted by placing the fraction $\frac{1}{2}$ above the number towards the right. Thus the square root of 25 is denoted either by $\sqrt{25}$ or $(25)^{\frac{1}{2}}$.

§9. Square root of 9 is 3 but the square root of 3 cannot be accurately determined. We may only write an approximate answer in decimals   Such an incomplete root is called **surd root** and the one of which the root is accurately determined is called a **rational root.** Thus the square root of 9 is 3, 3 is therefore a rational root, but the square root of 3 cannot be accurately ascertained, therefore $\sqrt{3}$ is a surd root

§10. **To ascertain the number of digits in the square root of a given whole number**

Since the square root of 1   is 1 ,

„   „   „   „   „ 100   is 10 ,

„   „   „   „   „ 10000 is 100 , &c. &c ,

it follows at once that the square root of a number consis-

ing of less than 3 digits will consist of only 1 digit, that of
a number of 3 and 4 digits will consist of 2 digits and so on.
Or if a dot ( ) be placed over every alternate digit beginning
with the digit in the unit's place of a number, *i e*, separate
the digits of the given number into periods of two, the num-
ber of dots thus placed, *i e*, the number of periods thus
marked will show the number of digits in the square root of
the number   Thus 8972 will be divided into two periods
and 48972 into three *e g*, 8972 and 48972   In the first
case 72 is the first period and 89 the second   In the second
case 72 is the first period, 89 the second and 4 the third.
It may be noted that if the given number consists of an
odd number of digits the last period will be of one
figure only.

§11. To find the square root of a number by
factors

Example 1   Extract the square root of 11025

Sol. $11025 = 7 \times 7 \times 5 \times 5 \times 3 \times 3$
                 $= 7^2 \times 5^2 \times 3^2$

the square root $= 7 \times 5 \times 3 = 105$.   Ans.

Note   It is not necessary to break the given number into prime
factors , we may conveniently break it into equal pairs of factors

EXERCISE 68

Extract the square root of —

1    $4 \times 4 \times 5 \times 5$ 2. $7 \times 7 \times 3 \times 3$
3    $11 \times 11 \times 3 \times 3 \times 2 \times 2$ 4 $13 \times 13 \times 5 \times 5 \times 2 \times 2$
5   1225.   6   2025.   7.   9216   8   17424.
9   19600   10   20736   11.   63504   12   81796.

13   Find the *least* numbers, which when multiplied
by the following numbers, will make the products perfect
squares — 75 , 180 , 1456 , 750750

14   What is the least square number which is divisible
by 4, 8, 15, 108 or 125 ?

§12. To find the square root of a whole number

When the factors of a given number are not conveni-
ently traceable we use the method corresponding to the
algebraical formula $\sqrt{a^2 + 2ab + b^2} = a + b$

Before we explain how to extract the square root of a
given number let us analyse the operation by finding the
squares of $a+b$ and $20+5$ when 20 stands for a and 5 for b

$$
\begin{array}{lll}
a \;+\; b & 20+5 & \text{Or } 400 \\
\underline{a \;+\; b} & \underline{20+5} & 200 \\
a^2+ab & 400+20\times 5 & \underline{25} \\
\underline{+ab\;\;+b^2} & \underline{+20\times 5+25} & 256 \\
a^2+2ab+b^2 & 400+2\times 20\times 5+25 \\
& =400+5(2\times 20+5)
\end{array}
$$

Now reverse the operation and find the square root of
625   First mark off the number into periods as explained
in Art. 10   As there are two periods in the number the
square root will consist of two digits   We also find that
the second period is less than 9 and greater than 4. There-
fore the left-hand side digit in the square root will be 2 and
the square root will be greater than 20 and less than 30.
We shall now proceed thus.—

$$
\begin{array}{r|l}
20 & 625\ (20+5 \\
   & \underline{400} \\
40 & 225 \\
   & \underline{200} \\
   & 25
\end{array}
$$

Exp   We have subtracted the
square of 20 from 625 and have
obtained a remainder 225   Now
doubling the 20, we get 40   Di-
vide the remainder 225 by 40.
It goes 5 times, leaving the re-
mainder 25 which is the square of 5   We conclude there-
fore that in order to make up the remainder 225 we have to
add 5 to 40 and then multiply the sum by 5   We shall
write the process thus

$$
(1)\qquad 20)\overset{\cdot}{6}2\overset{\cdot}{5}(20+5=25 \qquad\qquad (2)\quad 2\vert\overset{\cdot}{6}2\overset{\cdot}{5}(25
$$
$$
\underline{400}\qquad\qquad\underline{\vert 4}
$$
$$
40+5=45)\underline{225}\quad\text{Or briefly thus}\quad 45\vert 225
$$
$$
\underline{225}\phantom{\text{Or briefly thus}\quad 45}\vert\underline{225}
$$

Note   In the 2nd process zeroes have been omitted

The above process has been modelled on that of algebra.
For the sake of reference we write the process below

$$
\begin{array}{r|l}
a & a^2+2ab+b^2(a+b \\
  & \underline{a^2} \\
2a+b & 2ab+b^2 \\
      & \underline{2ab+b^2}
\end{array}
$$

§13. A Graphical Explanation

We have read in algebra that the difference of the squares of two numbers is equal to the product of the sum and difference of those numbers ; as

$$(27)^2 - (20)^2 = (27 + 20)(27 - 20)$$
$$= 47 \times 7$$
$$= (\text{twice } 20 + 7) \times 7$$

We conclude therefore that if from the square of a number we subtract the square of a part of it, the remainder = (twice that part + the second part) × the second part

With the help of this result let us find the square root of 729. It will be divided into two periods, the first period being 29 and the second 7 7 is greater than 4 and less than 9, therefore the left hand digit in the square root must be 2 and the square root will be between 20 and 30   Let us now draw a square supposing its area to be equal to 729

Fig I.   Fig II.   2nd part

$(20)^2$
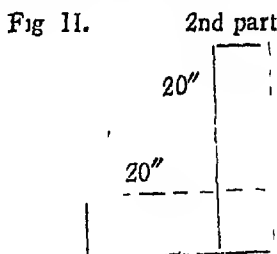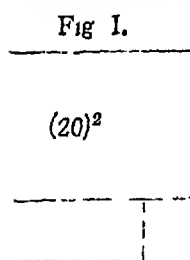
square inches.   Cut off a squre of 20 inches from it, the remaining figure as shown in Fig II is equal to 729—400 i e , 329 square inches.

Figure II can be put like the following rectangle, the area of which is equal to

(Twice the first part + the second part) × the second part = 329 square inches.
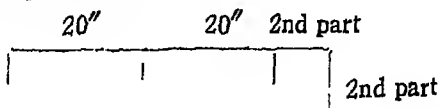
Now we have to find out the second part so that by adding it to twice 20 and multiplying the sum by it we may get 329.

Twice 20=40, when we divide 329 by 40 it goes 8 times, adding 8 to 40 and multiplying the sum by 8 we get the figure in the unit's place 4 instead of 9, we therefore reject this number and try another Let us try 7, adding 7 to 40 we get 47 and multiplying 47 by 7, we get 329. Therefore the second part is 7 and the side of the square is 20+7, i e, 27.

$$
\begin{array}{cl}
20 & 729(20+7 \\
   & \underline{400} \\
20\times2=40 & 329 \\
40+7=47 & \underline{329}
\end{array}
\qquad
\begin{array}{cl}
2 & 729(27 \\
  & \underline{4} \\
47 & 329 \\
   & \underline{329}
\end{array}
$$

From the above explanations we can extract the following

Rule *Mark off the given number into periods as explained in Art 10 Subtract the square of the greatest possible number (putting the number as the first figure of the root) from the first period at the left Bring down the remainder and the next period Divide the first two or three digits of the dividend thus formed by twice the part of the root already obtained (called trial divisor) Place the quotient as the second digit in the root and also to the right of the trial divisor Now multiply the newly formed divisor by the second digit of the root and put it as the subtrahend Continue the process till all the periods are exhausted*

Example 2. Extract the square root of 1452025

Sol.
$$
\begin{array}{rl}
1 & 1452025(1205 \\
  & \underline{1} \\
22 & 45 \\
   & \underline{44} \\
2405 & 12025 \\
     & \underline{12025}
\end{array}
$$
Ans

Exp. Mark off the periods, there being 4 periods we find that the square root will consist of 4 digits First period at the left is 1 and the square root of it is 1, subtracting 1 from 1, the remainder is zero Bring down the next

period 45 for the next dividend. The trial divisor is 1×2 or 2 and the first figure in the trial dividend is 4. The quotient will be 2   Place this 2 as the second digit of the root, and annex it to the trial divisor as well, making it 22. Multiply 22 by 2 and put the product 44 as subtrahend and then subtract it from 45   The remainder is 1   Bring down the next period 20   Now the trial divisor 12×2, i e., 24 cannot divide the first two digits 12 in the trial dividend 120, so the next digit in the root is zero   Place this zero in the root and also at the end of the divisor. Bring down the next period 25 and now we get 5 as the last digit of the required root.

Example 3   Find the least number which must be subtracted from or added to 89023 to make it a perfect square

Sol.
$$
\begin{array}{r|l}
2 & 89023\ (298 \\
  & 4 \\ \hline
49 & 490 \\
   & 441 \\ \hline
588 & 4923 \\
    & 4704 \\ \hline
    & 219
\end{array}
$$

Hence, (i) 219 is the least number to be subtracted from and (ii) $(299)^2 - 89023 = 378$ is the least number to be added to 89023 to make it a perfect square.

EXERCISE 69.

Find the square root of :—

1.	25600	2	28900	3	44100
4	50625.	5	73441.	6.	85849
7.	839056.	8	819025	9.	978121
10	2298256	11.	4515625	12.	9641025.
13.	80496784.	14.	72471169	15	81162081.
16.	244328161.	17	234733041	18.	464661136.
19.	26422502500.		20	850058184196	
21.	2250075000625.		22	49000350000625	

23.   A certain number of men subscribed as many pies each as there were subscribers, the whole subscription being Rs. 16 5a 4p   How many subscribers were there ?

24 A number of persons subscribed as many pice each as there were persons   The total subscription is Rs 206 $10a$ $3p$, find the number of persons.

25. The subscription to a certain fund amounted to Rs. 976. 9a , and each person subscribed as many annas as there were subscribers altogether.   Find the number of subscribers

26 A General arranged his men numbering 53824 into a solid square   How many men were there in the front ?

27 A General wishing to arrange his men numbering 276674 into a solid square found that there were two men less   How many men were there in the front ?

28 A General wishing to draw up his 251132 men into a solid square found that he had 131 men over , find the number of men in the front

29 Find the least numbers which must be subtracted from 8972 and added to 12723 to make them perfect squares.

30 Find the number whose third part multiplied by its seventh part gives 756

31 Find the least number of men in a regiment which can be drawn up in a hollow square 5, 8, 10 or 12 deep and also into a solid square

---

§14   To find the Square Root of a decimal Fraction

1	× ˑ1	= 01		√ 01	= 1.
01	× ˑ01	=ˑ0001		√ 0001	=ˑ01
001	× ˑ001	= 000001		√ 000001	=ˑ001.

From the above examples we note the   following points —

(1) If any decimal is squared there is always an even number of decimal places in the result   Consequently a decimal fraction must contain an even number of decimal places if it is a perfect square, i. e., if its square root can be exactly found   If the number does not contain an even number of digits a zero may be annexed

(2)   The pointing must begin from the place of *units* towards the right hand over every alternate figure as before and the number of such points will be the same as the number of decimal places in the square root.

(3)   The number of decimal places in the square root is always half the number of decimal places in the given number.

(4)   The square root is always greater than the given number

§15   We shall deduce the rule for extracting the square root of a decimal from the process of the following simple examples.

Example 1.   Find the square root of 23.

Note   It is clear that the square root of 23 cannot be accurately determined   An approximate answer in decimal may be given We shall therefore put the decimal point and annex some pairs of zeroes and proceed as before

Sol.   4 · 23 000000(4 795   **Ans.**
                              16
                    87   700
                          609
                   949   9100
                          8541
                  9585   55900
                          47925
          ·   7975

The process of extracting the square root of 23 000000 is really the process of extracting the square root of a number consisting partly of a whole number and partly of a decimal.

Example 2   Find the square root of ˙0023562

Note   The number of decimal places in the given number not even   in order to make them even we annex a zero

Thus 0023562 = 00235620
Sol.   0 | 00235620   (0485 . **Ans.**
                      4 | 16
                     88 | 756
                        | 704
                    965 | 5220
                        | 4825
          ● | 365.
          ●   ,

Rule   *Make the number of decimal places even by affixing a zero if necessary   Place a dot over the unit's figure and then over every alternate figure to its right and left to get periods of two and then proceed as in the case of a whole number   The number of periods in the integral part of the given number will show the number of digits in the integral part of the root and the number of periods in the decimal part will show the number of decimal places in the root*

Example 3   The product of two numbers is 105 625 and their quotient is 2 5 , find the numbers

Sol   Product—Quotient=(smaller No)²
                smaller No $=\sqrt{105\ 625-2\ 5}$
                        $=\sqrt{1056\ 25-25}$
                        $=\sqrt{42^{.}25}$
                        $=6\ 5$
and the greater No $=105\ 625-6\ 5$
                        $=1056\ 25-65$
                        $=16\ 25$

The required numbers are 16ˈ25 and 6ˈ5   **Ans.**

Example 4   The products obtained by multiplying together each pair of three given numbers are 8, 11 25, 14 4 respectively , find the numbers

Sol. Product of the three numbers$=\sqrt{8\times 11\ 25\times 14\ 4}$
                        $=\sqrt{8\times 162}$
                        $=\sqrt{4\times 2\times 2\times 9\times 9}$
                        $=36$

Nos. are (ı)   36—8   =4 5 ⎫
        (ıı)   36—11ˈ25 =3 2 ⎬   **Ans**
        (ııı)   36—14ˈ4 =2 5 ⎭

EXERCISE 70.

Find the square root of —

1.	ˈ09.	2	ˈ0064	3.	0081
4	7 29.	5	16 81	6.	34 81
7	9 3025	8.	84 8241	9	37 0881
10	150 0625	11	477 4225	12	225 6004
13	52 649536	14	ˈ015625	15	ˈ813604.
16	251953 8025	17.	227ˈ798649		

Find to four decimal places the square root of —

18 1   **19.** ˙00001   **20** ˙005.

21 15 013   **22.** 121 3654   **23** 256˙1034

Find to five decimal places the square root of —

24 11   **25.** 19.   **26** 31   **27** 101

28 501   **29** 605   **30.** 117

Find the square root of —

31 005329 **32.** ˙00053361 **33** ˙00000049112064

34 40000 400001

35. The product of two numbers is 078125 and their quotient is 5 find the numbers

36. The products obtained by multiplying together each pair of three given numbers are 7 14, 11 76, 19 04 respectively , find the numbers

---

§16 To find the square root of a vulgar fraction

Observe the following examples —

(*i*) $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$

(*ii*) $\sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{5}{3} = 1\frac{2}{3}$

(*iii*) $\sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{\sqrt{9}} = \frac{1\cdot414}{3} = 471$.

(*iv*) $\sqrt{\frac{3}{7}} = \frac{\sqrt{428571}}{\sqrt{3 \times 7}} = ˙654$

Or $\sqrt{\frac{3}{7}} = \frac{\sqrt{3 \times 7}}{\sqrt{7 \times 7}} = \frac{\sqrt{21}}{\sqrt{49}} = \frac{4\ 582}{7} = ˙654$

(*v*) $\sqrt{\frac{8}{9}} = \frac{\sqrt{8 \times 4}}{\sqrt{9 \times 4}} = \frac{\sqrt{32}}{\sqrt{36}} = \frac{5\ 656}{6} = ˙942$

The above examples give us the following

Rule (*i*) *If the numerator and the denominator of a fraction and of a mixed number when reduced to an improper fraction be perfect squares, then take the square root of the numerator for a new numerator and the square root of the denominator for a new denominator.* (*Exs i,ii*).

(*ii*) *If only the denominator of a fraction be a perfect square, the square root in that case is obtained by dividing the square root of the numerator by the square root of the denominator.* (*Ex iii*)

(*iii*) *If the numerator and the denominator be both imperfect squares, then convert the fraction into a decimal and extract its square root.* (*Ex iv*)

Or *Multiply both the numerator and the denominator by such a number as to make the denominator a perfect square and then proceed as in Ex iii* (*Exs iv, v*)

Example   Extract the square root of $\dfrac{3\ 75}{8\ 75}$

Sol. $\dfrac{3\ 75}{8\ 75} = \frac{375}{875} = \frac{3}{7}$ $\sqrt{\dfrac{3\ 75}{8\ 75}} = \sqrt{\tfrac{3}{7}} = 654$ **Ans.**

EXERCISE 71

Find the square root of :—

1.	$\frac{64}{81}$.	2	$\frac{121}{144}$	3	$\frac{225}{225}$	4	$\frac{256}{625}$.
5	$6\frac{1}{4}$.	6	$7\frac{3}{9}$	7	$5\frac{1}{16}$	8.	$5\frac{19}{25}$.
9.	$34\frac{15}{49}$	10	$84\frac{97}{121}$	11	$71\frac{9}{16}$	12	$29\frac{23}{49}$.
13.	$10\frac{161}{225}$.	14.	$21\frac{51}{169}$	15	$\frac{1\,44}{1\,69}$	16.	$\frac{16\,6}{62\,5}$.

Find to three decimal places the square root of —

17	$\frac{5}{16}$	18	$\frac{12}{25}$.	19	$\frac{7}{36}$	20.	$\frac{15}{49}$
21.	$2\frac{1}{9}$	22	$5\frac{3}{4}$	23	$3\frac{5}{16}$	24	$5\frac{1}{4}$
25.	$\frac{5}{8}$.	26	$\frac{1}{8}$	27	$\frac{5}{17}$	28	$2\frac{1}{15}$
29	$287\frac{5}{8}$.	30	$367\frac{3}{7}$.	31	$\frac{6\,05}{8\,47}$.	32.	$\frac{316\,8}{10\,08}$

33. The product of two numbers is 50 and their quotient is $1\frac{1}{8}$ Find the numbers.

34   The products obtained by multiplying each pair of the three given numbers are $7\frac{7}{12}, 12\frac{1}{18}, 16\frac{19}{24}$, respectively, find the numbers

§17   To find the square root of recurring deci-
mals

If a recurring decimal be a *perfect square*, it is con
venient to reduce it to a vulgar fraction and proceed as in
Art 12 (*i*), thus, $\sqrt{2\,7} = \sqrt{\frac{25}{9}} = \frac{5}{3} = 1\,6$

§18   But if the recurring decimal be *not a perfect
square*, we may extend the recurring part by repeating its
period and then proceed as in decimal questions

Thus to extract the square root of 4 273 to 3 places of
decimals, we can extend the recurring part 73 as long as
to enable us to get 3 decimal places in the square root, *e g*,
we may write 4 273 as 4 273737 and then proceed as in
Art 11

EXERCISE 72

Find the square root of —

1.	027.	2	28 4	3	4738 027
4	134	5	049382716	6	2˙361
7.	˙07	8	4 02981.	9.	˙00562

Cube root

§19   The cube root of a number is that number
which when multiplied by itself three times will produce
the given number   Thus 4 is the cube root of 64, since 64
is the cube of 4.

§20   The cube root of a number is indicated by the
symbol $\sqrt[3]{}$ placed before it.   It is sometimes denoted by the
symbol $\frac{1}{3}$ placed above the number towards the right   Thus
the cube root of 64 is denoted by $\sqrt[3]{64}$ or $(64)^{\frac{1}{3}}$

§21.   A number is said to be a **perfect cube** when its
cube root can be **exactly** found, *e g*, 125 is a perfect cube
but not 135, since there is no number, integral or fractional,
whose cube is **exactly** 135.

§22   Study the following table of the natural numbers
and their cubes —

Numbers	1	2	3	4	5	6	7	8	9
Cubes	1	8	27	64	125	216	343	512	729

Notice carefully that a cube may end with any digit even or odd. It is not so in the case of a square number

It may also be noted that when a cube ends in 1, 8, 7, 4, 5, 6, 3, 2, 9, its cube root ends in 1, 2, 3, 4, 5, 6, 7, 8, 9 respectively

§23   To ascertain the number of digits in the cube root of a given integral number.

Since the cube root of 1 is 1 ,
            the cube root of 1000 is 10
            the cube root of 1000000 is 100 , etc., etc ,

it follows at once that cube root of a number lying between 1 and 1000 must be between 1 and 10, *i e*, must be a number of one digit   Similarly the cube root of a number lying between 1000 and 1000000 must lie between 10 and 100, *i e*, must be a number of two digits, and so on Hence it follows that if a dot ( ) be placed over the unit's figure in the number and thence over every third digit to the left, the number of dots will evidently be the number of digits in the cube root of the number   This rule may also be applied to decimals.

§24.   To find the cube root of a given number.

Rule   *Place a dot over the unit's figure in the given number and then over every third figure to its left and also to its right if the number contains any decimal portion, adding zeroes if necessary to get periods of three digits   It may be noted that each period shall consist of three figures, except the first which may contain 1, 2 or 3 figures.*

Now find the number whose cube is either equal to, or next less than the first *period on the left hand and write it as the first figure of the required root   Subtract its cube from the first period and write down the remainder with the next period following it*

Multiply the square of the figure in the root just obtained by 300 and take the product as a trial *divisor,*

Then find how many times this divisor is contained in the dividend,—the quotient thus obtained will be the **next** *figure in the cube root.   Then, multiply this quotient by the product of previous figure in the root by 30 and place this result below the trial divisor   Below these, write the square of this last quotient and the sum of all the three will be regarded as a* complete *divisor   Multiply this complete divisor by the second figure of the root and subtract   Write down the remainder with the next period following it, which will form the next dividend*

Multiply the square of the root already obtained by 300 and take the product as a trial divisor and then find as before, how   many times this trial divisor is contained in the dividend,—the quotient thus obtained will be the next, i. e , the third figure in the root.   Then multiply the figures of the cube root already obtained by 30 and the product by the last quotient and place this product below the trial divisor   Below, these, write the square of the last quotient and add all the three together —the sum being taken as a complete divisor.   Multiply this divisor by the last figure in the root and subtract   Write down the remainder with the next period following it, which will form the next dividend   Proceed in this way, till all the periods are exhausted

Note —If at any stage of the process, the dividend is less than the trial divisor, put a zero in the root two zeroes to the trial divisor and bring down the next period

Example 1.   Find the cube root of 46656.

Sol   46656 (36 **Ans**
                  27
            _____
               |19656
$3^2 \times 300 = 2700$|
$3 \times 30 \times 6 = 540$|
$6^2 = 36$|
        _____
        3276|19656
           |_____

Exp   Here, first divide into periods, beginning with 6 , the first period on the left contains only 2 digits, viz, 46.   The trial divisor 2700 goes into the dividend 19656, 6 times.

Example 2. Find the cube root of 64481˙201.

Sol.   6⁴481 201 (40˙1   **Ans.**
                  64
                    ┃481201   **Exp** The first trial
$40^2 \times 300 = 480000$┃   divisor 4800 does not go
$40 \times 30 \times 1 = \ \ 1200$┃   into the dividend 481 and
$\ \ \ \ \ 1^3 = \ \ \ \ \ \ 1$┃   hence a zero is put after
                  481201┃481201   4 in the root

§25. If the given number is not a perfect cube, its
cube root may be obtained to any number of decimal places
by placing zeroes and bringing down periods of three zeroes
each

Example 3 Find the cube root of ˙3 to 3 decimal
places

Sol.   ˙300000000 (669   **Ans**
                  216
                    ┃84000
$6^2 \times 300 = $ 10800┃   **Exp**   Since the root is to be
$6 \times 30 \times 6 = $ 1080┃   found to 3 decimal places,
$\ \ \ \ 6^2 = $ 36┃   there must be 3 periods of 3
                  11916┃71496   figures each in the decimal
$66^2 \times 300 = 1306800$┃12504000   part, hence we affix 8 zeroes
$66 \times 30 \times 9 = $ 17820┃   to 3
$\ \ \ \ \ 9^2 = $ 81┃ 922309
                  1324701┃ 581691

§26 When one more than a half of the digits in the
required root is obtained by the ordinary method, the rest
can be found by contracted division as explained in Art 7,
Chapter IX

§27 Cube root of a fraction

If we are to find the cube root of a fraction, the deno
minator of which is a *perfect cube*, we may find the cube
roots of the numerator and denominator separately , as in

(1) $\sqrt[3]{\dfrac{64}{729}} = \dfrac{\sqrt[3]{64}}{\sqrt[3]{729}} = \dfrac{4}{9}$ **Ans.**

$(\imath\imath)$ $\sqrt[3]{\dfrac{29}{125}} = \dfrac{\sqrt[3]{29}}{\sqrt[3]{125}} = \dfrac{3\cdot0723}{5} - = \cdot6144$ **Ans.**

But if the denominator be not a *perfect cube,* then we may either reduce the given fraction to an equivalent fraction whose denominator is a perfect cube and then proceed as in previous case or we may convert the given fraction to a decimal and proceed in the ordinary way as in

$\sqrt[3]{3\tfrac{4}{7}} = \sqrt[3]{\tfrac{25}{7}} = \sqrt[3]{\tfrac{25}{7} \times \tfrac{49}{49}}\sqrt[3]{\dfrac{1225}{343}} = \dfrac{\sqrt[3]{1225}}{\sqrt[3]{343}} = \dfrac{10\,6}{7} = 1\cdot5$

Ans.

Or $\sqrt[3]{3\tfrac{4}{7}} = \sqrt[3]{3\cdot571428} = 1\,5$ Ans

EXERCISE 73

Find the cube roots of —

1 1331	**2** 21952.	**3.** 2048383
4 19034163	**5** 105823817	**6** 702121283072
7 12812 904	**8** 103 823.	**9.** ˙000512.
10 000012326391	**11.** 876467493	**12** $\frac{512}{729}$
13 $465\frac{31}{64}$.	**14** $\dfrac{132\,651}{64}$	**15** 18 609625.
16. ˙001030301	**17.** ˙037.	**18.** 1587˙962

Find the cube roots of the following up to 3 places of decimals —

19 $\frac{5}{7}$.	**20** $18\frac{7}{13}$.	**21.** ˙003 .
22 $\frac{1}{10}$	**23.** 7˙52	**24** $5\frac{4}{9}$.
25 Find the cube root of 1371742108367626890260631.		

§28   Extraction of some other roots.

The Fourth root of a number is found by finding the square root of the square root of the number, *e. g* ,

$\sqrt[4]{81} = \sqrt{9} = 3$ Ans

§29   The Sixth root of a number is found either by finding the cube root of the square root of the number or by finding the square root of the cube root of the number, $e\,g$,

$$\sqrt[6]{4096}=\sqrt[3]{64}=4 \text{ , or } \sqrt[6]{4096}=\sqrt{16}=4. \quad \textbf{Ans.}$$

§30   The Eighth root of a number is found by finding the square root of the square root of the square root of the number, i.e., the process of finding the square root is to be done thrice, $e.g$,

$$\sqrt[8]{256}=\sqrt[4]{16}=\sqrt{4}=2. \quad \textbf{Ans}$$

§31   The Ninth root of a number is found by finding the cube root of the cube root of the number, $e\,g$.

$$\sqrt[9]{19683}=\sqrt[3]{27}=3. \quad \textbf{Ans}$$

EXERCISE 74

Find the fourth roots of :—

1	1679616	**2**	1575 2961

Find the sixth roots of :—

3	531441	**4**	308·915776

Find the eighth roots of :—

5	214358881.	**6**	429981696

Find the ninth roots of :—

7	1953125.	**8.**	134217728

CHAPTER XII

METRIC SYSTEM AND DECIMAL COINAGE

§1. A Board of Mathematicians recommended a new system of weight and measure called the **Metric system** after the French Revolution of 1879 It was first originated in France and then was introduced in almost all the countries of Europe This system, the fundamental unit of which is a **Metre,** whence it derives its name, proceeds upon the principle of decimal division It is, therefore, being an easy calculation, largely used in scientific measurements in all parts of the world

§2. The unit of length is **Metre** which is equal to 39˙3707 inches nearly

TABLE

10 millimetres (mm)	=	1 centimetre (cm)
10 centimetres	=	1 decimetre (dm)
10 decimetres	=	1 Metre (m)
10 metres	=	1 decametre (Dm)
10 decametres	=	1 hectometre (Hm)
10 hectometres	=	1 kilometre (Km)
10 kilometres	=	1 myriametre (Mm)

4 inches.
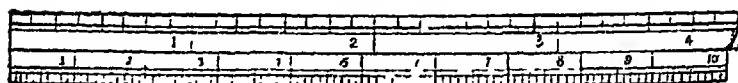
$\frac{1}{10}$m or 1 dm. or 10 cm. or 100 mm.

The root words in the Metric system are derived from the Greek and Latin

Greek			Latin
Deca means	10 times		Deci means one-tenth of.
Hecto „	100 „		Centi „ one-hundredth of.
Kilo „	1000 „		Milli „ one-thousandth of.
Myria „	10000 „		

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The following diagram will best illustrate the above Metric table —

Diagram.

Myriametre | Kilometre | Hectometre | Decametre | Metre. | Decimetre | Centimetre | Millimetre.

It will be seen that the value to the left of **Metre** increases tenfold and that to the right decreases tenfold.

Note Long distances are generally quoted in Kilometres, 1 kilometre = ⅝ English mile nearly

§3   Reduction It is clear from the above diagram that problems of reduction may be solved by multiplying or dividing by 10 as in the decimal system of measurement.

The following examples will best illustrate the Method of work —

Example 1   Express 9ʼ05 metres in millimetres.
Sol   9 05 metres = 90 5 decimetres
                         = 905 centimetres
                         = 9050 millimetres. **Ans.**

Example 2   Express 5915ʼ5 millimetres in metres
Sol.   5915 5 millimetres = 591 55 centimetres
                              = 59 155 decimetres
                              = 5ʼ9155 metres.
Evidently this answer may be read as
                         5 m 9 dm 1 cm 5 5 mm **Ans.**

Example 3.   Express 8972ʼ2 metres in kilometres.
Sol   8972 2 metres = 897 22 decametres
                         = 89 722 hectometres
                         = 8 9722 kilometres.
= 8 Km. 9 Hm. 7 Dm 2 m. 2 dm. **Ans.**

Example 4   Read as quickly as possible 25 1653894 myriametres into kilometres, hectometres, etc.

25 1653894 Mm = 25 Mm 1 Km 6 Hm 5 Dm. 3 m. 8 dm 9 cm. 4 mm. **Ans**

Example 5   Read as quickly as possible 893756
n illimetres into centimetres, etc
893756 mm =8 Hm 9 Dm 3 m 7 dm. 5 cm 6 mm. **Ans.**

Example 6   Multiply 2 26 metres by ·05 and read
the product in centimetres, etc

Sol   2 26
                        05
The product= ·113$\emptyset$ metres
            = 1 dm. 1 cm 3 mm.   **Ans**

Example 7   Divide   1262 5 kilometres by 32 and
read the quotient in kilometres, hectometres, etc.

Sol.   2) 1262 5
                    16)   631 25
The quotient=   39·453125 Km
=39 Km 4 Hm 5 Dm. 3 m 1dm 2cm 5 mm. **Ans**

EXERCISE 75

Reduce —

1   179 millimetres to decimetres, etc

2   1215 centimetres to decametres, etc.

3   1679 centimetres to metres, etc.

4   15 decametres to millimetres

5   15·5 kilometres to millimetres

Read the numbers in the following —

6   5 075 metres.   7   9 7018 kilometres

8   515 4 centimetres   9   8 972 metres

Simplify (10—12) and express the answer in metres, etc.—

10.   2125 millimetres + 5123 centimetres + 1215 deci-
metres + 21 metres

11.   156 035 metres — 149002 kilometres.

12   176005 kilometres — 142 056 metres.

13.   Multiply 1 15 metres by 003 and express the
result in millimetres

14   Multiply 13 15 decimetres by 2 5 and express the
product in metres, etc

15 Divide 1501'308 kilometres by 9 and express the quotient in millimetres

16 Divide 151300 millimetres by 1250 and express the quotient in metres.

17. The circumference of a wheel is 48 dms and it makes 2½ revolutions per second How long will it take to travel 48 miles ?

----

§4 Surface The unit of surface is the *square metre* In measuring land, the unit used is a *square deca-metre*. This is called an are

TABLE

10 centiares (sq m) make 1 deciare =11 96033 sq yds.
10 deciares   „   „   1 are   =119 6033 sq yds.
10 ares   „   „   1 dekare =1196 033 sq yds
10 dekares   „   „   1 hectare =11960 33 sq yds.
                                  =2½ Eng. acres nearly

Note Large surfaces are generally quoted in hectares

Example   16506 centiares=1650 6 deciares
                           =165'06 ares
                           =16 506 dekares
                           =1 6506 hectares
=1 hectare, 6 dekares, 5 ares, 6 centiares   Ans

§5 Volume The unit of volume is the *cubic metre*. In measuring wood this is called a stere

TABLE

10 decisteres make 1 stere   =35 317 cub ft.
10 steres   „   1 dekastere   =353 17 cub ft.
Example.   1685 decisteres   =168 5 steres
                                    =16'85 dekasteres.
      =16 dekasteres, 8 steres, 5 decisteres   Ans.

§6 Capacity The unit of capacity for liquid and dry goods is the **Litre** It is equal to a cubic decimeter.

TABLE

10 centilitres (cl) make 1 decilitre (dl)=1'714704 ch
10 decilitres   „   1 Litre (lit.) =1 07169 srs.

10 litres   make 1 dekalitre   (Dl)=10 7169.
10 dekalitres   „   1 hectolitre (Hl)=2 679 mds.
10 hectolitres „   1 kilolitre   (Kl)=26 79 mds.

Note   1 litre=1⅘ English pint nearly

Example.   15016 5 litre=1501 65 decalitre (Dl)
                          =150 165 hectolitre (Hl)
                          =15 0165 kilolitre (Kl)
                          =15 Kl 1 Dl 6 lit 5 dl. **Ans.**

§7   **Weight**   The unit of weight is the *gramme*
which is the weight of a cubic centimetre of distilled   water
at a certain fixed temperature   The weight of a *litre* of
such water is 1000 grammes, or one **kilogram.**

TABLE

10 milligrams (mg) make 1 centigram (cg)

10 centigrams   „   1 decigram (dg.)

10 decigrams   „   1 **gram** (gr)=1 028823 mashas

10 grams   „   1 decagram (Dg)= 8573527 tolas

10 decagrams   „   1 hectogram (Hg.)=8 573527 „

10 hectograms   „   1 kilogram (kg)=1 07169 seers

10 kilograms   „   1 myriagram (mg)=10 7169 „

10 myriagrams   „   1 quintal

10 quintals   „   1 millier.

§8   **To sum up**   We now give below the principal
measures in the metric system with their approximate
English equivalents and *vice versa*

(i) **Measures of Length**

1 metre   =39 3708 inches=1'0936 yds.
1 kilometre =·6214 mile   =1094 yds.

---

1 inch =   2 54 cm. │ 1 yard =   9144 metres.
1 foot =   3 05 dm   │ 1 mile   =   1 61 kilometres

Hence, a **kilometre** is nearly 5 furlongs or ⅝ of a
mile and a metre is nearly 1¹⁄₁₁ yards

(ii) Measures of Area.

1 sq metre = 10 7643 sq ft.

= 1˙1961 sq yds

and

1 sq inch = 6 4515 sq cm.

1 acre = 4047 hectares

Hence an hectare is a little less than 2½ acres.

(iii) Measures of Capacity.

1 litre = 0353 cu ft = 1 7608 pints

1 cu metre = 220 0967 gallons

and

1 cu inch = 16 3862 cu. cm

1 pint = 9465 litres

1 gal. = 4 5435 litres.

Hence a litre contains a little more than 1¾ pints.

(iv) Measures of Weight.

1 gram = 15 43 grains

and

1 grain = 0648 grams

Hence a gram is nearly 15½ grains.

§9 Money The unit of money is the Franc. It is about 9¾d. English

TABLE

10 centimes (c) make 1 decime

10 decimes ,, 1 franc (fr)

Accounts are kept in francs and centimes only; thus "20˙56 francs" is read as 20 fr and 56 c The franc is a silver coin composed of 9 parts of silver and one part of copper and weighs 5 grams. The *Napoleon* is a gold coin = 20 francs.

§10   The advantages of metric system

The great advantages of this system, as we observe from the above tables, are that (*i*) a compound quantity can be reduced to a single quantity and *vice versa* without any process, (*ii*) that the relative size of the different units are very easily understood

Note. In the examples given below, the specific gravity of a substance means the ratio which the weight of any volume of the substance bears to that of an equal volume of water.

EXERCISE 76.

1   Find the weight of 37 hectolitres of water.

2   How many fields each containing 2 hectares, 47 ares can be made from a farm of 313 hectares, 69 ares ?

3.   Find the weight of 20 cu cm of mercury, whose specific gravity = 13 5

4.   If a man walks 11 metres in 5 secs how many kilometres can he walk in an hour ?

5.   Find the weight in kilos of a bar of gold 10 cm. × 30 mm × 25 mm., its specific gravity being 19˙36

6.   A gentleman's income is reduced from 56085 fr. 50 c to 52720 fr 37 c by income tax   (1) How much tax does he pay in a franc ?   (2)   How much tax does he pay in the £, taking £1 = 25 francs ?

7.   After paying an income-tax of 15 c. in a franc, a gentleman's income is 365318 fr. 10 c. Express the income-tax paid by him as a fraction of his net income.

8   If the specific gravity of alcohol be ˙8, how many kilograms of alcohol will fill one-third of a tank whose measurements are 1 50 metres by 3˙20 metres by 80 cm ?

9   If the circumference of a wheel be 1 dekam 25 decim and if it makes 2½ revolutions in 1 second, how long will it take to travel 50 miles, taking 1 mile = 1⅗ kilom ?

10   If a sq. metre = 1550˙031 sq inches, find to five decimal places the number of sq cm. in a sq. inch

CHAPTER XIII

MEASUREMENT OF AREA

Note Some questions in this chapter are solved by the method of ratio and alligation   They may be omitted till those methods are learnt

§1. Observe the following figures —

```
┌─────────────────────┐         ┌──────────────┐
│                     │         │              │
│        Fig  I       │         │   Fig. II    │
│                     │         │              │
│                     │         │              │
└─────────────────────┘         └──────────────┘
```

Figure I is a rectangle and figure II is a square

Their surfaces are bounded by four straight lines making every angle a right angle   The opposite sides of the rectangle are equal while the square has all its sides equal.

The Perimeter of a rectangle or a square is the sum of all its sides   Hence the

Perimeter of a rectangle = $2 \times$ (length + breadth) and

Perimeter of a square   = $4 \times$ side

Note   The length and breadth of a rectangle are called its dimensions

§2   Observe also the following two more figures —

```
┌─────────────────────┐         ┌──────────────┐
│                     │         │              │
│        Fig  I.      │         │   F g  II.   │
│                     │         │              │
└─────────────────────┘         └──────────────┘
```
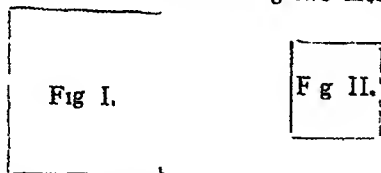
Each side in figure I is 1 inch and each side in figure II, 1 centimetre.   The first figure, therefore, represents one square inch and the second figure one square centimetre

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Thus a square the side of which measures 1 foot represents one square foot and so on

The area of a figure means the amount of surface enclosed by its bounding lines in square measures

§3   To find the area of a rectangle

The following examples will illustrate the rule of finding the area of a rectangle

Example 1   Find the area of a rectangle whose length is 10 inches and breadth 2 inches.

Let *ABCD* be a rectangle of which the length *AB* is 10 inches and the breadth *AD* is 2 inches.   Divide *AB* and
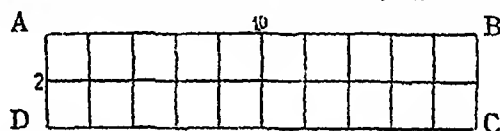
AD into 10 and 2 equal parts respectively and through the points of division draw lines parallel to *AD* and *AB* respectively   The rectangle has thus been divided into 20 equal squares each having one square inch surface Hence the area of the rectangle is 20 square inches.   Note that $10 \times 2 = 20$

Learn, therefore, the following formulæ —

Area of a rectangle = Length × Breadth
Whence Length   = area − breadth
and Breadth   = area − length

§4.   To find the area of a square

A square is also, as you have noticed by the figure, a kind of rectangle   The only difference is that it has all its sides equal.

Hence the area of a square = (side)²
Whence the side of the square = √area

Example 2   Find the area of a square whose side is 3 ft

Sol   .   area of a square = (side)²

       the reqd, area = $3 \times 3 = 9$ sq, ft **Ans**

Note   '9 square feet' denote an area 9 times as large as a square foot   But if we write "9 feet square" it will denote the area of a square whose side is 9 feet

Example 3   The length of a room is 15 ft 6 in and width 12 ft 6 in   Find its area

Sol   Area $=$ Length $\times$ Breadth

$$= 15\tfrac{1}{2} \times 12\tfrac{1}{2}$$
$$= \tfrac{31}{2} \times \tfrac{25}{2} = \tfrac{775}{4} \text{ sq ft}$$
$$= 193 \text{ sq ft } 108 \text{ sq. in } \textbf{Ans}$$

Example 4   The area of a room is 372 sq. ft , the breadth is 15 ft 6 in , find the length

Sol   Length $=$ area $-$ breadth

$$= 372 - \tfrac{31}{2}$$
$$= 372 \times \tfrac{2}{31} = 24 \text{ ft } \textbf{Ans}$$

Example 5   The area of the floor of a room is 36 sq yds 6 sq ft 112 sq in, the breadth is 19 ft 1 in. , find its length

Sol   Length $=$ area $-$ breadth

$$= 36 \text{ sq yds } 6 \text{ sq. ft. } 112 \text{ sq in.} - 19 \text{ ft} \quad 1 \text{ in}$$
$$= 330 \text{ sq ft } 112 \text{ sq in.} - 19 \text{ ft } 1 \text{ in.}$$
$$= \tfrac{2977}{6} \text{ sq. ft.} - \tfrac{229}{12} \text{ ft.}$$
$$= \tfrac{2977}{6} \times \tfrac{12}{229} \text{ ft.} \quad = \tfrac{52}{3} \text{ ft.}$$
$$= 17 \text{ ft. } 4 \text{ in } \textbf{Ans.}$$

§5   Diagonal of a rectangle and a square.

(i) $ABCD$ is a rectangle and BD is the diagonal.

Now BCD is a right-angled $\triangle$
$\therefore BD = \sqrt{(BC)^2 + (CD)^2}$
If $BC = 5$ ft and $CD = 12$ ft.
Then $BD = \sqrt{(5)^2 + (12)^2}$
$$= \sqrt{25 + 144} = \sqrt{169}$$
$$= 13 \text{ ft. } \textbf{Ans}$$
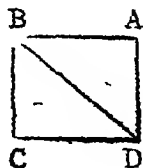
(11) *ABCD* is a square and *BD* is the diagonal.

Now *BCD* is a right angled $\triangle$
$$BD = \sqrt{(BC)^2 + (CD)^2}$$
If *BC* = *CD* = 5 ft
Then *BD* $= \sqrt{(5)^2 + (5)^2}$
$= \sqrt{25 + 25}$ $= \sqrt{50}$
$= \sqrt{25 \times 2}$ $= 5\sqrt{2}$
$= BC\sqrt{2}$

Since we see that the area of the square = $5 \times 5 = 25$ sq ft and the diagonal $= \sqrt{25 \times 2}$ we conclude that

(i) Diagonal = $\sqrt{2\ \text{area}}$ or side $\times \sqrt{2}$
(ii) Area = half the square of its diagonal

Example 6 A square recreation ground containing 33 acres 3765 square yards 4 square feet is to be enclosed with barbed wire placed at heights 2, 4, 6 feet above the ground. Find the length of wire if its length required for each round is $3\frac{1}{3}\%$ greater than the perimeter of the ground

Sol Area = 33 ac 3765 sq yds 4 sq. ft
      = 1471369 sq. ft.

perimeter = $\sqrt{1471369} \times 4$ ft
      = 4852 ft

∵ the wire is to make 3 rounds and the length of each round = $4852 \times \frac{310}{400}$ ft

∴ The reqd, length = $4852 \times \frac{310}{400} \times 3$ ft.
      = 15041 2 ft **Ans**

EXERCISE 77

Find the area of the rectangles having the following dimensions —

  1 18 ft by 10 ft
  2 12 ft 8 in by 10 ft 6 in.
  3. 6 ft. 6 in by 8 ft 8 in.

Find the area of the following in sq yards, sq feet and sq inches —

  4 A room 20 ft 8 in long, 12 ft 6 in broad
  5 A courtyard 19 ft 1 in long, 17 ft. 4 in broad.
  6 A courtyard 45 ft 6 in long, 28 ft 8 in b oad

7   Find the area of a rectangular field 440 yards long and 220 yards broad in acres

8   A square field is 550 yards long, find its area in acres

9   How many acres of land are there in a field 550 yards long, 220 yards broad ?

10.   How many sq yards of matting will be required for a rectangular room 45 ft. long, 18 ft 4 in. broad ?

11   Find the breadth of a room, the length of which is 35 ft and area is 46 sq yards 6 sq. ft.

12   A room contains 48 sq yards 6 sq. ft. What will be its length when the breadth is 18 ft ?

13   A rectangular field contains 2 acres   What will be its length when the breath is 22 yards ?

14   A square field contains 10 acres , find its side

15   A room is 20 metres in length and 10 metres in breadth   Find the number of square yards in the area of the floor, taking a metre as equal to 39 37 inches

16   The length of a field containing 21 acres, 3 roods, 25 sq poles $3\frac{7}{8}$ sq yards is twice its breadth   Find the length of the field

17   What is the area of a square whose diagonal is 21 ft ?

18.   The area of a square field is 162 sq. ft.   Find its diagonal

19   How long will a man take to run round the boundary of a square field containing 250 acres at the rate of 3 miles per hour ?

20   What length of wire will be required to enclose a square garden containing 62 5 acres placed at heights 1, 2, 3, 4 ft. above the ground, each circuit being 4% longer than the perimeter of the garden ?

§6   Carpeting or paving rooms, etc

The area of the carpet required to cover the floor of a room or the area of stones or bricks required for paving the floor of a room or courtyard is evidently equal to the area of its floor.   F.15

$$\text{length of the carpet} = \frac{\text{area of the floor}}{\text{breadth of the carpet}},$$

$$\text{and breadth of the carpet} = \frac{\text{area of the floor}}{\text{length of the carpet}}$$

$$\text{No of bricks or stones} = \frac{\text{area of the floor}}{\text{area of one brick or stone}}$$

Example 1. Find the cost of carpeting the floor of a room 35 ft long and 20 ft. broad at $2a$ per sq. ft.

Sol   Area of the floor $= 35 \times 20 = 700$ sq. ft
area of the carpet $=$ area of the floor
```
    ,,      ,,      ,,    = 700 sq ft.
    cost    ,,      ,,    = Rs ⅛ × 700
                          = Rs 87 8a    Ans.
```

Example 2   Find the cost of carpeting a room 30 ft long and 24 ft. broad with carpet 18 in wide at $2a\ 6p$. per yard.

Sol   Area of the carpet $= 30 \times 24 = 720$ sq ft
length of the carpet $= 720 - \frac{18}{12} = 480$ ft $= 160$ yds.
cost of the carpet $= \text{Rs.} \frac{5}{32} \times 160$
                                      $= \text{Rs } 25$ **Ans.**

Example 3   How many bricks 9 in by 4 in will be required for paving a room 36 ft by 21 ft ?

Sol   Area of the floor $= 36 \times 21 = 756$ sq. ft.
and area of one brick $= 9 \times 4$ $= 36$ sq in
                                         $= \frac{1}{4}$ sq ft

$\therefore$ No. of bricks reqd. $= \dfrac{\text{area of the floor}}{\text{area of the brick}}$

$= 756 - \frac{1}{4} = 3024.$ **Ans**

Example 4   A room 26 ft 3 in long and 23 ft 9 in broad is to be paved with equal square tiles , find the largest size of each tile and the number of tiles required.

Sol.   26 ft. 3 in. $= 315$ in
         23 ft 9 in. $= 285$ in
         G C M. of 315 and 285 $= 15$

the size of the tile is 15 in. square
and No of tiles $= \dfrac{315 \times 285}{15 \times 15} = 399$ $\Bigg\}$ **Ans**

Example 5　The length and breadth of a room are as 3 2 and the cost of matting it at 5a. per sq. yard is Rs 7. 8a. Find its dimensions.

Sol.　Re $\frac{5}{16}$ is the cost of 1 sq yard,

..　　Re. 1 is the cost of $\frac{16}{5}$ sq yards,

∴　Rs. $\frac{15}{2}$ is the cost of $\frac{16}{5} \times \frac{15}{2} = 24$ sq. yards.

Now the area of the floor of the room is 24 sq yards, or 216 sq ft. Consider a rectangle with dimensions 3 ft and 2 ft It will contain 6 squares each of 1 sq. ft.
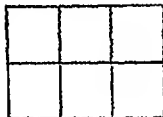
the area of one square $= 216 - 6 = 36$ sq. ft.

the side of the square $= 6$ ft

. the length of the room $= 6 \times 3 = 18$ ft $\Big\}$ **Ans**

and the breadth　„　$= 6 \times 2 = 12$ ft

Example 6　The cost of carpeting a room is Rs. 187 8a. Had the width of the room been 3 ft less the cost of the carpet would have been Rs 150 only Find the width of the room

Sol Diff between the costs $= 187\frac{1}{2} - 150 = $ Rs 37$\frac{1}{2}$,

i e, Rs 37$\frac{1}{2}$ is the cost of a carpet of a room 3 ft wide

.　Re 1　„　„　„　$3 \times \frac{2}{75}$ ft　„

Rs 187$\frac{1}{2}$ „　„　„　$3 \times \frac{2}{75} \times \frac{375}{2}$ ft　„

∴ width reqd $= 3 \times \frac{2}{75} \times \frac{375}{2}$ ft. $= 15$ ft　**Ans**

Example 7　A room is 20 ft long and 18 ft wide Had its length been 5 ft more and width 2 ft more the cost of the carpet would have increased by Rs 4 6a　Find the cost of the carpet

Sol　Area of the floor　$= 20 \times 18 = 360$ sq ft

Area in the 2nd case $= (20 + 5)(18 + 2)$ sq ft

$= 500$ sq ft

Diff in the areas　$= 500 - 360 = 140$ sq. ft

The cost of carpet 140 sq. ft in area $=$ Rs 4$\frac{3}{8}$

.　„　„　1　„　„　$=$ Rs $\frac{35}{8} \times \frac{1}{140}$

.　„　„　360　„　„　$=$ Rs $\frac{35}{8} \times \frac{1}{140} \times 360$

$=$ Rs. 11 4a　**Ans**

Example 8　The length of a room is 8 ft more than its breadth and its perimeter is 80 ft. Find the cost of carpeting it at 8a 6p per sq. yard

Sol.   Perimeter = (width + width + 8 ft.) × 2 = 80 ft.
∴   4 width + 16 ft = 80 ft.
∴   4 width = 64 ft
∴   width = 16 ft
and length = 16 + 8 = 24 ft.

Now solve the question further.

Example 9.   The sum of the length and breadth of a room is 27 ft and the area is 180 sq ft   Find its dimensions

Sol.   Consider any room whose dimensions are known to you and verify the following formulæ —

(length + breadth)^2 − (area × 4) = (length − breadth)^2
Hence (27^2 − 180 × 4) = (length − breadth)^2
∴ 729 − 720   = (length − breadth)^2
∴ Diff of the sides   = $\sqrt{9}$ = 3 ft.
but sum of the sides = 27 ft
∴ length = (27 + 3) ÷ 2 = 15 ft $\Big\}$ **Ans.**
and breadth = (27 − 3) ÷ 2 = 12 ft

EXERCISE 78.

Find the length of carpets required for the floor of (1—4):

1.   A room 15 ft long, 12 ft. broad , carpet 2 ft wide.

2.   A room 21 ft long, 18 ft broad , carpet 3 ft wide.

3.   A room 18½ ft. long, 13½ ft. broad , carpet 27 in. wide.

4.   A room 25½ ft. long, 16½ ft broad; carpet 44 in. wide.

5   How many pieces of carpet each 6 ft long 3 ft wide will cover the floor of a room 21 ft by 12 ft. ?

6.   How many pieces of carpet each 10 ft long, 27 in wide will cover the floor of a room 16 ft 8 in. by 13 ft. 6 in ?

7   How many paving stones each 2 ft long, 1½ ft. wide would be required to pave a rectangular courtyard 45 ft. by 25 ft ?

8   How many bricks each 9 in. long and 6 in. wide would be required to pave the floor of a room 18 ft. 6 in by 13 ft. 6 in. ?

9.   How many paving stones each 1½ feet long 9 in wide would be required to pave a square courtyard whose side is 30 ft. ?

10. Find the cost of carpeting a room 15 ft. 4 in by 14 ft 3 in. at 4𝑎 per square foot

11. Find the cost of matting a room 24 ft 9 in by 16 ft at 2𝑎. per sq yard

12. What will be the expenses of paving a court-yard 18½ ft. by 14½ ft. at 3𝑎 per sq. yard ?

13. Find the cost of carpeting a room 18 ft 8 in long, 16 ft 6 in. broad with carpet 2 ft. 4 in. wide at 8𝑎. per yard

14. Find the expenses of carpeting the floor of a room 21 ft 4 in by 14 ft. 3 in. with carpet 2 ft. wide at 12𝑎 per yard

15. What will be the expenses of paving a court-yard 50 ft by 33 ft with paving stones 1½ ft by 6 in. at Rs 16 per hundred ?

16. A hall is 48 ft long and the cost of carpeting it at 5𝑎. 4𝑝 per square yard amounts to Rs 53 5𝑎 4𝑝 Find its breadth

17. A hall is 45 ft long and the expenses of carpeting it at 8𝑎 per square yard amounts to Rs. 100 Find its breadth.

18. The cost of paving and varnishing the floor of a square room at 2𝑠 6𝑑. per square ft is £ 78 2𝑠. 6𝑑. Find its side

19. The length of a room is three times its breadth and the cost of matting it at 2𝑎. 3𝑝. per square yard is Rs 6. 12𝑎 Find its dimensions.

20. The length and breadth of a room are as 3 : 2 and the cost of carpeting it at 2𝑎. 6𝑝. per square yard is Rs. 23 7𝑎 Find its dimensions.

21. The length and breadth of a room are in the ratio of 5 4 and the cost of carpeting it at 2𝑎. 3𝑝. per sq. yard is Rs. 31 4𝑎 Find its dimensions.

22. A room is 25 ft by 21 ft and the cost of carpeting it at 4𝑎. per yard is Rs 21. Find the width of the carpet

23. A room is 24 ft by 16 ft and the cost of carpeting it with carpet 18 in wide is Rs 24. Find the cost of the carpet per yard

24. A courtyard is 40 feet long and the cost of car-

peting it at Rs 18 per 100 sq yards is Rs. 20.   Find the
cost of fencing it round at 12a. per yard

25   The expenses of carpeting a square courtyard
at 1a. per square ft is Rs 64.   Find the cost of fencing
it round at 4a per ft.

26. A room 38 ft 9 in by 30 ft. is to be paved with
equal square tiles , find the largest tile which will exactly
fit and also find the number of tiles required

27   A courtyard 37 ft 6 in. by 31 ft. 6 in. is to be
paved with equal square tiles , find the largest tile which
will exactly fit and also find the number of tiles required

28   The area of a rectangular field whose breadth
is 500 yards is 100 acres   Find the cost of cultivating it
at Rs 3. 2a 8p per 100 sq. yards and also the cost of
fencing it round at Rs. 2 8a. per yard

29   The length of a rectangular field is twice its
breadth   If the rent of the field at £ 3 7s 6d an acre be
£ 151 17s 6d , find the cost of surrounding it with a
fence at 4¾d per yard

30   A rectangular courtyard the sides of which are
as 5   11 costs Rs. 144. 6a. for paving at 10a 6p per square
yard   Find the length of its sides.

31   The cost of carpeting a room is Rs 106. 10a 8p
If the width of the room had been 4 ft. less, the cost of
carpeting would have decreased by Rs. 26. 10a 8p. ,
find the width of the room

32   A room is 16 ft long and 12 ft broad. Had its
length and breadth been 2 ft. and 1 ft. more respectively,
the cost of carpeting would have increased by Rs. 5. 4a.
Find the cost of carpeting it.

33   A room is 16¼ ft. long and 12½ ft. broad
Had the length been 3½ ft. more and breadth 1½ ft. less,
the cost of carpeting it would have increased by Rs. 3. 7a.
Find the cost of carpeting it

34   The length of a room is 10 ft. more than its
breadth and its perimeter is 100 ft , find the cost of car
peting it at 7a. 6p per sq. yard.

35   The sum of the length and breadth of a room
is 41 ft. and its area is 400 sq ft. , find its dimensions.

§7   Verandah round a room and Path round a garden

Example 1. A room 30 ft. long, 20 ft broad is surrounded by a verandah 5 ft. wide   Find the area of the verandah and also the cost of paving it at 2a. per sq. yard.

Sol. It is clear from the diagram that the area of the verandah is equal to the area of the outer rectangle *minus* the area of the inner rectangle
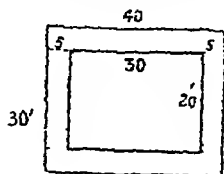
Area of the outer rectangle $=40 \times 30$ $= 1200$ sq ft.

  „   „   inner   „ $= 30 \times 20$ $= 600$ sq ft.

   area of the verandah $= 1200 - 600$ $= 600$ sq. ft.

   Hence, cost $= \text{Re } \frac{1}{8} \times 600$ $= \text{Rs. } 75$ Ans

Aliter   The length of verandah
   $= 30 + 30 + (20 + 10) + (20 + 10)$
   $= 120$ ft., breadth $= 5$ ft.
  area $= 120 \times 5 = 600$ sq ft
   Hence cost $= \text{Rs. } \frac{1}{8} \times 600$
        $= \text{Rs. } 75$ Ans.
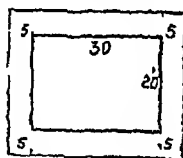
Aliter   Area of $ABGE = 40 \times 5 = 200$ sq. ft

Area of $FMDC = 40 \times 5$
        $= 200$ sq ft.
Area of $ELPF = 20 \times 5$
        $= 100$ sq ft
Area of $GRQM = 20 \times 5$
        $= 100$ sq. ft.
Total area $= 200 + 200 + 100 + 100$
        $= 600$ sq. ft
Cost $= \text{Re. } \frac{1}{8} \times 600 = \text{Rs } 75$ Ans
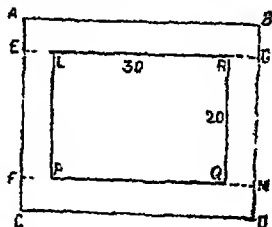
Example 2   A lawn 150 ft long, 120 ft. broad has a path 10 ft. wide inside running round it.   Find the cost of covering the path with flag stones at 2s 6d. per square yard.

Sol.
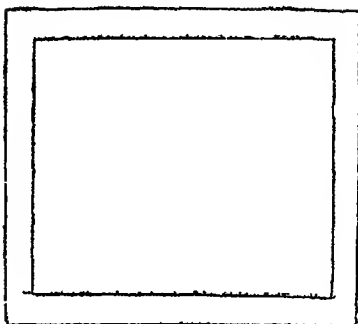
Area of the outer rectangle $= 150 \times 120 = 18000$ sq. ft.

    ,,   ,, inner rectangle $= 130 \times 100 = 13000$ sq. ft.

    ∴ area of the path $= 18000 - 13000$ sq. ft.

                         $= 5000$ sq. ft. $= \frac{5000}{9}$ sq. yds.

    ∴ cost $= £\frac{1}{8} \times \frac{5000}{9} = £\frac{625}{9}$

                         $= £69 \ 8s \ 10\frac{2}{3}d$ **Ans**

The second and third methods are left for the student as exercises

Note   In the first example twice the width of verandah was *added* to the length and breadth of the room because the verandah was *outside* it

In the second example twice the width of the path was *subtracted* from the length and breadth of the lawn because the path was running *inside* it

Example 3   A rectangular lawn 80 ft. by 60 ft. has two roads each 10 ft. wide running in the middle of it, one parallel to the length and the other parallel to the breadth   Find the cost of gravelling them at $3a$. per square yard.

Sol.
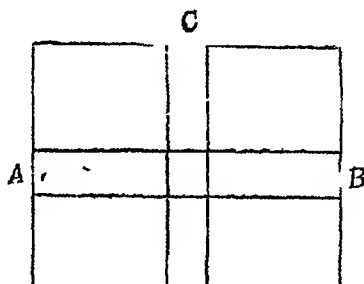
Area of the path AB $= 80 \times 10$ $= 800$ sq. ft
Area of the path CD $= 60 \times 10$ $= 600$ sq ft.
Total area $= 800 + 600 = 1400$ sq ft.

But area of the common portion $= 10 \times 10 = 100$ sq. ft.

∴ area of the roads to be gravelled

$= 1400 - 100 = 1300$ sq. ft.

∴ cost $= \text{Re } \frac{3}{16} \times \frac{1300}{9} = \text{Rs. } \frac{325}{12}$

$= \text{Rs } 27. 1a \ 4p.$ **Ans.**

Example 4   A path 9 ft wide, running all round within a square garden has an area of 3 acres   Find the area of the part of the garden enclosed by the path

Sol $ABCD$ is a garden, $abcd$ is the part enclosed by the path Produce the sides as shown in the diagram.
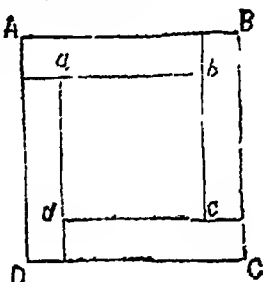
Now area of the four equal rectangles $= 3$ acres.

∴ area of one rect $= \dfrac{3 \times 4840}{4}$ sq yds

$= 3630$ sq. yds

∵ breadth of the rectangle $= 3$ yds.

∴ length of the rectangle $= 3630 \div 3 = 1210$ yds

∴ $ab = 1210 - 3 = 1207$ yds.

∴ the area of the sq $abcd = 1207 \times 1207$ sq yds.

$= 1456849$ sq yds.

$= 301$ acres 9 sq. yds. **Ans.**

Example 5   A path of uniform width running *outside* a rectangular garden contains 2800 sq ft   If the dimensions of the garden be 150 ft and 120 ft , find the width of the path

Sol.   In example 9 of Art. 5 we have stated a formula that $(\text{length} + \text{breadth})^2 - (\text{area} \times 4) = (\text{length} - \text{breadth})^2$, whence (*i*) $(\text{length} - \text{breadth})^2 + \text{area} \times 4 = (\text{length} + \text{breadth})^2$ and (*ii*) $(\text{length} + \text{breadth})^2 - (\text{length} - \text{breadth})^2 = \text{area} \times 4$.

It is also evident that the dif-
ference between the sides of the
outer rectangle must be the same
as the difference between the sides
of the inner rectangle.
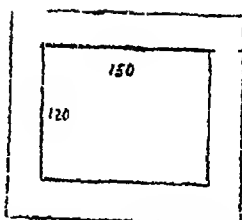
∴ the difference between the
sides of the inner rectangle is 30 ft

difference between the sides of the outer rectangle is
also 30 ft.

also area of the outer rectangle $=(150 \times 120) + 2800$ sq ft
$$=20800 \text{ sq ft.}$$

Now apply formula (i)

∴ $(30)^2 + 20800 \times 4 = (\text{length} + \text{breadth})^2$

Or $\sqrt{900 + 83200} = \text{length} + \text{breadth}$

Or length + breadth $= 290$ ft.

but length − breadth $= 30$ ft

∴   length $= (290 + 30) \div 2 = 160$ ft

i. e, the length of the outer side $= 160$ ft

∴   path $\times 2 = 160 - 150$ $= 10$ ft.

∴   path $= 5$ ft **Ans.**

Aliter. Since the difference between the length and
breadth of the inner rectangle is 30 ft therefore the differ-
ence in the dimensions of the outer rectangle is also 30 ft.
The area of outer rect. is $(120 \times 150) + 2800 = 20800$ sq ft.

Now find out 2 factors of 20800 whose difference is 30.

∴ $20800 = 160 \times 130$

∴ length of the outer rectangle $= 160$ ft.

∴ twice the width of the path $= 160 - 150 = 10$ ft.

∴   width „ $= 5$ ft. **Ans**

Example 6 A square garden $ABCD$ contains 1'8
acres. It is to be crossed by two paths along the diagonals
AC, BD. Each path is to be 8 yards wide with its centre
line along the diagonal Find the cost of making and
paving the paths at 6s. 3d per square yard.

Sol. '' Area=1 8 acs.

AC^2=1 8 × 2 acs.
    =3 6 × 4840 sq. yds

$AC = \sqrt{3\ 6 \times 4840}$
    =6 × 22 or 132 yds.

Now EF=8yds and G is the mid pt of E,
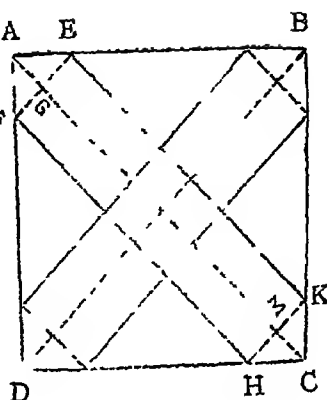
' $AG=GE=GF=4$ yards.   [AG bisects rt $\angle A$.
Similarly, $CM=4$ yards.

GM or $EK=132-(4+4)$ or 124 yards.

Area of the path $AEKCHF=\frac{1}{2} \times 8 \times 4 \times 2 + 124 \times 8$
                    or 1024 sq yards

Likewise area of the other path = 1024 ,,   ,,

Area of the centre sq =8 × 8 or 64 sq yards.

∴ Area reqd. for paving=1024 + 1024 − 64
                    =1984 sq. yds

Hence cost=£$\frac{5}{16}$ × 1984 = £620. **Ans**

Example 7 In the centre of the room 40 ft. square there is a square of carpet and the rest of the floor is covered with cloth. The price of the carpet is $12a$ per sq. yard and that of the cloth $4a\ 6p$ per sq. yard and the total cost on both is Rs 96 $14a$, find the width of the cloth.

Sol Area of the floor=40 × 40 = 1600 sq ft. Had there been only carpet on the floor the cost would have been Rs. $\frac{3}{4} \times \frac{1600}{9}$
                    =Rs. 133. 5a. 4b

i e , the cost would have been increased by Rs 133. 5a 4p. − Rs. 96. 14a = Rs. 36 7a. 4p.=Rs $\frac{875}{24}$.
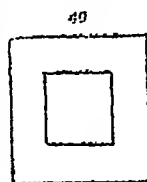
Again, if there had been only cloth on the floor the cost would have been Rs. $\frac{9}{32} \times \frac{1600}{9} =$ Rs. 50.

i.e., the cost would have been decreased by Rs. 96. 14a.—Rs 50=Rs. 46 14a = Rs $\frac{375}{8}$.

Hence the ratio between the areas of the carpet and the cloth would be inverse, *i.e.*, $\frac{375}{8} : \frac{375}{24} = 1125 \quad 875 = 9 : 7$

the area of the carpet $= 1600 \times \frac{9}{16} = 900$ sq. ft.

∴ The length of the carpet $= \sqrt{900} = 30$ ft. but the length of the room $= 40$ ft.

twice the width of the cloth $= 10$ ft.

width of the cloth $= 5$ ft   **Ans.**

Aliter. The area of the room $= \frac{1600}{9}$ sq. yards.

The mean $\Big\}$ $= \dfrac{\text{Rs. } 96 \ 14a \ \times 9}{1600}$
cost per sq yard $\Big\}$

$= \frac{279}{32}a$

carpet	cloth
$12a$	$\frac{9}{2}a$.
$\frac{279}{32}a$	
$\frac{279}{32} - \frac{9}{2}$	$12 - \frac{279}{32}$
$\frac{135}{32}$	$\frac{105}{32}$
9	7

By the Alligation Rule we got the ratio between the area of the carpet and the area of the cloth. Now proceed further.

EXERCISE 79.

1 A rectangular courtyard 100 ft. long, 80 ft. wide has within it a gravel path 8 ft wide running round it Find the area of the path and the cost of gravelling it at 5a 3p per square yard.

2. The area of a square garden is 10 acres. On the inside of the garden and along the four sides of it there is a gravel path 5 ft. wide Find the cost of constructing the path at 1a. 6p. per sq yard

3. Find the cost of paving a pathway 6 ft. wide round and immediately outside a flower garden 21 yards long and 10 yards broad, at $5\frac{4}{7}$ pies per sq. yard

4 A field is 110 yards long and 90 yards broad. On the inside of it along the four sides there is a path 5 yards wide. The cost of gravelling the path is 4a. per sq. yard and the cost of planting grass on the remaining field is Re 1 8a. per 100 sq yards. Find the total cost.

5   A field contains one acre, its length is 110 yards, on the inside of it along the four sides a road 5 ft wide has been constructed and the remaining field has been planted with grass at 2a 3p per sq yard. Find the cost

6   A room is 24 ft 4 in long and 14 ft 3 in broad It is surrounded by a verandah 4 ft. wide   Find the cost of carpeting it at 13a 6p per square yard

7.   A garden is 350 yds long and 250 yds broad It has two roads each 8 ft wide running in the middle of it, one parallel to the length and the other parallel to the breadth   Find the cost of gravelling them at 1a 3p. per sq. yard.

8   A garden 220 yards by 110 yards, has two roads 6 ft and 4 ft. wide running in the middle of it   The first road runs parallel to the length and the second parallel to the breadth.   Find the area of the roads.

9   How many paving stones, each of them 1 ft long, 9 in wide, will be required for paving a street 30 ft. wide, surrounding the outside of a square grass plot, the area of the grass plot being 10 acres ?

10.   A road 6 ft wide has been constructed in the middle of a garden parallel to the length   The area of the road is 420 square yards and the area of the garden is 31500 square yards, find the dimensions of the garden

11   A garden contains 2 acres and its breadth is 88 yards.   A road has been constructed in the middle of it parallel to the length of the garden.   If the area of the road be 1650 sq. ft, find its breadth

12.   A rectangular court is 50 yards long and 30 yards broad.   It has paths 6 ft in breadth joining the middle points of the opposite sides and also a path of the same breadth running all round it.   The remainder is covered with grass.   If the cost of pavement be 1s 8d per sq ft, and the turf 3s. per sq yard, find the cost of laying out the court

13.   A marginal walk all round the *inside* of a rectangular park 48 ft by 36 ft occupies 608 sq ft., find the width of the path

Exp If a diagram of the four walls of a room be drawn side by side, it will form a rectangle as shown below.

15′		The four	walls	
	20′	30′	2′?′	30′
A	B	C	D	A
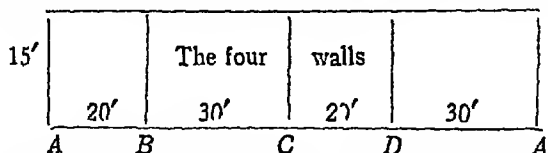
The length of this rectangle is equal to twice (the length + breadth) and the breadth is equal to the height of the room.

∴ the area of this rectangle is equal to the area of the four walls,

$$\text{Area of the four walls} = 2(\text{length} + \text{breadth}) \times \text{height}$$
$$= \text{Perimeter} \times \text{height}$$

Or,

Area of the wall length-wise = length × height
Area of the wall width-wise = width × height

∴ area of the two walls = length × height + width × height
$$= \text{height} (\text{length} + \text{breadth})$$

∴ area of the four walls = height (length + breadth) × 2
$$= \text{height} \times \text{perimeter}.$$

Thus the area = 2(30 + 20) ft × 15 ft.
$$= 1500 \text{ sq. ft} \quad \text{Ans}$$

Example 2. A room is 25 ft. long, 20 ft. broad and 15 ft. high. It has two windows each 4 ft by $2\frac{1}{2}$ ft, two doors each 6 ft. by 4 ft. Find the cost of papering the walls with paper 27 in. wide at $1a$ $1\frac{1}{2}p$ per yard

Sol. Area of the four walls   = 2(25 + 20) × 15 sq ft.
                                  = 1350 sq ft.

Area of the two windows   = $4 \times \frac{5}{2} \times 2 = 20$ sq. ft.
Area of the two doors   = 6 × 4 × 2 = 48 sq. ft.
Area of the doors and windows   = 20 + 48 = 68 sq. ft.
Area to be covered with paper   = 1350 − 68 = 1282 sq.ft
∴ paper reqd to cover the walls = $\frac{1282}{9} \times \frac{36}{27} = \frac{5128}{27}$ yds.

∴ cost   = Re $\frac{8}{128} \times \frac{5128}{27}$
      = Rs $\frac{641}{48}$
      = Rs. 13   5a   8p. **Ans.**

§9. Height of the room

Example 3   A room is 31 ft long, 21 ft broad and the cost of papering the walls with paper 26 in. wide at $5a$ $4p$. per yard is Rs 93 $5a$ $4p$ Find the *height* of the room

Sol   Re $\frac{1}{3}$ is the price of 1 yard paper

. Re. 1 is the price of 3 yards paper

∴ Rs $\frac{280}{3}$ is the price of 280 yards paper

paper required to cover the walls = 280 yards

. area of the paper $= 280 \times 3 \times \frac{26}{12}$ sq. ft
$$= 1820 \text{ sq ft.}$$

Since the area of the paper = the area of the walls,
$$1820 \text{ sq ft} = 2(31 + 21) \times \text{height}$$
$$\text{height} = \frac{1820}{2(31 + 21)} \text{ ft}$$
$$= \frac{35}{2} = 17\frac{1}{2} \text{ ft}   \text{ Ans}$$

Note   From the above solution we can deduce the following rules —

$$\text{Height} = \frac{\text{area of the walls}}{2(\text{length} + \text{breadth})}$$
$$\text{Length} = \frac{\text{area of the walls}}{2 \times \text{height}} - \text{breadth}$$
$$\text{Breadth} = \frac{\text{area of the walls}}{2 \times \text{height}} - \text{length}$$

Example 4   Length, breadth and height of a room are in the ratio of 3 2.5 and the area of its four walls is 1800 sq ft   Find its length, breadth and height

Sol.   The area of the four walls of a room when the length, breadth and height are 3, 2, 5 ft respectively $= (3 + 2) \times 2 \times 5$
$$= 50 \text{ sq. ft}$$
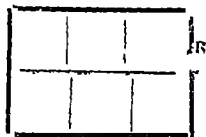
When the area of the four walls is 50 sq ft the area of the floor of one square as shown in the figure is 1 sq ft, when the area of the four walls is 1800 sq ft., the area of the floor of one square would be
$$1800 - 50 = 36 \text{ sq ft}$$

.. side of the   square $= \sqrt{36} = 6$ ft
∴ length $= 6 \times 3$ $= 18$ ft ⎫
   breadth $= 6 \times 2$ $= 12$ ft ⎬ **Ans**
and height $= 1800 - [(18 + 12) \times 2] = 30$ ft ⎭

EXERCISE 80.

Find the area of the four walls of a room (1—4) —

1   18 ft. long, 15 ft broad and 12 ft high.
2 ͡ 21 ft long, 18 ft broad and 15 ft. high
3.   21 ft 8 in long, 17 ft 4 in broad and 15 ft. high
4   24 ft 3 in long, 16 ft 3 in. broad and 12 ft high

Find the length of paper required for the walls of a room (5 and 6) —

5   18 ft. long, 17 ft broad, 12 ft high , paper 28 inches wide.

6   18 ft. 5 in long, 15 ft. 7 in broad, 12⅓ ft high paper 25 in. wide

Find the cost of papering the walls of a room (7, 8) —

7   21 ft 6 in long, 16 ft 6 in broad, 12 ft. 6 in, high , paper 1½ ft. wide at 4a per yard.

8   23 ft 4 in long, 17 ft. 2 in. broad, 18 ft high , paper 33 in wide at 1a 3p per yard, allowing 358 sq. ft for doors and windows

9   The length, the breadth and the height of a room are 25 ft. 7 in., 20 ft 5 in and 14 ft respectively   Its walls are papered at 3s. 6d a sq yard and its ceiling painted at 1s 2d. a sq ft   Find the total cost

10   Find the cost of white-washing a room 22½ ft by 12 ft and 11 ft high at one anna per square yard, making allowance for four windows each 4 ft × 2½ ft and two doors each 8½ ft. × 4 ft

11   The cost of papering the walls of a room 22 ft. long, 18 ft. broad, and 14⅓ ft. high at 4a per yard is Rs 40. Find the width of the paper

12   A room is 19 ft 6 in long, 17 ft. broad and 13 ft. high. The cost of papering the walls with paper 26 in, wide is Rs 73. Find the cost of the paper per yard

13. The cost of papering and white-washing the walls of a room 16 ft. long and 14 ft broad at 5a per square yard is Rs 25   Find the height of the room.

F 16

14. A room is 17 ft 6 in long and 14 ft high, the cost of papering and white-washing its walls at 3*a* per sq ft is Rs 170. 10*a*   Find the breadth of the room

15. Find the area of the four walls of a room, the length of which is 15 ft and the height 13 ft , if the cost of matting its floor at 2*a* per square yard is Rs 2 8*a*

16 The cost of matting a room 16 ft. broad and 12 ft high at 3*a* per square yard is Rs. 7. 9*a*. 4*p*   What will be the cost of papering its walls at the same rate, allowing for 6 doors, each 6 ft by 3 ft ?

17 The length of a room is 32½ ft   The cost of painting the walls at Re 1 14*a* per square yard is Rs 308 2*a* and the cost of carpeting the room at Rs 2 4*a* per square yard is Rs 150 5*a*. Find the height and width of the room

18 A room is 25 ft high and its length is thrice its breadth   The cost of white-washing its four walls at 5*a*. per sq yard is Rs 62 8*a* , find the cost of carpeting it at 12*a* per sq yard

19 The length and breadth of a room are in the ratio of 5 4. The cost of papering its four walls with paper 27 in wide at 1½*a* per yard is Rs 20 and the cost of carpeting it at 7*a* 6*p* per sq. yard is Rs 26 8*p* , find its length, breadth and height

20 The length, breadth and height of a room are in the ratio of 5 4 3, and the area of its four walls is 384 sq yards   Find its length, breadth and height.

MISCELLANEOUS EXERCISE 81

1 A room is 15 ft. 6 in long, 12 ft 6 in, broad and 14 ft. high   Find the cost of white-washing its walls at 1*a* per square yard

2 A room is 21 ft long, 16 ft. 6 in broad and 15 ft. high. Find the cost of papering its walls at 1*a*. 6*p* per sq. yard, allowing 72 sq ft for doors and windows.

3 A room is 17 ft 8 in long, 15 ft 4 in. broad and 16 ft high   Find the cost of papering its walls with paper 18 in. wide at 4*a* 6*p*. per piece of 10 feet.

4. My room is in a very-bad condition   It is 16 ft. long, 12 ft broad and 15 ft high   It has two doors each 6 ft by 3 ft, two windows each 4 ft by $2\frac{1}{2}$ ft. I intend to have the doors and windows painted and the walls white-washed, the cost of painting is $1a\ 6p$ per sq ft. and of white-washing $2a\ 3p$ per square yard   Calculate the total cost

5 ✓ A room of my house is 21 ft 6 in long and 15 ft broad   The cost of painting and repairing the four walls at $4a.\ 6p.$ per sq. yard amounts to Rs 36. $8a$ Find the height of the room

6   The cost of white-washing a room 17 ft broad and 13 ft 8 in. high at $2a$ per sq yard amounts to Rs. 13 $10a.$ $8p$ Find the length of the room

7. The length of a room is twice its breadth and its height is 15 ft   The cost of white-washing its four walls at $1a\ 3p$ per sq yard is Rs 9 $6a$ Find the length of the room

8. A room is 25 ft by 15 ft   It has 4 doors each 6 ft. by $3\frac{1}{2}$ ft, 4 windows each 4 ft by $2\frac{1}{2}$ ft, one fire-place 2 ft by 1 ft   Allowing for doors, etc, the cost of white-washing at $1a\ 3p.$ per square yard is Rs 11 $6a\ 6p.$ Find the height of the room.

9   The length and breadth of a room are 3 2 and the height is 21 ft.   The cost of papering the four walls at $1a$ per sq yard is Rs 11 $10a\ 8p$ Find the dimensions of the room

10   A room is 23 ft 8 in long and 19 ft 4 in broad. The cost of papering its walls after allowing 109 sq ft for windows, etc., with paper $1\frac{1}{2}$ ft wide at $4a$ per yard amounts to Rs 89 $8a.$ Find the height of the room.

11   Find the number of bricks, each 9 in by 4 in, required for paving a courtyard 36 ft by 21 ft.

12. A room is 13 ft. 6 in. broad and the cost of carpeting it at $12a.$ per sq yard is Rs. 36.   Find the length of the room.

13   2640 yards by 2 ft paper is required for papering the walls of a room the length of which is twice its breadth,

How many yards of carpet 4 ft wide will be required for the floor of the same room, the height of the room being 33 ft ?

14   The length of a room is thrice its breadth and the height is 25 ft   The cost of papering its walls at $5a$ per square yard amounts to Rs $62. 8a$, find the cost of carpeting it at $12a$ per sq. yard.

15   The length of a room is twice its breadth and the height is 20 ft   The cost of papering the walls with paper $2\frac{1}{2}$ ft. wide at $4a$. per yard amounts to Rs 64   Also the carpet required for the floor is 42 yards 2 ft.   Find the width of the carpet

16   The breadth of a room is 15 ft   The cost of papering its walls at $2p$ per square ft   is Rs. 16 $10a$ $8p$. and the cost of carpeting it at $2a$ per sq ft is Rs. 46 $14a$. Find the length and the height of the room.

17.   The length of a room is 30 ft   The cost of carpeting it at $2a$ $3p$. per square yard is Rs 11. $11a$ $6p$ and the cost of repairing its walls at $9a$. per sq. yard is Rs 137 $8a$.   Find the breadth and the height of the room.

18   The cost of carpeting the floor of a room is Rs 75   If the breadth of the room had been 3 ft less, the cost would have been Rs 60   Find the breadth of the room.

19   The cost of carpeting a room is Rs. 125   If the length of the room had been 4 ft less, the cost would have been Rs 100   Find the length of the room

20   The cost of carpeting a room is £7 4s. and of papering the same room with paper at $2\frac{1}{2}d$. per square ft is £10 12s. 6d.   The length of the room is 18 ft. and if the width had been 4 ft less, the cost of the carpet would have been £1. 16s. less.   Find the height of the room.

21.   A room is 16 ft by 12 ft   Had it been 2 ft. longer and 1 ft. wider the cost of carpeting it would have been increased by Rs. 5 $4a$.   Find the cost of the carpet

22   A room is 18 ft by 16 ft   Had it been 2 ft. longer and 1 ft less wide the cost of carpeting it would have been increased by Rs 2. $4a$ Find the cost of the carpet

CHAPTER XIV

AREA OF SOME SIMPLE FIGURES.

§1   In the previous chapter we have explained how to find out the area of a rectangle and a square. Now in this chapter we shall explain how to find out the area of some other simple figures.

§2   **Area of a right-angled triangle.**

Let *ABCD* be a rectangle with *BD* its diagonal   It is clear from the diagram that the diagonal divides the rectangle into two equal right-angled triangles *BCD* and *ABD*
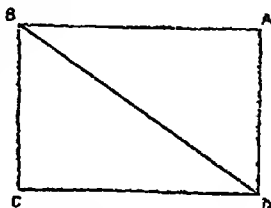
Since the area of the rectangle $= BC \times CD$

the area of the right-angled $\triangle\ BCD = \dfrac{BC \times CD}{2}$,

where *BC* and *CD* are the perpendicular and the base of the right angled triangle respectively.

the area of a right-angled triangle $= \dfrac{\text{perpendicular} \times \text{base}}{2}$

hence   perpendicular $= \dfrac{\text{area} \times 2}{\text{base}}$ and base $= \dfrac{\text{area} \times 2}{\text{perpendicular}}$.

In the right-angled $\triangle$ the side opposite to the right angle is called the Hypotenuse

It may be noted that hypotenuse $= \sqrt{(\text{perp})^2 + (\text{base})^2}$.

§3.   **Area of any triangle**

Let *ABCD* be a rectangle   Take a point *P* on *AB* and join *CP* and *DP*   Thus we have a triangle   *CDP*   within the rectangle   *PM* is the height of the triangle and *CD* the base   It is clear from the diagram that the area of the triangle is half the area of the rectangle

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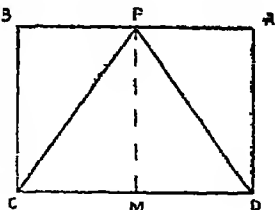
Since the area of the rectangle $= CD \times AD$,

the area of the triangle $= \dfrac{CD \times AD}{2} = \dfrac{CD \times PM}{2}$

We shall express this result in words as follows —

Area of a triangle $= \dfrac{\text{base} \times \text{height}}{2}$

whence,   Base $= \dfrac{\text{area} \times 2}{\text{height}}$ and height $= \dfrac{\text{area} \times 2}{\text{base}}$

Aliter   Area $= \sqrt{s(s-a)(s-b)(s-c)}$,

where $s =$ semi-perimeter and a, b, c, the sides of the triangle   The formula is proved in geometry

§4   Area of an equilateral triangle

An equilateral triangle is one which has all its sides equal   Let ABC be an equila-
teral triangle whose side is equal
to 4 inches   By application of the
second formula of Art 3 we shall
proceed thus
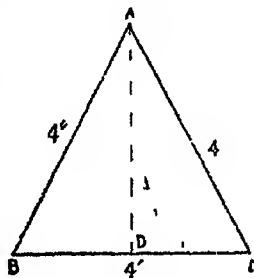
Semi-perimeter $= \tfrac{1}{2}(4 + 4 + 4) = 6$

$$
\begin{aligned}
\text{area} &= \sqrt{6(6-4)(6-4)(6-4)} \\
&= \sqrt{6 \times 2 \times 2 \times 2} \\
&= \sqrt{2 \times 2 \times 2 \times 2 \times 3} \\
&= \sqrt{4 \times 4 \times 3} \\
&= 4 \times 1\ 732 \\
{}^{*}&= 4 \times 4 \times \ 433.
\end{aligned}
$$

Also height $AD = \dfrac{\text{area} \times 2}{\text{base}}$

$$= \dfrac{4 \times 4 \times \ 433 \times 2}{4}$$

$${}^{*}= 4 \times \ 866$$

From the asterisked lines we conclude the following formulæ

Area of an equilateral triangle $=$ side $\times$ side $\times$ 433
Height or altitude $=$ side $\times$ 866

§5   Area of a quadrilateral.

Quadrilateral is a plane figure bounded by four sides

(a) If four sides and one diagonal be given we shall find out the areas of the two triangles thus formed by the formula

Area$=\sqrt{s(s-a)(s-b)(s-c)}$ and then add

(b) If one diagonal and the two offsets from this diagonal to the opposite corners be given the area of the quadrilateral is found by the formula $\frac{1}{2}$ d $(P_1 + P_2)$

Example   Find the area of the quadrilateral whose diagonal is 213 ft , offsets 97 and 103 ft

Sol   Area$=\frac{1}{2} \times 213$ $(97 + 103)=21300$ sq ft **Ans**

§6   Area of a parallelogram

Parallelogram is a four-sided figure having its opposite sides equal and parallel

Let $ABCD$ be a parallelogram

Produce DC and draw AL and BM perpendiculars on DC

Now $ALMB$ is a rectangle the area of which is equal to the area of the given parallelogram, since $\triangle ADL$
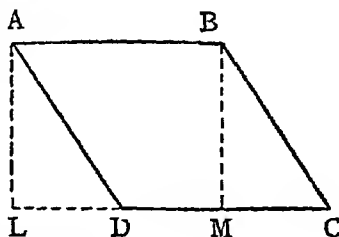
is equal to $\triangle BCM$ in all respects.

Now area of the rectangle $ALMB=AL \times LM$
area of the parallelogram $=AL \times LM$
$=AL \times DC$
[$LM=DC$.

We express this result in words thus —

Area of a parallelogram$=$height$\times$base,
whence height or altitude$=$area$-$base
and base $=$area$-$height

§7   Area of a rhombus

Rhombus is a four-sided figure, having all sides equal and the diagonals of which bisect each other at right angles.

Let $ABCD$ be a rhombus with diagonals $6''$ and $8''$, $i\ e$, $BD=6''$ and $AC=8''$

∵ area of $\triangle ABC$

$=\dfrac{8\times3}{2}$ sq. inches

and area of $\triangle ACD$

$=\dfrac{8\times3}{2}$ sq. inches

area of the rhombus

$=\dfrac{8\times3\times2}{2}$ sq. inches

$=\dfrac{8\times6}{2}$ sq. inches
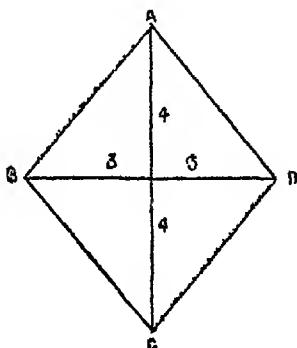
The result may be expressed in words thus —

Area of a rhombus $=\dfrac{\text{diagonal}\times\text{diagonal}}{2}$.

§8. Area of a cylinder.

Take a rectangular piece of paper and join the edges of its breadth. It will represent the figure of a cylinder The length of the rectangular paper will represent its circumference and breadth, its height

Since the area of a rectangle
$$=\text{length}\times\text{breadth},$$

∴ Area of a cylinder $=$ circumference $\times$ height

whence circumference $=$ area $\div$ height,

and height $\qquad=$ area $\div$ circumference

§9. Circle.

Draw some circles of different radii Mark points on the circumferences near each other and measure the

These parts make a figure similar to a parallelogram
Had the circle been divided into 32 equal parts, lines AB
and CD would have been more straight,
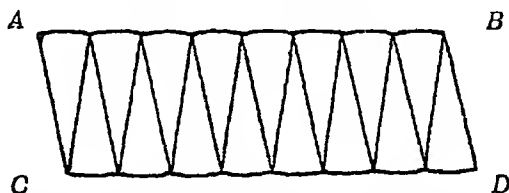
It is clear that the area of the parallelogram is equal
to the area of the circle   We also observe that the base
of the parallelogram is equal to the semi-circumference of
the circle and height is equal to the radius

<div align="center">

Area of the circle $=$ semi-circumference $\times$ radius.

But semi-circumference $=$ radius $\times \frac{22}{7}$ [Art 9]

Area of a circle $=$ radius $\times$ radius $\times \frac{22}{7}$

</div>

EXERCISE 82

Note   The first ten questions to be done mentally

1   The base of a right-angled triangle is 8 yards and
perpendicular is 6 yards , find the area.

2   The base and the height of a triangle are 7 and
3 yards respectively , find the area

3   The base of a parallelogram is 15 ft. and height is
5 ft , find its area

4   The circumference of a cylinder is $5\frac{1}{2}$ yards and
height is 8 yards , find its area

5   The diagonals of a rhombus are 15 ft and 12 ft ,
find its area

6   What would be the area of a rhombus the diagonals
which are $4\frac{1}{2}$ ft and 6 ft ?

7   The circumference of a circle is 88 ft , find its
diameter

8   The circumference of a circle is 154 ft , find its
radius

circumference with threads as accurately as possible
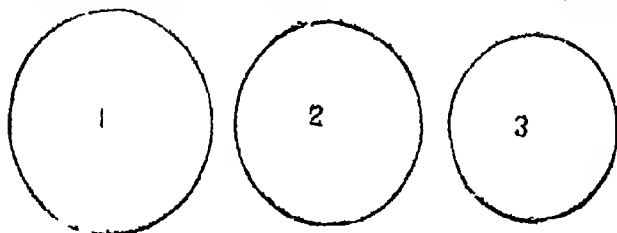
from one point to another till the whole circumference is measured. Now measure the threads used in each circle in inches and fill u the following table :—

No.	Circumference in inches	Diameter in inches	Ratio between circumference and diameter
1			
2			
3			

You will now find that the circumference in each case is $\frac{22}{7}$ times the diameter, that is to say

circumference $=$ diameter $\times \frac{22}{7}$ and

semi-circumference $=$ radius $\times \frac{22}{7}$.

§10. Area of a circle.

Draw a circle on a thin cardboard. Draw diameters so as to divide the circle into 16 equal parts. Cut off these parts and arrange them in a figure as shown on the next page.
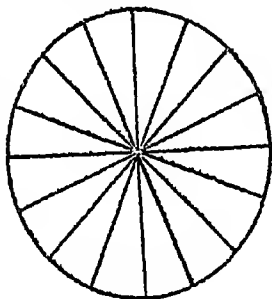
are respectively 18 ft 2 in and 11 ft 7 in , if the painting of the walls at the same cost per sq yd amount to Rs 74. 6a ?

34   The sum of the length and breadth of a room is 26 ft and its area is 160 sq ft , find its dimensions

35   A rectangular field is 80 ft long and 60 ft broad A path of uniform width whose area is 1300 sq ft has been constructed *within* along the four sides , find its width

36   A path of uniform width running *outside* a rectangular garden contains 3200 sq ft   If the length and breadth of the garden be 80 ft and 60 ft respectively, find the width of the path.

√37.   Find the cost of lining with zinc, at 6s 9d per sq yd the sides and bottom of a cistern whose dimensions are length 7 ft 10 in., breadth 5 ft 4 in. and depth 1 ft. 9 in   [*Burma, 1923*]

38   A room is 21 ft 4 in long, 15 ft 9 in broad, the doors and windows together occupy 65 sq ft   The cost of papering the remaining part of the surface of the walls with paper 25 in wide at 3s 9d per piece of 12 yds is £2. 8s 8d , find the height of the room   [*Burma, 1924*]

39   A lawn tennis ground is half as long again as it is wide   The cost of levelling it at 5a per square yard is Rs 1470 , find the cost of enclosing it with an iron railing at Rs 4 per yard.   [*Patna, 1926*]

40   The length of a room is 20 ft., the cost of papering the walls with paper 2½ ft wide at 4a. per yd is Rs 30 6a 8p , and that of carpeting the room at Rs 3 5a. 4p per sq yd is Rs 122 3a 6⅔p , find the height of the room

41   The length of a square plot of land is 110 yards It has two paths, each 2½ yards wide with its centre line along the diagonal   Find to the nearest penny the cost of covering the remaining plot with grass at 2d per square yard.

23   The length of a room exceeds its breadth by 12 ft and its perimeter is 76 ft   Find the cost of carpeting it at 4*a* per sq ft.

24   The length of a room is greater than its breadth by 8 ft. and its perimeter is 64 ft.   Find the cost of carpeting it at 12*a* per sq yard

25   A room is 18 ft broad.   Leaving a passage 2 ft wide along the sides, a carpet has been spread, the cost of carpeting at 2*a* per square ft being Rs 28 , find the length of the room

26   A room is 30 ft long   Leaving a passage 2½ ft. wide along the sides, a carpet worth Rs 14 13*a* 6*p*. at 4*a* 6*p* per square yard has been spread   Find the cost of papering the ceiling at 1*a* 3*p*. per sq yard

27   A garden is 220 ft by 160 ft   A road 5 ft. wide has been constructed within along the sides   Find the cost of gravelling it at 9*a*. per 100 sq. ft

28   A garden is 300 ft by 250 ft.   It is surrounded by a path 10 ft wide   Find the cost of gravelling this path at 3*a*. per sq yard

29.   A field is 150 yards by 120 yards,   It has two roads each 10 ft. wide running in the middle of it, one parallel to the length and the other parallel to the breadth Find the area of these roads, also find the cost of planting grass at 7*a* per 100 sq ft on the remaining portion.

30   A path 6 ft wide, running all round a square garden has an area of 2 acres.   Find the area of that part of the garden enclosed by the path

31.   Find the cost of papering the walls of a room 22 ft long, 18 ft. wide and 20 ft high, with rolls of paper 21 inches wide, at Rs 2. 10*a* per roll of 12 yards.

32   The area of a rectangular field whose breadth is 500 yards is 100 acres.   Find the cost of cultivating it at Rs. 3 2*a* 8*p*. per 100 sq. yds and also the cost of fencing it round at Rs. 2 8*a*. per yard.

33   The cost of painting a room 9 ft. 6 in high, 15 ft 3 in long and 10 ft broad is Rs 47 8*a* , what must be the height of another room whose length and breadth

9   The radius of a circle is 7 ft., find its area

10   The sides of an equilateral triangle is 4 ft.; find its area

11   The area of a right-angled triangle is 46 sq ft 96 sq in. and the base is 8 ft 9 in, find its height

12   The area of a right-angled triangle is 18 sq. ft 48 sq in and its height is 3 ft 4 in, find its base.

13   The hypotenuse of a right-angled triangle is 120 ft and one side is 24 yards, find the other side

14   The sides of a triangle are 15, 25, 30 ft, find its area to 3 decimal places

15.   The base of a triangle is 35 ft and the height is 12 4 ft, find its area

16.   The area of a triangular field is 140 square yards and the base is 63 ft, find its altitude.

17   The sides of a triangle are 16 yds. 2 ft, 26 yds. and 37 yds 1 ft respectively, find the perpendiculars drawn from the angles on the opposite sides

18   The side of an equilateral triangle is 50 ft, find its area

19.   The area of an equilateral triangle is 270 sq. ft. 90 sq in., find its side

20   The side of an equilateral triangle is 12 ft. 6 in, find its altitude

21.   The circumference of a cylinder is 15 ft. and the height is $3\frac{1}{2}$ ft, find its area.

22   The circumference of a cylinder is 21 ft and the height is 3 ft 4 in, find its area.

23.   The area of a cylinder is 52 sq. ft. 72 sq in and the height is 4 ft. 8 in, find its circumference

24   The base of a parallelogram is 21 ft 4 in. and the height is 7 ft 6 in, find its area

25   The area of a parallelogram is 148 sq. ft. 32 sq. in and the height is 9 ft 8 in, find its base

26   The diagonals of a rhombus are 6 ft. 8 in. and 5 ft 3 in. respectively, find its area

27   The area of a rhombus is 50 sq ft., one of the diagonals is 6 ft 8 in, find the other diagonal

28   The diameter of a circle is 10 ft. 6 in., find 1 circumference

29.   The radius of a circle is 7 ft, find its circumference.

30.   The circumference of a circle is 88 ft., find its radius

31   The circumference of a circle is 14 yards 2 ft., find its diameter

32   The radius of a circle is $3\frac{1}{2}$ ft., find its area.

33.   The circumference of a circle is 220 yards, find its area

34.   The diagonals of a rhombus are 6 ft. and 8 ft, find its side

35.   The area of a rhombus is 2400 sq. ft. and the side is 50 ft, find its diagonals.

36.   The side of a rhombus is 20 ft and one of its diagonals is 24 ft., find the other diagonal

37.   Find the area of a ring the outer and inner radii of which are $3\frac{1}{2}$ ft. and $2\frac{1}{2}$ ft.

38   The area of a circle is equal to that of a square. Compare their perimeters

39.   In a quadrilateral $ABCD$, the diagonal AC measures 2 ft. 9 in. and the offsets from this diagonal to B and D measure 1 ft. 7 in. and 11 in respectively, find the area of the quadrilateral

40   In a quadrilateral $ABCD$, the sides AB, BC, CD, DA, measure 20, 13, 17, 10 lks. respectively and the diagonal AC 21 lks, find the area of the quadrilateral.

CHAPTER XV

MEASUREMENT OF VOLUME

§1   A thing which occupies some space is called a Solid and the space which a solid occupies is called its Volume, Capacity or Cubic Content

§2   Observe the following two figures. You have often seen such solids and you can note that both the solids
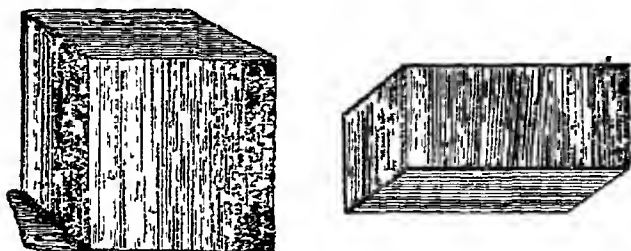
are bounded by six faces   The only difference you can find is that in one solid, each face is a square and in the other each face is a rectangle   The one which has square faces is called a cube, and the one which has rectangular faces is called a rectangular solid or a cuboid.

§3   A rectangular solid measuring an inch each way is called a cubic inch, measuring a foot each way is called a cubic foot , and so on.

Note   A rectangular solid is said to have three dimensions, viz , *length, breadth* and *thickness* (or height or depth)

§4   To prove that
one cubic yard = 27 cubic feet and
one cubic foot = 1728 cubic inches,

Take 27 wooden rectangular solids, each measuring a cubic foot, and place them as shown in the figure

It will be seen that the model thus prepared is a cubic yard   It has three layers, each layer has three lines and each line has three wooden rectangular solids. Hence it is proved that a cubic yard $= 3 \times 3 \times 3 = 27$ cubic feet.
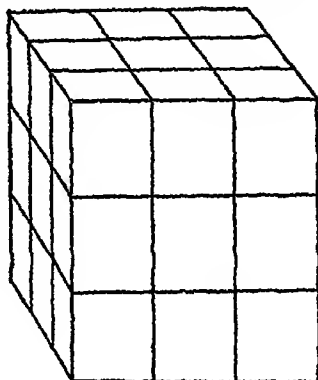
Similarly by the aid of the diagram given below, it
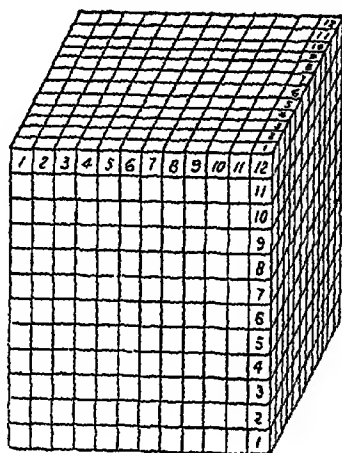
will be seen that a cubic foot contains $12 \times 12 \times 12 = 1728$ cubic inches.

§5. To find the volume of a cuboid and a cube.

(*i*) Let us take some wooden rectangular solids again, each measuring 1 cubic ft. Make a layer containing 4 solids in its length and 3 in its breadth. Make another similar layer and place it on the first layer as shown in the annexed figure. The model now has two layers, each layer has three lines and each line has four
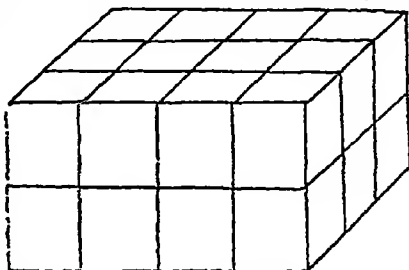
wooden rectangular solids. It is clear, therefore that the model contains $4 \times 3 \times 2 = 24$ cubic ft., where $4'$ is the length, $3'$ is the breadth and $2'$ is the height. Hence the

Volume of a cuboid = length × breadth × height,

or briefly $v = l \times b \times h$

(*ii*) Since in a cube, length, breadth and height are equal,

Volume of a cube = (length)^3 or (edge)^3

Rule. *Express the length, breadth and height in units of the same denomination, their product will give the volume in cubic units of the same denomination.*

§6 Diagonal of a cuboid and a cube.

(*i*) Diagonal of a cuboid $= \sqrt{l^2 + b^2 + h^2}$

Let $l = 5$ yards, $b = 4$ yards, and $h = 3$ yards, then the

diagonal $= \sqrt{5^2 + 4^2 + 3^2}$

$= \sqrt{25 + 16 + 9}$

$= \sqrt{50} = 7.07$ yds   **Ans**

Note. This is the longest rod that can be placed in a room having dimensions 5, 4 and 3 yds.

(*ii*) Diagonal of a cube $= \sqrt{l^2 + l^2 + l^2}$

$= \sqrt{3l^2} = l\sqrt{3}$

F. 17

Let the side of a cube be 5 in , then

the diagonal $=5\sqrt{3}$

$$=5\times1\cdot732=8\cdot66 \text{ inches. } \textbf{Ans.}$$

§7   To find the surface of a cuboid and a cube.

(*i*) Let the dimensions of a cuboid be 5, 4, 3 in respectively   Since the cuboid has six rectangular faces and the two opposite ones are equal,

the surface of the cuboid $=2(5\times4+5\times3+4\times3)$
$$=94 \text{ sq in}$$
i e., surface $\qquad\qquad =2(lb+lh+bh)$

(*ii*) Since a cube has six equal faces

the surface of a cube $=6\times(\text{edge})^2$

Example 1   A log is 15 ft long, 2 ft 3 in broad and 1 ft, 6 in thick , find its volume and its surface.

Sol (*i*)   Volume $=$ length $\times$ breadth $\times$ thickness

$$=15\times\tfrac{9}{4}\times\tfrac{3}{2}=\tfrac{405}{8} \text{ cubic ft}$$

$$=50 \text{ cubic ft 1080 cubic in }\quad\textbf{Ans.}$$

(*ii*)   Surface $\ =2(lb+lh+bh)$

$$=2(15\times\tfrac{9}{4}+15\times\tfrac{3}{2}+\tfrac{9}{4}\times\tfrac{3}{2}) \text{ sq. ft.}$$
$$=2(\tfrac{135}{4}+\tfrac{45}{2}+\tfrac{27}{8})=\tfrac{477}{4} \text{ sq ft}$$
$$=119 \text{ sq ft 36 sq in. Ans}$$

Example 2   A box measures 3 ft 6 in each way , find its volume and also find the cost of varnishing it at $4p$ per sq ft

Sol. (*i*) Volume $=$ length $\times$ breadth $\times$ height

$$=\tfrac{7}{2}\times\tfrac{7}{2}\times\tfrac{7}{2}=\tfrac{343}{8} \text{ cubic ft}$$

$$=42 \text{ cubic ft 1512 cubic in }\quad\textbf{Ans.}$$

(*ii*) Since the box has six surfaces and each surface is a square, therefore

the area of one surface $=\tfrac{7}{2}\times\tfrac{7}{2}=\tfrac{49}{4}$ sq ft

  „   „ six surfaces $=\tfrac{49}{4}\times6$ sq ft

.   cost $\qquad\qquad =$ Rs $\tfrac{49}{4}\times6\times\tfrac{4}{105}=$ Rs. $\tfrac{49}{32}$
$$=\text{Rs. 1 } 8a. 6p\quad\textbf{Ans.}$$

Example 3   A platform 8 ft. long, $5\frac{1}{2}$ ft broad and 3 ft high is intended to be constructed , find the cost of its construction at $2a$ per cubic ft and also find how many bricks each 9 in long, 4 in broad and 3 in. thick will be required for it.

Sol.   Volume $=$ length $\times$ breadth $\times$ height

$=8\times\frac{11}{2}\times3=132$ cubic ft

(i) Cost $=$ Rs $132\times\frac{1}{8}=$ Rs 16 8a. $\Big\}$ **Ans**

(ii) No. of bricks $=\dfrac{132\times1728}{9\times4\times3}=2112$

Example 4   A tank is 30 ft long, 12 ft broad and $4\frac{1}{3}$ ft deep , how much water will it contain if the weight of one cubic ft. of water is 1000 ounces ?

Sol.   Volume of the tank $=$ length $\times$ breadth $\times$ height

$=30\times12\times\frac{9}{2}=1620$ cubic ft

weight of water $=\dfrac{1620\times1000}{16}=101250$ lbs

$=45$ tons 4 cwt 2 lb **Ans**

Example 5.   A box with a lid measuring on the outside 3 ft 2 in in length, 2 ft 2 in in breadth and 1 ft 8 in in height is made of wood 1 in thick , find how many cubic ft of air it contains and also find the weight of the box if a cubic foot of the wood weighs 27 lbs

Sol   The wood being 1 in thick the inner dimensions are 3 ft , 2 ft and $1\frac{1}{2}$ ft. respectively

(i)   the volume of the solid occupied by the air

$=3\times2\times\frac{3}{2}$ cubic ft

$=9$ cubic ft **Ans**

(ii) The volume of the box $=3\frac{1}{6}$ ft $\times2\frac{1}{6}$ ft $\times\frac{5}{3}$ ft.

$=\frac{19}{6}\times\frac{13}{6}\times\frac{5}{3}=\frac{1235}{108}$ cubic ft.

. the volume of the wood $=\left(\frac{1235}{108}-9\right)$ cubic ft

$=\frac{263}{108}$ cubic ft

and . weight of the box $=\frac{263}{108}\times27$ lb $=\frac{263}{4}$ lb

$=65\frac{3}{4}$ lb **Ans.**

Example 6   In the above example, find the expenses of painting the box inside and outside at $5a. 4p$ per square yd

Sol. The exterior surface
$$=2(\tfrac{19}{6}+\tfrac{13}{6})\times\tfrac{5}{3}+\tfrac{19}{6}\times\tfrac{11}{6}\times 2$$
$$=2\times\tfrac{32}{6}\times\tfrac{5}{3}+\tfrac{19}{6}\times\tfrac{13}{6}\times 2$$
$$=\tfrac{160}{9}+\tfrac{247}{18}=\tfrac{63}{2}\text{ sq ft}$$

and the interior surface $=2(3+2)\times\tfrac{3}{2}+3\times 2\times 2$
$$=2\times 5\times\tfrac{3}{2}+3\times 2\times 2$$
$$=15+12=27\text{ sq ft.}$$

∴ the whole area to be painted $=\tfrac{63}{2}+27$
$$=1\tfrac{117}{2}\text{ sq. ft }=1\tfrac{13}{2}\text{ sq yds.}$$

∴ the cost required $=\text{Rs }\tfrac{13}{2}\times\tfrac{1}{3}=\text{Rs }1\tfrac{1}{6}$
$$=\text{Rs. 2 } 2a.\ 8p\quad\text{Ans}$$

Example 7. Water flows into a reservoir 24 ft long, 15 ft. broad and 11 ft deep through a pipe of 9 sq. in , at the rate of 4 miles an hour. How long will it take to fill the reservoir ?

Sol Volume of the reservoir $=24\times 15\times 11$ cubic ft.
Volume of water that flows into the reservoir
$$=4\times 1760\times 3\times\tfrac{9}{144}=1320\text{ cub ft. per hour}$$

∴ time reqd $=\dfrac{24\times 15\times 11}{1320}=3$ hours. **Ans**

Example 8 A reservoir measuring inside 8 ft in length, 5 ft in breadth and 4 ft in depth contains 60 cubic ft. of water Bricks each measuring $9''$ by $6''$ by $2\tfrac{3}{4}''$ are put in the reservoir till it is brimful , find the number of bricks put in it, supposing each brick absorbs $\tfrac{1}{17}$ th of its own volume of water

Sol Volume of the reservoir $=8\times 5\times 4=160$ cubic ft.
Volume of water $=60$ cubic ft

Therefore bricks can occupy $160-60=100$ cubic ft of place if they do not absorb any water But as the bricks absorb $\tfrac{1}{16}$ th of its own volume of water, consequently there will be more than 100 cubic ft of place for them

volume occupied by bricks $=100\times\tfrac{17}{16}=104$ cubic ft
But volume of a brick $=\tfrac{9}{12}\times\tfrac{6}{12}\times\tfrac{11}{48}=\tfrac{1}{12}$ cubic ft
∴ number of bricks $=104\div\tfrac{1}{12}=1248$ **Ans**

EXERCISE 83

Note   The first 11 questions are to be done mentally

Find the cubic contents of the rectangular solids having the following dimensions :—

1	4 ft , 3 ft , 1½ ft.	**2**	3 ft , 2 ft., 1 ft.
3.	9 in , 6 in., 2 in.	**4**	10 in , 5 in , 1½ in.
5	3 ft , 2½ ft , 1½ ft	**6**	8 cm., 6 cm , 5 cm
7	18 in., 12 in , 8 in	**8**	4 ft , 3 ft , 2¼ ft
9	5 yds , 4 yds , 1½ ft.	**10**	5⅝ yds , 2 ft , 1¼ ft

11.   Find the volume of a cube whose edge is
(a) 4 inches, (b) 6 inches, (c) 3 yds.

12   Find the longest rod that can be placed in a room having dimensions 16 ft., 12½ ft and 18 ft

13   Find the diagonal of a cube whose edge is 9 ft.

Find the surface of the rectangular solids in Examples 14 and 15, whose dimensions are the following —

14   Length 1 ft 3 in , breadth 11 in and height 8 in

15   Length 4 ft 6 in , breadth 2 ft. 4 in. and height 1 ft 6 in

16   The edge of a wooden cube is 2 ft 2 in   Find its surface

17   Find the surface of a cube whose edge is 4 ft. 3 in

18   A solid is measuring 3 ft. 4 in each way, find its capacity

19   A room is 18 ft. long, 12 ft. broad and 9 ft. high, how many cubic yds of air does it contain ?

20   A room is 16 ft 8 in. long, 12 ft 4 in broad and 9 ft high , how many cubic yards and cubic feet of air does it contain ?

21   How many bricks each 9 in by 5 in by 1½ in., are required for a wall 18 ft long, 15 ft high and 2¼ ft thick ?

22   How many bricks, each 8 in. by 6 in by 2 in are required for a platform 21 ft long, 12 ft. broad and 4½ ft. high ?

23   How many bricks, each 8 in by 4 in by 2 in.
are required for a wall 15 ft long, 12 ft high and $2\frac{1}{2}$ ft.
thick, leaving in it a doorway 9 ft by 4 ft. ?

24.   Find (correct to the nearest pie) the cost of
painting the surface of a box, each edge of which is 3 ft
4 in at 8p per sq ft

25   Find the cost of painting the surface of a box
the dimensions of which are, length 4 ft., breadth 3 ft and
height $2\frac{1}{2}$ ft at 6p per square ft

26   A log of wood 16 ft long, $2\frac{3}{4}$ ft broad and 1 ft
thick is purchased at 2a 3p per cubic ft and the cost of
varnishing it is 4p per sq. ft   Find the total cost.

27.   A reservoir 21 ft long, 15 ft broad and 6 ft
deep contains water   Find the weight of the water when
a cubic ft of water weighs 1000 ounces

28   A cubic foot of water weighs 60 lbs.   Find in
tons, etc., the weight of a rainfall of 2 inches over a field 15
yards long and 8 yards wide

29   A cubic foot of water weighs $6\frac{1}{2}$ gallons   Find
in gallons the water of a rainfall of $1\frac{1}{2}$ inches over a park
18 ft by 16 ft

30   A ditch is 12 ft long, 8 ft. wide and 4 ft. deep
Find the cost of filling it with earth when 48 cubic ft of
earth cost Rs 5 12a

31   A box with a lid measuring on the outside 3 ft
$1\frac{1}{2}$ in in length, 2 ft. $7\frac{1}{2}$ in in breadth and 1 ft 5$\frac{1}{2}$ in in
height is made of wood $\frac{3}{4}$ in thick , how many cubic ft. of
air does it contain ?

32   The interior dimensions of a box made of wood
of 1 in in thickness are 3 ft. 4 in. long, 2 ft 10 in. broad
and 2 ft. 4 in high   Find the weight of the box if a cubic
ft of the wood weighs 36 lbs.

33   The exterior dimensions of a box made of wood
of $1\frac{1}{2}$ in. in thickness are 4 ft. long, 3 ft broad and $2\frac{1}{2}$ ft.
high.   Find how many books 6 in. long, 3 in broad and
$\frac{3}{4}$ in thick are required to fill it

34   A cistern 22 yds long, 15 yds wide and 2 yds

deep is to be filled by a pipe whose sectional area is 24 sq
in   How long will it take to fill it when the water flows at
5 miles an hour ?

35. A reservoir 33 ft long, 21 ft. wide, $7\frac{1}{2}$ ft deep is
filled in 2 hours 37 min 30 sec. by a pipe whose bore is
4 in × 3 in.   How fast does the water flow in the pipe ?

36. A cistern 6 ft long, 3 ft wide and 4 ft. deep
contains 42 cubic ft of water   How many bricks 8 in by
4 in by 3 in can be put in it, when a brick is supposed to
absorb $\frac{1}{7}$ of its own volume of water ?

37 A crow wishing to quench its thirst came to a
vessel which contained 23 cubic inches of water   The
crow being unable to reach the water picked up several
small stones, each three quarters of a cubic inch in size,
and let them drop into the vessel until the water came to
the top of it   If the size of the vessel was such that it
could exactly hold 73 cubic inches of water, find the number
of stones dropped in by the clever crow,

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§8 In the foregoing exercises we have thoroughly
practised the formulæ

> Length × breadth × height = volume,
> whence   Length = volume − (breadth × height),
>   breadth = volume − (length × height),
> and   height = volume − (length × breadth)

Example 1 The volume of a cuboid is 61 cubic ft.
432 cubic in , its length is 5 ft. 3 in, and the thickness is
2 ft 6 in   Find the breadth

> Sol · breadth   = volume − (length × thickness)
> reqd breadth = $61\frac{1}{4} - (5\frac{1}{4} \times 2\frac{1}{2})$ ft
> $= \frac{245}{4} \times \frac{4}{21} \times \frac{2}{5}$ ft
> $= \frac{14}{3}$ or 4 ft. 8 in   **Ans**

Example 2. A rectangular solid costs Rs. 5 10*a* at
2*a* 3*p*. per cubic ft   Find its thickness if its length be 8 ft.
and breadth 2 ft 6 in

> Sol. $\frac{9}{4}a$ is the cost of 1 cubic ft.
>   1*a*   ,,   ,, $\frac{4}{9}$ cubic ft
>   90*a*   ,,   ,, $\frac{4}{9} \times 90$ or 40 cubic ft

Now 40 cubic ft is the volume of the rectangular solid

thickness = volume ÷ (length × breadth)

∴ reqd thickness = $\frac{40 \times 2}{8 \times 5}$ = 2 ft. **Ans**

EXERCISE 84

1   The cubic content of a rectangular solid is 30 cubic ft , the length is 4 ft and the breadth 3 ft   Find its height.

2   The cubic content of a rectangular piece of wood is 40 cubic ft , the length is 5 ft and the breadth 4 ft   Find its thickness.

3   What will be the width of a rectangular piece of wood, the volume of which is 15 cubic ft , the length 5 ft. and the thickness $1\frac{1}{2}$ ft. ?

4   Find the thickness of a rectangular stone the capacity of which is 25 cubic ft , the length 5 ft and the breadth 3 ft

5   The length and the thickness of a rectangular stone are 8 ft and $3\frac{1}{2}$ ft respectively   Find its breadth when its volume is 112 cubic ft.

6.   The cubic content of a log of wood is 64 cubic ft 1008 cubic in , its length is 15 ft 6 in. and the breadth 2 ft. 6 in   Find its thickness.

7.   The volume of a heap of coal is 16 cubic yds 6 cubic ft 1296 cubic in   What will be its height if the length be 15 ft and breadth $6\frac{1}{2}$ ft ?

8   The volume of a heap of coal 16 ft. long, $8\frac{1}{2}$ ft. broad is 27 cubic yds 19 cubic ft   Find its breadth.

9   The cubic content of a log of wood 28 ft. long and $1\frac{3}{4}$ ft thick is 105 cubic ft   Find its breadth

10   A cistern 10 ft long and 6 ft. 8 in. broad contains 14 cubic yds 22 cubic ft of water   Find its depth.

11.   A log of wood whose edge is a square contains 36 cubic ft   Find its width, if the length be 16 ft.

12.   A log of wood whose edge is a square contains 28 cubic ft 216 cubic in., its length is 18 ft.   Find its thickness.

13 A log of wood whose edge is a square of 1 ft 4 in. side contains 1 cubic yd 5 cubic ft Find its length

14 A solid stack 5 ft by 5 ft by 3 ft contains 1728 bricks, each 10 in long and 5 in broad, find the thickness of each brick.

15 A solid stack 8 ft. by 4 ft contains 2560 bricks, each 9 in × 4 in × 1½ in , find the height of the stack

16 A cistern 24 ft long 15 ft. broad is filled in 3 hours by a pipe whose sectional area is 9 sq in. The water flows in the cistern at 4 miles an hour , find the depth of the cistern

17 A piece of ground is 21 yds long and 18 yds wide. To what uniform height must earth be spread upon it, that it may cost the owner Rs 210 at Rs. 3 5a 4p per cubic yard ?

18 A cistern containing 600 gallons measures externally 7 ft by 2 ft 11 in by 5 ft 6 in , the sides are 1½ in thick , find the thickness of the bottom, supposing 6¼ gallons=1 cubic ft

19 In a box which measures internally 4 ft by 3 ft , I can pack 2880 books each 6 in long, 3 in wide and 1½ in. thick Find the depth of the box

20 In a box measuring internally 3½ ft by 3 ft. by 2⅓ ft I can pack 2016 books each 6 in long and 3 in. wide. Find the thickness of each book

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§9 Volume of a prism.

Definition A right prism is a solid bounded by plane faces, the two ends of which are congruent parallel figures and the side faces are rectangles.

If we take a wooden cuboid with dimensions—length 5″, breadth 4″ and height 3″ and draw two diagonals parallel to each other, one on the lower surface and the other on the upper surface of it and with a saw cut the cuboid into two equal parts across these diagonals, each part will then represent a right prism, the base of which will be a right-angled triangle.

Since the volume of the cuboid $= 5 \times 4 \times 3$ cub in

„   „   „ prism $= \dfrac{5 \times 4 \times 3}{2} = 10 \times 3$ „ „

where 10 sq ft is the area of the right-angled triangle or the area of the base of the prism   Therefore

Volume of a prism = Area of the base × height

The base of the right prism may be an equilateral triangle, a trapezium, a pentagon, a parallelogram, etc etc., but the method of finding the volume in each case is the same as mentioned above.

§10   Volume of a right circular cylinder

Since the upper and lower surfaces of a right circular cylinder are equal and parallel to each other,

Volume of a cylinder = Area of the base × length and
area of its curved surface = circumference × height

Example 1   The area of the base of a right prism is 9 sq in and its height is 5 in   Find the volume.

Sol   Volume = area of the base × height
$= 9 \times 5 = 45$ cubic inches.   **Ans**

Example 2   The base of a right prism is an equilateral triangle whose side is 2 ft   If the height be 5 ft., find the volume

Sol   Volume = area of the base × height
$= (2 \times 2 \times 433) \times 5$ cub ft.
$= 1\ 732 \times 5$ cub ft
$= 8\ 66$ cub. ft.

Example 3   Find the volume and the curved surface of a cylinder, the height of which is 8 inches and a base of diameter 1 in

Sol   (i)   Volume = area of the base × height
$= (\frac{1}{2} \times \frac{1}{2} \times \frac{22}{7}) \times 8$ cub in
$= \frac{11}{14} \times 8 = \frac{44}{7}$ cub in
$= 6\frac{2}{7}$ cub in   **Ans**

(ii) Curved surface = circumference × height
$= (1 \times \frac{22}{7}) \times 8$ sq. ft
$= \frac{22}{7} \times 8$ or $\frac{176}{7}$ sq. ft $= 25\frac{1}{7}$ sq ft. **Ans**

Example 4   The external diameter of hollow cylindrical tube made of iron $\frac{1}{2}$ in thick is 3 inches. If the length of the tube be 21 ft, find the number of cubic inches of iron in it

Sol   External radius of the tube $=1\frac{1}{2}$ in

Internal   „   „ $=1\frac{1}{2}-\frac{1}{2}=1$ in

volume of the tube $=\frac{5}{2}\times\frac{5}{2}\times\frac{22}{7}\times21\times12$ cub in

$=1782$ cub in

and internal volume of the tube $=1\times1\times\frac{22}{7}\times21\times12$ cub in.

$=792$ cub in

Volume of iron $=1782-792$

$=990$ cub in   **Ans**

Example 5   Two cylindrical iron pipes, each open at both ends, have equal internal volumes. The external and internal diameters of one are 11 and 10 inches, and those of the other are $5\frac{3}{4}$ and 5 inches   Compare the quantity of iron used in each pipe

Sol   Let h and H inches be the respective lengths of the pipes

Then the internal volume of the 1st $=25\pi h$

and   „   „   „   „   2nd $=\frac{25}{4}\pi H$

$\frac{25}{4}\pi H=25\pi h$ [Given

$H=4$ h

quantity of iron in the 1st $=[(\frac{11}{2})^2-5^2)]\times\pi h$

and   „   „   „   2nd $=[(\frac{23}{8})^2-(\frac{5}{2})^2]\times\pi H$

1st   2nd $=\frac{21}{4}\pi h$ $\frac{129}{64}\pi H$

$=28$ 43.   [H=4h

EXERCISE 85

Note   The first 5 quetions to be done mentally

1   The area of the base of a right prism is 21 sq ft. and its height is 5 ft   Find the volume

2   The base of a right prism is a parallelogram whose adjacent sides are 12 ft and 8 ft   Find the volume of the prism if its height be 10 ft.

3   The area of the base of a cylinder is 15 sq in and its height is 10 in   Find the volume

4   Find the volume of a cylinder which has a height of 10 in and a base of radius 7 in

5   Find the volume of a cylinder the height of which is 14 in and the base of radius 1 in

6 The base of a right prism is a regular hexagon, whose area is 3'125 sq in. If the height is 8 in., find the volume.

7. The area of the base of a right prism is 12 sq. ft. 1 125 sq in and its height is 8 in   Find the volume

8 The base of a right prism is a rectangle whose adjacent sides are 12 ft and 9 ft   If the height of the prism is 7 ft , find the volume

9 The base of a right prism is a rhombus whose diagonals are 10 ft and 8 ft   If the height of the prism be 6 ft , find the volume.

10 The base of a right prism is a right-angled triangle, the sides making the right angle are 4 ft and 3 ft.   If the height of the prism be 5 ft , find the volume.

11. The radius of a cylindrical tube is $\frac{3}{4}$ in and its height is 28 ft   Find the quantity of water it can contain

12 The diameter of a cylindrical tube is 2 in and its height is 21 ft , how much water can it contain ?

13 The radius of a well is 7 ft and its depth is 30 ft. How much earth was taken out when it was excavated ?

14. The radius of a well is $4\frac{3}{8}$ ft   How much water does it contain if the depth of the water is 32 ft. ?

15. A well contains 1760 cubic ft of water and its radius is 4 ft   Find the depth of the water

16 The depth of water in a well is 25 ft.   Find its radius if it contains $962\frac{1}{2}$ cubic ft of water

17 The radius of a well is $3\frac{1}{2}$ ft. and its wall is 1 ft thick   If its depth be 21 ft find the volume of its wall

18. A well, $4\frac{1}{2}$ ft inside radius is to be sunk 28 ft deep with a brick lining 1 ft thick. How many bricks each 9 in by 4 in by 3 in are required to construct it ?

19. A hollow cylindrical tube open at both ends is made of iron 13 in thick   If the external diameter be 2 ft. 3 in. and the length of the tube be 14 ft , find the number of cubic feet and inches of iron in it

20. The external diameter of a hollow cylindrical tube (open at both ends) made of iron 2 in. thick is 1 ft. 10 in. The length of the tube is 21 ft., find the number of cubic ft., etc , of iron in it

21   A cylindrical vessel contains $111\frac{3}{5}$ cubic inches of water and its radius is $2\frac{1}{4}$ in   Find the depth of the vessel.

22.   A cylindrical vessel contains 11 lbs of water and its radius is $3\frac{1}{2}$ in   Find the depth of the vessel if a cubic ft of water weighs $61\frac{5}{7}$ lbs.

23   A tube is 110 yards long and its radius is 8 in How many gallons of water can it contain when a cubic ft. of water $= 5\frac{1}{4}$ gallons ?

24   A well 7 ft inside diameter has been sunk 21 ft deep   Earth taken out of it forms an embankment of uniform width of $10\frac{1}{2}$ ft round the well.   Find the height of the embankment

25   A well has been   excavated   10 ft   deep   The earth taken out from it has been spread all round it to a uniform width of 7 ft. to form an embankment   Find the height of the embankment when the diameter of the well is 14 ft.

26   The external diameter of an open cylindrical pipe made of iron $\frac{1}{4}$ inch thick is 13 inches and the internal diameter of another made of the same metal $\frac{3}{8}$ inch thick is 6 inches, compare the quantities of metal in the pipes if they have equal internal volumes

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§9   We now close this chapter with the recapitulation of the mensurational formulæ.

Mensurational Formulae

1   Rectangle

   (i) Area $=$ length $\times$ breadth.

   (ii) Length $=$ area$-$breadth.

   (iii) Breadth $=$ area$-$length

2   Square

   (i) Area $=$ (side)^2 or half the square of its diagonal.

   (ii) Side $= \sqrt{\text{area}}$.

   (iii) Diagonal $=$ side $\sqrt{2}$ or $\sqrt{2\,\text{area}}$

3. **Parallelogram**
  (*i*) Area =base×height
  (*ii*) Base =area÷height
  (*iii*) Height =area÷base.

4 **Triangle.**

(*a*) Right-angled △ —

  (*i*) Area$=\dfrac{\text{base} \times \text{perpendicular}}{2}$.

  (*ii*) Base= (*a*) $\dfrac{\text{area} \times 2}{\text{perpendicular}}$

      (*b*) $\sqrt{(\text{hypotenuse})^2 - (\text{perpendicular})^2}$

  (*iii*) Perpendicular$=(a)\dfrac{\text{area} \times 2}{\text{base}}$

      (*b*) $\sqrt{(\text{hypotenuse})^2 - (\text{base})^2}$.

  (*iv*) Hypotenuse$=\sqrt{(\text{base})^2 + (\text{perpendicular})^2}$.

(*b*) Isosceles Right-angled △ —

  (*i*) Base$=\dfrac{\text{hypotenuse}}{\sqrt{2}}$

  (*ii*) Perpendicular$=\dfrac{\text{hypotenuse}}{\sqrt{2}}$

(*c*) Equilateral △ —
  (*i*) Area=side×side דּ'433
  (*ii*) Height=side× 433
  (*iii*) Side=height÷'433

(*d*) Any triangle —

Area (*i*) $\dfrac{\text{base} \times \text{height}}{2}$.

  (*ii*) $\sqrt{s(s-a)(s-b)(s-c)}$, where *s* denotes semi-perimeter and *a, b, c,* the sides of the △.

5 **Rhombus**

(*i*) Area=(*a*) ½×product of its diagonals,
    (*b*) Base×height

(*ii*) Diagonal$=\dfrac{2 \times \text{area}}{\text{other diagonal}}$.

6 Quadrilatetal.

(*i*) Area $= \frac{1}{2}$ diagonal $\times$ (sum of its offsets),

(*ii*) Diagonal $= \dfrac{2 \times \text{area}}{\text{sum of its offsets}}$.

(*iii*) Area of a quadrilateral inscribed in a circle $=$ $\sqrt{(s-a)(s-b)(s-c)(s-d)}$, where s is the semi-sum of sides a, b, c, d.

7 Trapezoid

(*i*) Area $= \frac{1}{2} \times$ sum of parallel sides $\times$ height

(*ii*) Height $= \dfrac{2 \times \text{area}}{\text{sum of parallel sides}}$

8 Regular Polygon.

(*i*) Area $= \dfrac{\text{No. of sides}}{2} \times$ side $\times$ radius of inscribed circle

(*ii*) Side $= \dfrac{2 \times \text{area}}{\text{No. of sides} \times \text{radius of inscribed circle}}$.

(*iii*) Area of Hexagon $= \dfrac{3a^2 \sqrt{3}}{2}$ where a is the side.

(*iv*) Area of Octagon $= 2a^2(1 + \sqrt{2})$,,   ,,

9 Circle

(*i*) Area $=$ radius$^2 \times \pi$, $[\pi = \frac{22}{7}$

(*ii*) Radius $= \sqrt{\text{area} \times \frac{7}{22}}$

(*iii*) Diameter $=$ circumference $\times \frac{7}{22}$

(*iv*) Circumference $=$ diameter $\times \frac{22}{7}$

10. Arc

Arc of $D^0 = \dfrac{D}{360} \times$ circumference.

11. Sector of D^0.

(*i*) Area $= \dfrac{D}{360} \times$ area of circle ,

(*ii*) Area $= \frac{1}{2}$ arc $\times$ radius

12. **Segment**
   Area=sector−triangle.

13 **Cuboid.**
   (i)   Surface$=2(lb+lh+bh)$,   where l denotes length, b breadth and h height
   (ii) Volume$=l \times b \times h$

14. **Cube**
   (i)   Surface$=6(\text{edge})^2$,
   (ii) Volume$=(\text{edge})^3$

15. **Prism**
   (i) Lateral surface$=$(perimeter of base)$\times$ height
   (ii) Volume $=$area of base $\times$ height.

16 **Cylinder**
   (i) Curved surface$=2r\pi h$
   (ii) Volume $=r^2\pi h$.

17 **Cone**
   (i) Volume$=\frac{1}{3}$ area of base $\times$ height$=\frac{1}{3}r^2\pi h$

   (ii) Curved surface$=\frac{1}{2}$ perimeter of base $\times$ slant height$=\pi r l$, where l is slant height

   (iii) Whole surface$=\pi r l + r^2 \pi = \pi r(l+r)$

Again, if h, the height of the cone, be given
$$l=\sqrt{h^2+r^2}$$

   whole surface$=\pi r(\sqrt{h^2+r^2}+r)$.

18 **Sphere**   (i) Surface$=4\pi r^2$, (ii) Volume$=\frac{4}{3}\pi r^3$.

19. **Pyramid**   Volume$=\frac{1}{3}$ area of base $\times$ height.

20   If the sides opposite the angles A, B, C of the $\triangle$ ABC be a, b, c, $\triangle$ its area, R circumradius, r inradius r_1, r_2, r_3, ex-radii, then

   (i) $\triangle = \sqrt{s(s-a)(s-b)(s-c)}$, where $s=\frac{1}{2}(a+b+c)$,

   (ii) R$=\dfrac{abc}{4\triangle}$.   (iii) $r=\dfrac{\triangle}{s}$

   (iv) $r_1=\dfrac{\triangle}{s-a}$, $r_2=\dfrac{\triangle}{s-b}$, $r_3=\dfrac{\triangle}{s-c}$.

CHAPTER XVI
UNITÀRY METHOD AND CHAIN RULE
I Unitary Method

§1   If we know the price, weight, length of any number of units, we can by division, find the price of one unit of the same substance and when that is found we can, by multiplication, find the price, weight, length, etc , of any number of units of the same substance   The process by which we combine the two processes described above, is called the **unitary method**

Example 1.   If 2 books cost ~~Rs~~ 6, what will one book cost ?

Sol   The cost of 2 books = ~~Rs~~ 6

     „   „   1 book = ~~Rs~~ 6—2 = ~~Rs~~ 3 **Ans**

Example 2   If 1 book costs Rs 3, what will 7 books cost ?

Sol   The cost of 1 book = ~~Rs~~ 3

     „   „   7 books = ~~Rs~~ 3 × 7 = ~~Rs~~ 21   **Ans**

Now if we combine the two examples given above we form the following

Example   If 2 books cost Rs 6, what will 7 books cost ?

Sol.   The cost of 2 books = ~~Rs~~ 6

     „   of 1 book = ~~Rs~~ 6—2 = ~~Rs~~ 3

     „   of 7 books = ~~Rs~~ 3 × 7 = ~~Rs~~ 21   **Ans**

First the price of one article is found by division and then the price of the required number of articles is obtained by multiplication

Observe also that in every unitary question two of the three terms are like terms and the 3rd is an unlike term, it is similar to the term we obtain as answer

Thus in the above example, 2 books and 7 books are like terms and the 3rd term Rs 6 is an unlike term

F 18

In solving such questions the 3rd unlike term is placed last in the first line as shown in the examples solved above   The following solved examples will further illustrate the method clearly.

Example 3   If 7 yards of cloth cost Rs 15 5a , what will 11 yds. of cloth cost ?

Sol.   Cost of 7 yards = Rs 15 5a

„   „ 1 yard = Rs. 15. 5a ÷7 = Rs 2 3a

„ „ 11 yards = Rs 2 3a × 11 = Rs 24 1a **Ans**.

Example 4   If $\frac{2}{15}$ of an estate be worth Rs 85 8a , what is the value of $\frac{13}{15}$ of it ?

Sol $\frac{2}{15}$ of an estate is worth Rs. 85 8a

$\frac{1}{15}$ of   „   „   Rs 85. 8a ÷2
                            = Rs 42. 12a.

∴ $\frac{13}{15}$ „   „   „   Rs 42 12a × 13
                            = Rs 555. 12a **Ans.**

Example 5.   6 men or 9 women earn Rs. 75 in a certain time , how much will 10 men and 6 women earn in the same time ?

Sol   6 men earn as much as 9 women,

1 man earns as much as $\frac{9}{6}$ women,

10 men earn as much as $\frac{9}{6}$ × 10 = 15 women,

∴ 10 men and 6 women earn as much as 15 + 6 = 21 women

Now 9 women earn Rs 75,

∴   1 woman earns Rs $\frac{75}{9}$,

21 women earn Rs $\frac{75}{9}$ × 21 = Rs 175. **Ans.**

Example 6   10 horses and 15 cows eat grass of 5 acres in a certain time , how many acres will feed 15 horses and 10 cows for the same time, supposing a horse eats as much as 2 cows ?

Sol   Since 1 horse eats as much as 2 cows,

10 horses will eat as much as 20 cows,

∴ 10 horses and 15 cows will eat as much as 20 + 15 = 35 cows

Similarly 15 horses and 10 cows will eat as much as 40 cows

Now 35 cows eat grass of 5 acres,

1 cow will eat grass of $\frac{5}{35}$ acres,

40 cows „   „   „ $\frac{5}{35} \times 40 = \frac{40}{7}$ acres,

$= 5\frac{5}{7}$ acres   **Ans.**

EXERCISE 86

1   If 9 articles cost Rs. 17. $7a$, what will one article cost ?

2   Railway fare for 41 miles is $6a$ $10p$, what is the fare for 66 miles ?

3.   If I run a mile in 7 min 12 sec , how long will I take to run 440 yards ?

4   If one article costs Rs 2 $3a$, what will 12 articles cost ?

5   If 6 chairs cost Rs. 32 $12a$, what will 3 dozen cost ?

6   If 21 yards of cloth cost Rs 43 $5a$, what will one yard of cloth cost ?   And also what will 15 yards of cloth cost ?

7   If 16 seers of sugar cost Rs 6 $8a$, what is the cost of 9 seers ?

8   52 yards of cloth cost Rs 39 , what will 8 yards of cloth cost ?

9   A man earns Rs 33 $12a$ in 15 days , what will he earn in 20 days ?

10   A man walks $4\frac{1}{2}$ miles in 72 min , find his rate per hour in metres

11   Find the cost of 37 yards of silk if 25 yards cost Rs 65 $10a$.

12   16 men can reap 85 bighas of field, how many bighas will 24 men reap ?

13   If the railway fare for 75 miles be Rs 4 $11a$, what is the fare for 49 miles ?

14   The price of 10 bullocks is equal to the price of 15 sheep, how many bullocks can be purchased for the price of 39 sheep ?

15   If 37 chairs cost £8 11s $1\frac{1}{2}d$, how many can be had for £70 6s ?

16. If 15 lbs of tea cost 18s. 9d., how much would 1 qr of tea cost ?

17. What would be the simple interest on Rs. 625 if the interest on Rs 100 is $3\frac{1}{3}$?

18 If $2\frac{1}{2}$ cwt cost £1 9s 2d , what will 21 lbs cost ?

19. If 15 seers of sugar cost Rs 3. 12a , find the price of $1\frac{3}{4}$ maunds

20 A man runs a mile in 5 minutes 30 seconds, how long will he take to run 440 yards ?

21. If 3 md 12 sr. 8 chk of sugar cost Rs 16 9a., what will 1 md. 7 sr 5 chk cost ?

22 I travel 3 miles 7 fur 20 po in 1 hour , how far can I go in 6 hrs. 14 min. 40 sec. ?

23 If 24 men can reap 16 acres in a certain period, how many men will reap 40 acres in the same time ?

24 If $\frac{3}{17}$th of an estate be worth Rs 110, 1a , what is the value of $\frac{1}{12}$th of the estate ?

25 If $\frac{1}{5}$th of a ship is worth £70. 19s. 6d., what part of her is worth £250 10s

26 If $\frac{5}{8}$th of a property be worth Rs 141 12a., find the value of $\frac{3}{4}$th of that property

27 The value of $\frac{4}{13}$th of a cargo is Rs. 576 12a , what is the value of $\frac{7}{15}$th of that cargo ?

28 If $\frac{6}{8}$th of a house be worth Rs. 856 3a., find the value of $\frac{5}{13}$th of that house.

29 If $\frac{5}{16}$th of a cargo is worth Rs. 1055. 5a , what is the value of $\frac{7}{24}$th of that cargo ?

30. A man rides $46\frac{1}{2}$ miles in 5 hours 10 minutes, how much time will he take to ride $28\frac{1}{2}$ miles ?

31 If 6 men do as much work as 8 women, how many men will do as much work as 12 women ?

32 If 6 bullocks or 15 sheep cost Rs 225, what will 9 bullocks and 12 sheep cost ?

33 A railway train runs at the rate of 22 miles an hour , how far will it go between 6-15 A M and 2-35 P M. ?

34   6 cows and 5 horses graze 5 acres of grass in a certain time, how many acres of grass will 8 cows and 9 horses graze for the same time, supposing 2 cows to eat as much as 3 horses ?

35   If 6 bullocks and 16 sheep eat the grass of 2 acres in a certain time, how many acres will feed 15 bullocks and 6 sheep for the same time, supposing a bullock eats as much grass as 3 sheep ?

36   A man employed a servant on the condition that he would get Rs. 45 and a horse after a month, but after 12 days he dismissed the servant, giving him only a horse, find the value of the horse. (*Hint* Salary for 18 days = Rs 45)

37   A man employed a servant on the condition that he would get a horse and Rs 40 after a month, but after 18 days he dismissed him giving him only a horse, find the value of the horse.

———

§2   An increase in the number of workmen corresponds to a diminution in the number of days and *vice versa*

Example 1.   If 15 men can do a piece of work in 10 days, how long will it take one man to do it ?

Sol.   15 men can do the work in 10 days
. 1 man   „   „   in 10×15 days
    i.e., 150 days.   **Ans.**

Example 2   If one man can do a piece of work in 150 days, how long will it take 25 men to do it ?

Sol.   1 man can do the work in 150 days
    25 men   „   „   „   in 150—25 days
        i.e., 6 days   **Ans.**

Now combine the above two examples thus :—

Example 3   If 15 men can do a piece of work in 10 days, how long will it take 25 men to do it ?

Sol   15 men can do the work in 10 days
        1 man   „   „   in 10×15 days
        25 men   „   „   in $\dfrac{10 \times 15}{25}$ days
            i.e., 6 days. **Ans**

Example 4   If 10 men can mow a field in 5 days, how many men will be required to mow the same field in $12\frac{1}{2}$ days ?

Sol   In 5 days the field is mowed   by 10 men

.   in 1 day   „   „   by 10×5 men

'   in $\frac{25}{2}$ days   „   „   by $\dfrac{10 \times 5 \times 2}{25}$ men

$$i \, e, \text{4 men. Ans.}$$

Example 5   A garrison of 1500 men is provisioned for 60 days   After 25 days the garrison is reinforced by 500 men , how long will the remaining provisions last ?

Sol   Since the garrison is reinforced by 500 men therefore there are $(1500 + 500)$ or 2000 men now.

The provisions left would last 1500 men 35 days

.   „   „   „   1   man 35×1500 days

.   „   „   „   2000 men $\dfrac{35 \times 1500}{2000}$ days

$$i \, e, 26\frac{1}{4} \text{ days   Ans.}$$

Note   Sometimes it so happens, that in a question, at first sight, it would appear that more than 3 terms are given but such an example in some cases   come under the same rule, as in the following example

Example 6.   A contractor undertook to do a certain work in 55 days and employed 48 men to do it.   In 11 days, only $\frac{1}{5}$ of the work was done.   How many extra men should he employ in order to complete the work in time ?

Sol   In 11 days only $\frac{1}{5}$th of the work was done   It is evident therefore, that the remaining $\frac{4}{5}$th will be finished in 55 days , but he wants to finish the remaining work in $55 - 11 = 44$ days

In 55 days the   rem   work can be finished by 48 men
in 1 day   „   „   „   by 48×55 men

in 44 days   „   „   „   by $\dfrac{48 \times 55}{44}$ men

$$i \, e, \text{ 60 men}$$

he should employ $60 - 48$ or 12 men more   **Ans.**

Example 7. 5 men and 6 boys finish a piece of work in 4 days, 4 men and 3 boys in six days In how many days would 3 men and 6 boys finish the same work ?

Sol 5 men and 6 boys finish the work in 4 days, 20 men and 24 boys will finish it in 1 day.

also 4 men and 3 boys finish it in 6 days, 24 men and 18 boys will finish it in 1 day.

If follows therefore, that the work of 20 men and 24 boys is equal to the work of 24 men and 18 boys.

Or the work of 4 men=the work of 6 boys,
" " 2 men= " 3 boys
the work of 5 men and 6 boys=the work of 9 men
also the work of 3 men and the work of 6 boys=the work of 7 men

Now the question is ' "9 men can finish a piece of work in 4 days. In how many days would 7 men finish it" ?

EXERCISE 87

Note Some of the examples in this exercise are cases of direct method

1 If 8 men can finish a piece of work in $17\frac{1}{2}$ days. how many men are required to do the same work in 7 days ?

2. If 15 men can reap a certain field in 17 days, in how many days could 9 men reap the field ?

3 If 28 men can finish a piece of work in 48 days, in how many days could 35 men do it ?

4 If 9 maunds of grass are eaten by 21 cows in 14 days, how long will it take 7 cows to eat the same grass ?

5 If 8 men or 12 women can do a piece of work in 15 days, how long will it take 6 men and 9 women to do the same work ?

6 If 15 bullocks or 25 horses can mow a field in 10 days, in what time could 9 bullocks and 15 horses mow the same field ?

7. If 7 oxen or 11 horses eat the grass of a field in 37 days, how long will it take 5 oxen and 8 horses to eat it ?

8   9 men and 15 boys can reap a field in 15 days, in what time could 15 men and 16 boys reap the same field, supposing 3 men do as much work as 4 boys ?

9   If 7 boys earn as much as 4 men and 48 men with the help of 14 boys earn Rs 423 8a, how many boys must help 20 men, in order to earn Rs 272 4a in the same time ?

10   A garrison of 2100 men has provisions for 36 days how many men must be turned out, so that the provisions may last for 54 days ?

11   A garrison of 2700 men has provisions for 51 days, how long would they last if the garrison be increased by 900 men ?

12.   A garrison of 1400 men is provisioned for 54 days, if after 14 days the garrison be increased by 600 men, how long will the remaining provisions last ?

13   A ship leaves a port with food enough to last 14 weeks, 6 of the crew absconded and the voyage lasted 16 weeks when it was found that the food had just exhausted.   Find the number of the full crew

14   A besieged garrison have 4 months' provisions at the rate of 18 chataks per man per day   How long would they be able to hold out, if each man were allowed only 12 chataks per day ?

15   A train travelling 40 miles an hour reaches its destination in 3 hours 15 minutes, how long will it take to reach the same place, travelling at the rate of 25 miles ?

16.   A contractor undertook to do a certain work in 75 days and employed 60 men to do it, after 25 days he found that only $\frac{1}{4}$th of the work was done.   How many more men must he employ in order that the work may be finished in time ?

17   A contractor undertook to do a certain work in 80 days and employed 72 men to do it, after 20 days he found that $\frac{2}{3}$rd of the work has been finished   How many men should he dismiss in order that the work may be finished on the date agreed upon ?

18   A contractor takes up the work of making a road 189 miles long in 15 months   He engages 256 men, but after $\frac{1}{5}$th of the time finds that only $\frac{1}{6}$th of the work has been completed   How many more men should he   now employ to complete the contract in time ?

19.   If the 6d loaf weighs 3˙4375 lbs when the wheat is 55 5s a quarter, what is the price of wheat when the loaf weighs 2 8125 lb ?

20   If the 4d loaf weighs 3 chataks when the price of wheat is Rs 3 a maund, what should its weight be when wheat is at Rs 3 6a a maund ?

21   A piece of gold at Rs 38 15a per oz. is worth Rs 1500 , what should be the price of a piece of silver of equal weight at Rs 27 4a per lb ?

22   If 17 men can do a piece of work in 89 days and if after working for 33 days, 3 men leave, in how many days in all will the work be done ?

23   I take 55 min to walk to cantonment by the road and 58 min 40 sec to return by the fields, walking at the same rate   The distance by the road is $3\frac{3}{4}$ miles , what is it by the fields ?

24   6 men and 7 boys can finish a piece of work in 21 days   In how many days could 12 men and 6 boys finish the same work, supposing 2 men work as much as 3 boys

25   The Bombay Express travels 27 miles an hour including stoppages and 30 miles an hour when it does not stop   In what distance will it lose 45 minutes by stopping ?

26   25 men were employed to do a piece of work in 24 days   After 15 days, 10 more men were engaged and the work was finished a day too soon   In what time could they finish the work if extra men were not employed ?

27   A fort is provisioned for 75 days , after 25 days a reinforcement of 400 men arrived and the food will now last only 40 days   How many men were there in the fort ?

28   7 men and 5 boys can finish a piece of work in 12 days , 9 men and 25 boys in 5 days   In how many days could 6 men and 5 boys do the same work ?

29.   6 men and 8 boys finish a piece of work in 10 days , 12 men and 4 boys in 8 days.   In how many days would 3 men and 2 boys finish it ?

§3   Double Unitary Method

Example 1   If 7 men working 9 hours a day can finish a piece of work in 14 days, in how many days will 12 men working 7 hours a day, finish the same work ?

Note   The answer required is number of days   therefore put 14 days at the end of the first line

Sol   7 men, 9 hrs a day, will finish work in 14   days

  1   man, 9 hrs. „   „   „ in 14×7 days

  ∙   1 „ 1 hr „   „   „ in $14 \times 7 \times 9$ days

  12 men, 1 hr „   „   „ in $\dfrac{14 \times 7 \times 9}{12}$ days

  ∙   12 „ 7 hrs „   „   „ in $\dfrac{14 \times 7 \times 9}{12 \times 7}$ days

         $= \frac{21}{2}$ i.e., in $10\frac{1}{2}$ days. **Ans.**

Example 2   If 5 men working 8 hours a day can compose a book of 240 pages in 21 days, how long will it take 3 men to compose a book of 360 pages working 10 hours a day ?

Sol   5 men, 8 hrs. a day comp 240 pages in 21   days,

  1 man 8 hrs.   „   „ 240 „ „ 21×5 „

  1 „ 1 hr   ,   „ 240 „ „ $21 \times 5 \times 8$ „

 ∴ 1 , 1 hr   „   „ 1 page „ $\dfrac{21 \times 5 \times 8}{240}$ „

  ∙   3 „ 1 hr   „   „ 1 „ „ $\dfrac{21 \times 5 \times 8}{240 \times 3}$ „

  3 men 10 hrs   „   „ 1 „ „ $\dfrac{21 \times 5 \times 8}{240 \times 3 \times 10}$„

  3 , 10 hrs   „   „ 360 pages „ $\dfrac{21 \times 5 \times 8 \times 360}{240 \times 3 \times 10}$„

          i e in 42 days **Ans.**

EXERCISE 88.

1   If 5 men working 8 hours a day can finish a piece of work in 20 days, how long will it take 8 men, working 12 hours a day to do the same work ?

2   If 9 men reap a field of 8 acres in 12 hours, how many men will reap a field of 28 acres in 18 hours ?

3   If 6 men can dig a trench in 5 days, working 12 hours a day, how many men will dig the same trench in 4 days working 10 hours a day ?

4   If 13 men can do a piece of work in 12 days of 8 hours, how many hours a day must 9 men work to do the same work in 16 days ?

5   If the wages of 12 men for 25 days amount to Rs. 450, what will be the wages of 20 men for 16 days ?

6   What will be the wages of 15 men for 10 months when 9 men receive £261 15s for 8 months ?

7   If 40 maunds of rice be carried 150 miles for Rs 12 8a, for what sum of money will 50 maunds of rice be carried 320 miles ?

8. If a man travels 51 miles in $1\frac{1}{2}$ days by walking 9 hours a day, in how many days will he travel 170 miles by walking $7\frac{1}{2}$ hours a day ?

9. If 10 masons build a house in 25 days of 6 hours, in how many days of 8 hours will 15 masons build the house ?

10   If 144 men in 5 days of 11 hours each can dig a trench 132 yds long, 5 ft wide and 2 ft deep, in how many days of 9 hours each, can 56 men dig a trench 210 yds long, 8 ft wide and 3 ft deep ?

11   If Rs. 500 bear an interest of Rs 40 in 2 years, interest at the same rate will Rs 625 bear in $1\frac{1}{2}$ years ?

12   If with a capital of Rs 10000, a person gains Rs 500 in 16 months, in how many months will he gain Rs 1250 with a capital of Rs 4000 ?

13   If 200 men can make an embankment 5 miles long in 25 days, how much over time must 60 men work to finish an embankment 2 miles long in 32 days, 12 hours being a day's work ?

14   If 5 men can reap a field 800 ft long and 700 ft. broad in $3\frac{1}{2}$ days of 14 hours each, in how many days of 18 hours each can 7 men reap a field 1800 ft. long and 960 ft broad

15   If 39 men can reap a field 65 yards long and 55 yards broad in 2 days, how many men are required to reap a field 77 yards long and 75 yards broad in 14 days ?

16   If 12 men working 8 hours a day take 15 days to do a piece of work, how long will 16 boys working 10 hours a day take to do double the work, the work of one man being equal to that of two boys ?

17   If 12 men or 15 women or 18 boys can do a piece of work in 15 days of 8 hours, find how many men assisted by 5 women and 6 boys will finish the same work in 16 days of 9 hours each

18   If 7 men or 8 women or 10 boys can finish a piece of work in 24 days of 9 hours, find how many men with the help of 4 women and 5 boys can finish it in 18 days of 6 hours

19   When wheat is sold at Rs. 6 a maund, Rs 27 8a maintain a family of 10 men, what sum of money will maintain a family of 15 men, when wheat is sold at Rs 5 a maund ?

20   When rice is 10 seers a rupee, a sum of money maintains a family of 18 men for 15 days, how long will the same amount of money maintain a family of 6 men when rice is 14 seers a rupee ?

21.   If a penny loaf weighs 6 oz when wheat is 5s 6d a bushel, what should be the weight of a shilling loaf when wheat is 8s 3d a bushel ?

22   If 12 pumps can raise 1218 tons of water in 11 days of 9 hours each, how many pumps will raise 2030 tons of water in 12 days of 11 hours each ?

23   If 5 pumps each having a length of stroke of 3 feet, working 15 hours a day for 5 days, empty the water out of a mine, how many pumps with a length of stroke of 2½ ft working 10 hours a day for 12 days will be required to empty the same mine, the strokes of the former pumps being performed 4 times as fast as those of the other ?

24   A besieged garrison consists of 300 men, 120 women and 40 children, and has provisions enough for 200 men for 30 days   If a woman eats ⅔rd as much as a man

and a child half as much as a man and if after 6 days 100 men with all the women and children escape, for how long will the remaining provisions last the garrison ?

25   If when wheat is at Rs 3 per maund, the 4a loaf weighs 8 chk , what should be the price of wheat per maund, when 3 sr 2 chk of bread cost 12a 6p. ?

26   If Rs 240 be paid for bread for 49 persons for 20 months, when wheat is at Rs 3 per maund, how long will Rs 234 supply bread for 91 persons, when wheat is at Rs 3 8a per maund ?

27   If 44 cannon firing 30 rounds an hour for 3 hours a day, consume 300 barrels of powder in 5 days, how long will 400 barrels last 66 cannon, firing 40 rounds an hour for 5 hours a day ?

28   If 60 guns firing 5 rounds in 6 min kill 350 men in $1\frac{1}{4}$ hrs how many guns firing 7 rounds in 9 min will kill 980 men in 25 min at the same rate ?

29   If 5 horses require as much corn as 8 ponies and 15 quarters of corn last for 12 ponies for 64 days, how long may 25 horses be kept for £41. 5s , when corn is 22s. a quarter ?

30   If 240 men working 10 hrs a day can dig a trench 300 yds long, $3\frac{1}{2}$ ft deep and $2\frac{1}{2}$ ft wide in 6 days, in how many days would 80 men, working 9 hrs a day dig a trench 500 yds long, 3 ft. wide and $2\frac{1}{4}$ ft. deep ?

31   If 10 compositors, in 16 days of 14 hrs each can compose 20 sheets of 24 pages in each sheet, 50 lines in a page and 40 letters on the average in a line, in how many days of 7 hrs each can 20 compositors compose a volume to be printed in the same letter, containing 40 sheets, 16 pages in a sheet, 60 lines in a page and 50 letters in a line ?

32   If 7 women earn as much as 4 men and 48 men assisted by 14 women earn 121 guineas in 17 days, what number of women with the help of 20 men will earn £21. 3s 6d in one-third of the time ?

33   If the wages of 25 men amount to Rs 766 10a 8p in 16 days, how many men must work for 24 days to earn Rs 1035, the daily wages of the latter set being half of the former ?

34   If 17 men working 8 hrs a day made an excavation 121 ft 6 in long, 25 ft 6 in broad and 24 ft deep in 54 days, how many hours a day must 18 men work during 51 days in order that they may make an excava tion whose length and breadth are 1 ft. 6 in. less and depth 1 ft 6 in greater than the preceding one, supposing 9 men of the latter group do as much work as 10 of the former ?

35   If 38 men working 6 hours a day can do a piece of work in 12 days, find in what time 57 men working 8 hrs a day can do a piece of work twice as great, if 2 men of the first set can do as much work in 1 hour as 3 men of the second set can do in 1½ hours

36   A contractor agrees to finish a piece of work in a certain time   He engages 55 men who work 9 hrs a day   But after ¼th of the time   he found that only $\frac{4}{7}$th of the work had been done   How many more men must he now employ in order that all the men working 11 hours daily may finish the work in the fixed time ?

37   A town is garrisoned with 10000 troops who have provisions enough for 6 months, giving a ration of 3½ lbs. daily to each man   How many men must be sent away, so that by giving ½ lb less to each man, the provisions may last for 8 months ?

38.   If a family of 9 men in Lahore spend Rs 16380 in a year, what must be the expenses of 8 men in Calcutta who live in the same style for 7 months, assuming that the prices at Calcutta are $\frac{3}{8}$th of what they are in Lahore ?

39   A farmer engages 30 men and 45 women to cut down his crop in 20 days of 12 hours each , but after 12 days' work, he intends to finish the work in 4 days more of 10 hrs each   How many more men must be employed assuming that 2 men do as much work as 3 women ?

40   6 men or 9 women or 12 boys working 10 hours a day can finish a piece of work in 12 days   How many men with 3 women and 4 boys working 5 hours a day would finish the same work in 4 days ?

II Chain Rule

§4   If we wish to express one quantity A in terms of another R and have data of the following type, viz,

$$aA = mM,$$
$$bM = nN,$$

$$cN = pP, \text{ etc etc}, \text{ then } A = \frac{mnp}{abc} P, \text{ i. e },$$

the quantity required is obtained by dividing the product of all numbers on the right-hand side by that of the numbers on the left hand side.

Example   2 horses cost as much as 5 cows, 6 cows as much as 8 oxen, 10 oxen as much as 50 sheep, 14 sheep as much as 9 goats. If the cost of one goat be Rs 7, how much will one horse cost ?

Sol.   Reqd No of rupees = one horse,
$$2 \text{ horses} = 5 \text{ cows,}$$
$$6 \text{ cows} = 8 \text{ oxen,}$$
$$10 \text{ oxen} = 50 \text{ sheep,}$$
$$14 \text{ sheep} = 9 \text{ goats,}$$
$$\text{One goat} = \text{Rs. } 7.$$

$$\text{required No of rupees} = \frac{1 \times 5 \times 8 \times 50 \times 9 \times 7}{2 \times 6 \times 10 \times 14 \times 1}$$
$$= \text{Rs } 75 \quad \text{Ans.}$$

This method is known as **Chain Rule**   The given relations are put in the form of an equation in such a manner that each denomination occurs on each side of the sign of equality, which means "are worth," and then the product of the numbers on the right-hand side is divided by the product of the numbers on the left-hand side   The quotient is the required value

EXERCISE 89

1   2 horses cost as much as 6 cows, 8 cows as much as 9 oxen, 3 oxen as much as 8 sheep   If 5 sheep cost Rs 40, what will one horse cost ?

2   If 12 rupees are worth 4 dollars, 3 dollars are worth 18 francs, 15 francs are worth 10 marks, how many marks can be exchanged for Rs 36 ?

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3   If 6 rupees are worth 10 shillings, 15 shillings are worth 20 francs and 25 francs are worth 5 dollars, how many dollars can be exchanged for Rs 81 ?

4   5 ducks are worth 6 chickens, 3 chickens are worth 2 geese and 9 geese are worth 8 turkeys, what is the price of 3 ducks when 2 turkeys cost Rs 15 ?

5   If 8 lb of tea be worth 6 lb of coffee, 5 lb of coffee be worth 7 lb of chicory and 9 lb of chicory be worth 12 lb of sugar, how many pounds of tea can be given in exchange for 14 lb of sugar ?

6   If 15 maunds of wheat be worth 18 maunds of grams, 16 maunds of grams be worth 25 maunds of barley and 20 maunds of barley be worth 12 maunds of rice, how many maunds of wheat can be given in exchange for 27 maunds of rice ?

7   If 2 horses cost as much as 11 cows, 3 cows as much as 7 sheep, 14 sheep as much as 15 goats, what will one horse cost when a goat costs Rs 5 ?

8   A can do as much work in 5 days as B can do in 8 days, and B can do as much in 4 days as C in 9 days, in what time will C do a piece of work which A can do in $2\frac{1}{3}$ days ?

9.   If A can do as much work in 3 days as B can do in $2\frac{1}{2}$ days and B can do as much in 4 days as C in $4\frac{1}{2}$ days, in what time will C do a piece of work which A can do in 8 days ?

10   If 3 kilog of tea be worth 5 kilog of cocoa and 7 kilog of cocoa be worth 11 kilog of coffee, what will 5 kilog of tea be worth, if 1 kilog of coffee is worth 6 fr 30 c ?

CHAPTER XVII.

RATIO AND PROPORTION.

I. RATIO

§1 In ratio we compare two quantities of the *same kind* and consider what multiple, part or parts one is of the other In comparing 8 with 4 we observe that it is 2 times 4 This comparison can be represented as 8—4 or $\frac{8}{4}$. Hence

Def Ratio is that relation between two numbers which is expressed by the fraction, the numerator of which is the measure of the first quantity and denominator is the measure of the second quantity.

The expression $\frac{8}{4}$ is otherwise read as the ratio of 8 to 4 and is further usually expressed by the notation 8.4.

§2 The two numbers which form the ratio are called its **terms**, the first number is called the **antecedent** and the second number the **consequent**

§3 If the terms of a ratio be multiplied or divided by the same quantity the value of the ratio is not altered.

Thus 2 5 is the same as 8 20

and 12 16 is the same as 3 4.

§4 **Comparison of ratios.**

We can compare the ratios thus —

Example Compare the ratios 3.5 and 7.8

Sol. 3 5$=\frac{3}{5}$, 7 8$=\frac{7}{8}$

$\frac{3}{5}, \frac{7}{8}=\frac{24}{40}, \frac{35}{40}.$

Hence 7.8 is greater than 3 5

§5 **Kinds of ratio**

(1) If the antecedent in a ratio is greater than the consequent, the ratio is called one of **greater inequality** as 5 3,

(*ii*)   If the antecedent is equal to the consequent, the ratio is called one of **equality** as 3 3.

(*iii*)   If the antecedent is less than the consequent the ratio is called one of the **less inequality** as 3 5

§6.   A ratio of greater inequality is diminished and of less inequality increased by adding any same quantity to both its terms.

If 1 be added to the terms of the ratio of greater inequality 7·4 it becomes the ratio 8 5 which is less than the former.

If 1 be added to the terms of the ratio of less inequality 4 7 it becomes 5 8 which is greater than the former

Conversely   A ratio of greater inequality is increased and of less inequality diminished by subtracting any same quantity from both its terms

If 1 be subtracted from the terms of the ratio of greater inequality 5 3 it becomes 4 2 which is greater than the former

If 1 be subtracted from the terms of the ratio of less inequality 3 5 it becomes 2 4 which is less than the former.

§7.   A ratio compounded of other ratios

Ratios are compounded by multiplying together the antecedents for a new antecedent and the consequents for a new consequent

Take the following ratios 2 3, 5 6, 7 8   Then the ratio $2 \times 5 \times 7$ $3 \times 6 \times 8$ or 70 144 is said to be the ratio compounded of the three given ratios

When the ratio is compounded with itself once it is called the *duplicate ratio*, thus 2^2 3^2 *i e*, 4 9 is the duplicate ratio of 2·3 conversely 2 3 is the *sub duplicate ratio* of 4 9.

When the ratio is compounded with itself twice it is called the *triplicate ratio*   Thus 8·27 is the triplicate ratio of 2 3 and conversely 2:3 is the *subtriplicate ratio* of 8 27

§8. Inverse or reciprocal ratio.

If 4 5 be the given ratio then $\frac{1}{4}$ $\frac{1}{5}$ or 5.4 is the inverse or reciprocal ratio

§9 We shall now solve some examples.

Example 1 Find the ratio of $3a$ $4p$ to $5a$

Sol. $3a$ $4p = 40p$

$5a = 60p$

ratio $= \frac{40}{60} = \frac{2}{3} = 2$ 3 **Ans.**

Example 2 Which of the two ratios is greater 5˙6 or 7 8 ?

Sol 5 $6 = \frac{5}{6} = \frac{20}{24}$

7 $8 = \frac{7}{8} = \frac{21}{24}$

∴ 7 8 is the greater **Ans**

Example 3 A, B, C are three quantities of the same kind, their ratios are A $B = 3$ 4, B $C = 5$ 6, find the ratio between A and C.

Sol $\frac{A}{B} = \frac{3}{4}$ and $\frac{B}{C} = \frac{5}{6}$

∴ $\frac{A}{B} \times \frac{B}{C} = \frac{3}{4} \times \frac{5}{6}$

$\frac{A}{C} = \frac{5}{8}$

A $C = 5$ 8 **Ans**

Example 4 A, B, C, D are four quantities of the same kind Their ratios are A $B = 3$ 4, $B˙C = 5.6$ and C $D = 8$ 9 Compare them

Sol. A $B = 3$ 4

$B.C = 5.6 = 1$ $\frac{9}{5}$

$= 4 \cdot \frac{24}{5}$

Also C $D = 8$ $9 = 1.\frac{9}{8}$

$= \frac{24}{5}$ $\frac{9}{8} \times \frac{24}{5} = \frac{24}{5}$ $\frac{27}{5}$

A B C $D = 3$ $4 \cdot \frac{24}{5}$ $\frac{27}{5}$

$= 15 \cdot 20$ 24 27. **Ans**

Example 5   Three numbers are in the ratio of $3.4.5$ and the sum of their squares is 450, find the numbers

Sol   Suppose that the numbers are $3x$, $4x$, $5x$.

. $\quad 9x^2 + 16x^2 + 25x^2 = 450$

$\qquad\qquad$ or $50x^2 = 450$

$\qquad\qquad$ or $\quad x^2 = 9$

$\qquad\qquad$ or $\quad x = 3$

. reqd numbers are $3 \times 3 = 9$
$\qquad\qquad\qquad\quad 3 \times 4 = 12$ $\Big\}$ **Ans.**
$\qquad\qquad\qquad\quad 3 \times 5 = 15$

EXERCISE 90.

Simplify the following ratios :—

1   9 3   **2**   12 16.   **3**   20·25

4.   125·225   **5.**   144.156.

Find the ratio of —

6.   2 yards and 2 feet.

7   One acre and 1760 sq yards

8.   One mile and 440 yards

9   4 centimetres and 6 decimetres.

Which of the ratios is greater ?—

10   3 4 or 8 9.   **11**   5.6 or 3.4

12   15 20 or 11 16

Form the ratio compounded of the ratios —

13   5 6, 8.9, 12 15

14   Find the duplicate ratio of 9 8

15   Find the triplicate ratio of 6 7.

16.   Find the sub-duplicate ratio of 81 144.

17   (a)   What is the antecedent if 24 be the consequent and $\frac{1}{6}$ the ratio ?

$\qquad$ (b)   What is the consequent if 13 5 be the antedent and 4 the ratio ?

18   If $A = 3\frac{1}{3}$ of B and $C = 5\frac{1}{2}$ of B, find the ratio of A to C

19   Compare the rates of two trains, one of which goes $397\frac{5}{8}$ miles in $11\frac{2}{5}$ hours and the other goes $262\frac{4}{13}$ miles in $8\frac{4}{9}$ hours

20   Three quantities A, B, C are of the same kind such that $A.B = 7:8$ and $B:C = 9.14$. Find the ratio between A and C.

21.   Four quantities A, B, C, D are of the same kind such that $A.B = 5.6$, $B:C = 7\ 8$ and $C:D = 9:10$ (ı) find the ratio between A and D, (ıı) compare A, B, C, D.

22   If B makes a profit of Rs 3, when A makes Rs. 2, C makes a profit of Rs 10 when B makes Rs 9 and D makes a profit of Rs 5 when C makes Rs 4, compare the profits of $A, B, C,$ and D.

23.   Two numbers are in the ratio of 4.5 and the difference of their squares is 225, find them

24   Two numbers are in the ratio of 3.5 and the difference of their cubes is 784, find the numbers

25   Three numbers are in the ratio of 3.6:5 and the sum of their squares is 3430, find the numbers

26   The sum of two numbers is 36 and their difference is 4, find their ratio.

27   7 of one number is equal to 075 of another number, find the ratio.

28   Two numbers are in the ratio of 3.5 but when they are increased by 10 their ratio is 5 7, find the numbers.

29.   A greyhound pursues a hare and takes 5 leaps for every 6 leaps of the hare, but 4 leaps of the hound are equal to 5 of the hare, compare the rates of hound and hare

30   Find the continued ratios between A, B, C, D if A and B are in the ratio 1.2, B and C are in the ratio 3 4 and C and D are in the ratio 5.6, also find the ratio between A and D.

II PROPORTION

§10   Compare the two ratios 12.16 and 18 24.

$$12 \ 16 = \tfrac{12}{16} = \tfrac{3}{4},$$

and $18 \ 24 = \tfrac{18}{24} = \tfrac{3}{4}$

We find that the two ratios are equal , this equality of two ratios is named **proportion**

Def   Four quantities are said to be in proportion when the first bears to the second the same ratio as the third bears to the fourth   Thus, 12, 16 and 18, 24 are said to be in proportion since the ratio 12.16 is equal to the ratio 18 24

The proportion is expressed thus —

$$12 \text{:} 16 \quad 18 \text{:} 24 \text{ or } \tfrac{12}{16} = \tfrac{18}{24}$$

The first and the fourth terms are called the **Extremes** and the second and the third are called the **means**, here 12,. 24 are the extremes and 16, 18 the means.

Note   Here 24 is also called the fourth proportional to 12, 16, 18

§11.   When four quantities are in proportion the product of the extremes is equal to the product of the means.

Let 12, 15, 16, 20 be four quantities in proportion.

$$\tfrac{12}{15} = \tfrac{16}{20}$$

$$12 \times 20 = 15 \times 16$$

Hence any three terms in proportion being given the fourth may be determined by the formula proved above.

Conversely.   If the product of two quantities be equal to the product of the two others, the four are proportionals, making the factors of one product, the means and the factors of the other the extremes.

§12.   Three quantities are said to be in continued proportion when the first bears to the second the same ratio as the second bears to the third   Thus 2, 4, 8 are in continued proportion since $2 \ 4 \quad 4 \ 8$

Note   In this case 8 is called the third proportional to 2 and 4, and 4 is called the mean proportional between 2 and 8

Example 1. Find a fourth proportional to 6, 10 and 9.

Sol Let x be the fourth proportional, then by definition, 6 10   9 x

product of the extremes=product of the means

$$6x=90$$

i.e., $x=15$ **Ans**

Example 2. Find a third proportional to 8 and 12.

Sol Third proportional to 8, 12 means fourth proportional to 8, 12, 12   Let it be equal to x.

We have therefore, 8 12   12 x

$$8x=144, i e , x=18 \quad \textbf{Ans.}$$

Example 3. Find the mean proportional between 4 and 9.

Sol. Let x be the required mean proportional.

We have therefore, 4 x x 9

or $x^2=36$

· $x=6$ **Ans.**

Note It is clear that the mean proportional between two numbers is equal to the square root of their product

EXERCISE 91.

Find the fourth proportional of ,—

1. 5, 6, 10.   **2.** 8, 12, 6.   **3** 6, 15, 12.

Find the third proportional of —

4. 8, 12.   **5** 9, 12   **6.** 21, 42

Find the mean proportional between —

7. 12, 147.   **8.** 36, 81.   **9.** 64, 225.

Find the value of x in the following proportions —

10. 6'7 '' 3'x.   **11.** 12 18 ∶ x:24.

12 x.16 ∶ 9'12.

Are the following proportions true ?

13. 5 7 '   8 9   **14.** 16'8 · 10:5.

15   One man adds 3 seers of water to 21 seers of milk and another adds 4 seers of water to 27 seers of milk , compare the amount of milk in the two mixtures

16.   Find the number which has to 40 the ratio of 3·75 to 3 and complete the proportion of which the first. second and fourth terms are $\frac{1}{10}$, 35 and $3\frac{3}{4}$.

17   If 3 men and 11 boys working together, can do 5 times as much work per hour as a man and a boy together, compare the work of a boy with that of a man.

18.   270 sheep and 14 horses eat 101 acres of grass in 30 days , 155 sheep and 21 horses eat 185 acres of grass in 75 days. Compare the amounts eaten by a sheep and a horse in the same time

19   Find x in 12 yds   4 yds.$=£9$ $£x$.

20   Find x in 1·5   7 5$=x$ lbs   3 lbs.

---

§13. Simple Proportion or Rule of Three.

When any three terms of a proportion are given, we can find the fourth (Art 10) This method of finding the fourth term is called the method of **simple proportion** or the **Rule of Three** The modern tendency is to discourage the method of "Rule of Three" and to encourge the "unitary method" But problems, which we have solved by the unitary method can very *easily* be solved by this method A great mathematician thus writes —

"The unitary method was originally introduced as a protest against the too mechanical method of the old 'Rule of Three' It was contended that while by the old method a pupil was merely taught to write down his three terms in certain places and then multiply or divide them by a fixed rule learnt by heart, by a new method he would be compelled to think out the step for himself, but as a matter of fact, the unitary method has tended to become even more mechanical than the old 'rule of three' and there is at the present time a reversion to the method of proportion " (J. T. Best)

§14   Consider the following examples —

Example 1.   The price of 8 chairs is Rs 40, find the price of 15 chairs

Note   If the number of articles is increased the price will also be increased in the same ratio, therefore the ratio of the two articles is equal to the ratio of the two corresponding prices

chairs   chairs   Rs

Hence   8 .   15   ∵ 40   Rs x,

where x stands for the required answer.

. $8x =$ Rs 15×40 $i\, e$, Rs. 600

or $x =$ Rs 75   **Ans.**

This is an example of what is called the **Direct Rule of Three** In this method the greater quantity requires the greater answer and the less quantity requires the less answer

Example 2   20 men can do a piece of work in 10 days , in how many days could 25 men do the same work ?

Note   Here we observe that an increase in the number of workmen corresponds to a decrease in number of days and *vice versa* Therefore the inverse ratio of the numbers of men is equal to the ratio of the corresponding numbers of days

Hence 25 men   20 men .∵ 10 days . x, where x stands for the required number of days.

. . $25x = 20 \times 10$ $i\, e$, 200 days.

. $x = 8$ days   Ans

This is an example of what is called the **Inverse Rule of Three**   In this method a greater number requires a less answer and a less number requires a greater answer.

§15.   In every question of simple proportion or **Rule of Three**, two of the terms are of the same kind and the third term is of the kind as the required answer   And the second term is evidently greater or less than the first term according as the fourth term, $i\, e$, the required answer, is greater or less than the third.

Hence to arrange the terms in a question of *Rule of Three* we give the following

Rule. *Denote the answer by the letter x and put it down as the 4th term   Of the three given quantities, put down in the 3rd place that which is of the same kind as the answer   Next from the nature of the question determine whether the answer will be greater or less than the third term ; if the answer is greater, then put down in the 2nd place the greater of the two remaining quantities, but if the answer is less, then put down in the 2nd place the smaller of the two remaining quantities   Having thus arranged the terms, x, i.e , the required answer is obtained by dividing the product of the 2nd and 3rd terms by the first term*

Note   The first two terms in the proportion must be expressed by numbers which measure them by the *same unit*

Example 3   If the third class railway fare for 110 miles is Rs 1. 11*a* 6*p*, what is the fare for 350 miles ?

Note   Here x is the required fare and hence Re 1 11*π* 6*p* is put in the 3rd place   Now . the fare for 350 miles is greater than that for 110 miles, . we put in the second place 350 which is the greater of the two remaining quantities, 110 and 350, and hence in the 1st place we put the remaining quantity, 110   This is evidently a Direct Rule of Three and should be written thus —

Sol.   mi mi   Rs. *a. p*
              110, 350 : 1 11 6 . *x*
  i e,   11: 35 : 1 11 6 : *x*

$$x = \frac{\text{Re. } 1.\ 11a\ 6p\ \times 35}{11} = \frac{\text{Rs } 60\ 2a.\ 6p}{11}$$

$$= \text{Rs. } 5.\ 7a\ 6p \quad \textbf{Ans}$$

Example 4   400 men have food enough to last 23 weeks , if they are joined by 60 men, how long will the provisions last ?

Note   The number of men is increased from 400 to 460 Putting *a* in the 4th place, evidently 23 must be put in the 3rd place The required answer will be clearly less than 23, since when the number increases, the food must last for a shorter period and therefore we put 400 in the 2nd place and 460 in the first place, This is, therefore, the Inverse Rule of Three and should be written thus —

Sol.   Men men   wks.
              460 ' 400   ::   23 ' *x* ,

$$\therefore \quad x = \frac{400 \times 23}{460} = 20 \text{ weeks. } \quad \textbf{Ans}$$

Note   The above examples show how very easily we can do the questions which were done by the Unitary Method, if we do them by the Rule of Three

EXERCISE (Repetition)

Students should do the questions in Exercises 86 and 87 by the Rule of Three

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§16. Compound Proportion or Double Rule of Three.

Complex problems which require two or more applications of the Rule of Three can be solved by a shorter method known as the Double Rule of Three

Example 1. If 10 masons can build a wall 50 ft long in 25 days of 8 hours each, in how many days of 6 hours each will 15 masons build a wall 36 ft long ?

Now consider the following points —

1.   First wall   10 masons are working.   Second wall : 15 masons are working

.   less days are required.   (Inverse proportion).

2   First wall is 50 ft long   Second wall is 36 ft long
.   less days are required   (Direct proportion).

3.   First wall, work 8 hrs. a day   Second wall   work 6 hrs   a day

.   more days are required   (Inverse proportion).

The solution will be arranged as follows .—

Masons 15˙10⎱
Length 50˙36⎰ ˙˙ 25 days   No of days required.
Hours   6   8⎰

$$\therefore \text{ days required} = \frac{10 \times 36 \times 8 \times 25}{15 \times 50 \times 6} = 16. \text{ Ans.}$$

Such a statement of the question in which two or more applications of the Rule of Three are involved is called **Compound Proportion or Double Rule of Three.**

Example 2   If 15 men can dig a trench 15 yards long, 9 ft broad and 5 feet deep in 12 days of 8 hours each, in how many days will 24 men dig a trench 25 yards long, 6 ft broad and 9 ft. deep working 6 hours a day ?

Note that :—

1   First trench, 15 men are working , second trench 24 men are working

.. less days are required.   (Inverse proportion).

2. First trench, 15 yds long , second trench, 25 yds. long
        more days are required   (Direct proportion)

3. First trench, 9 ft wide , second trench, 6 ft. wide
        less days are required   (Direct proportion)

4. First trench, 5 ft deep , second trench, 9 ft. deep
        .. more days are required.   (Direct proportion).

5   First trench, work 8 hours a day , second trench, work 6 hours a day

        more days are required (Inverse proportion)

Hence the following

Sol.   Men   24 : 15 ⎫
        Length 15 . 25 ⎪
        Breadth 9   6 ⎬ ∴ 12 days : No. of days reqd.
        Depth   5. 9 ⎪
        Hours   6. 8 ⎭

∴ No. of days $= \dfrac{15 \times 25 \times 6 \times 9 \times 8 \times 12}{24 \times 15 \times 9 \times 5 \times 6} = 20$ Ans.

The examples solved above give us the following

Rule. *Denote the required answer by x and put it in the 4th place. The quantity which is of the same kind as the required answer should then be put in the 3rd place. Then take any pair of the quantities of the same kind and arrange these terms, comparing with the 3rd term, as in a separate question on the Rule of Three Similarly all pairs of quantities of the same kind are to be arranged. Then the required answer is obtained by dividing the product of all terms in the 2nd and 3rd places by the product of all terms in the first places.*

§17.   There is another method of arranging the several terms in a question of Rule of Three or Double Rule of Three and students will find that in most cases, this method is not only simpler and more convenient but it saves much time also, though for some reason or other,

modern writers generally neglect to explain this method. We, however, take the liberty of clearly explaining the method with illustrations.

In every question, we should find out the causes and the effects produced by these causes Assuming as an axiom that effects have the same relation or ratio to each other as the causes which produce them, it is evident that in any two causes of the same kind we shall have the following proportion :—

First cause second cause First effect:second effect and then as explained in Art 10, we can find any one term if the three others are given

§18 We may denote the required term or quantity by x, which must be considered as and treated in the same way as any other number This x may occupy any place in the proportion either by itself or as one of the factors with other given numbers as shown in the following examples —

Example 1 If 15 men can reap a field of 12 acres in a certain time, how many acres will 10 men reap in the same time ?

Sol   Here $\left. \begin{array}{l} 15 \text{ men} \\ 10 \text{ men} \end{array} \right\}$ are the first and second causes.

and $\left. \begin{array}{l} 12 \text{ acres} \\ x \text{ acres} \end{array} \right\}$,, ,, ,, ,, effects

and ∴ 15 men   10 men .. 12 acres x acres

∴ $15 \times x = 10 \times 12$,

whence $x = \dfrac{10 \times 12}{15} = 8$ acres **Ans**

Example 2 If the three-penny loaf weighs 4 oz , when wheat is 10s a bushel, what should be the price of wheat when the two-penny loaf weighs 6 oz. ?

Sol   In the former case, the price of 1 oz is $\frac{3}{4}d.$,

In the latter ,, ,, ,, ,, ,, is $\frac{1}{3}d.$

we have the following proportion —

$\frac{3}{4}d$ $\frac{1}{3}d$ $10s$ $x s$,

$\frac{3}{4} \times x = \frac{1}{3} \times 10$,

whence $x = \frac{1}{3} \times 10 \div \frac{3}{4} = \frac{40}{9} = 4\frac{4}{9}s$ **Ans.**

In these two examples, the causes are simple terms being dependent upon only one magnitude.

Example 3   If 8 men can do a piece of work in 15 days, how many days will 20 men take to do a piece of work twice as much as the former ?

Sol   Here the causes are to each other as 8×15 is to $20 \times x$ and the effects may be taken as 1 and 2,

$$\cdot \quad 8 \times 15 \quad 20 \times x \quad 1 \quad 2,$$

whence $20 \times x = 8 \times 15 \times 2,$

$$x = \frac{8 \times 15 \times 2}{20} = 12 \text{ days   Ans}$$

Example 4   If 10 cannon which fire 3 rounds in 5 minutes, kill 270 men in $1\frac{1}{2}$ hours, how many cannon which fire 5 rounds in 6 minutes, will kill 500 men in 1 hour, at the same rate ?

Sol   In the former case $\frac{3}{5}$ round is fired in 1 min and in the latter case $\frac{5}{6}$ in 1 min and hence in this case,

the causes are $10 \times \frac{3}{5} \times \frac{3}{2}$ and $x \times \frac{5}{6} \times 1$

and the effects are 270 and 500 men, therefore,

$$10 \times \frac{3}{5} \times \frac{3}{2} \quad x \times \frac{5}{6} \times 1 \quad 270 \quad 500,$$

whence $x = \dfrac{10 \times 3 \times 3 \times 500 \times 6}{5 \times 2 \times 5 \times 270}$

$$= 20 \text{ cannon.   Ans.}$$

In the exercise given below, students will be well advised if they solve many of them if not all, by the method explained above

EXERCISE (Repetition)

Students should do the questions in Exercise 88 by the method of the Double Rule of Three

CHAPTER XVIII.

MISCELLANEOUS PROPOSITIONS

1   TIME AND WORK.

§1.   Suppose a field of grass is divided into 5 equal parts   If a man can reap it in 5 days, he will reap $\frac{1}{5}$ of it in one day   Conversely, if he can reap $\frac{1}{5}$ of it in one day he can reap the whole in 5 days   In solving the problems of time and work, these two fundamental laws always regulate our steps.
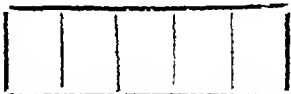
EXERCISE (Oral)

1.   If A can do a piece of work in 10 days—

(a) what part of it will he do in 1 day ? and

(b) what part of it in 7 days ?

2   If B can finish $\frac{1}{8}$ part of a work in one day, how long will he take to finish it ?

3   If A can do a piece of work in 6 days and B can do in 5 days—

(a) what part of it will A do in one day ?

(b) what part of it will B do in one day ?

(c) what part of it will A and B together do in one day ?

(d) how long will they both take to do it ?

4 A can build a wall in 10 days and B in 15 days—

(a) what part of it will A build in one day ?

(b) what part of it will B build in one day ?

(c) what part of it will both build in one day ?

(d) how long will they both take to build it ?

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§2 Some solved examples

Example 1   A can do a piece of work in 8 days and B can do it in 12 days , how long will both working together take to do it ?

Sol   ∵ A can do the work in 8 days,

A can do $\frac{1}{8}$ of it in 1 day ,

∵ B can do the work in 12 days,

B can do $\frac{1}{12}$ of it in 1 day ,

A and B together can do $(\frac{1}{8} + \frac{1}{12})$ of it in one day

or   ,,   ,,   ,, $\frac{5}{24}$,,   in one day

∴   A and B together can do the whole work in

$\frac{24}{5}$ days, i.e , $4\frac{4}{5}$ days. **Ans.**

Example 2   A can finish $\frac{3}{8}$ of a work in 15 days and B can finish the remaining work in 10 days   In what time can A and B together do the same work ?

Sol   (i)   A can do $\frac{3}{8}$ of the work in 15 days,

A can do $\frac{3}{8} \times \frac{1}{15} = \frac{1}{40}$ of the work in one day.

Remaining work $= 1 - \frac{3}{8} = \frac{5}{8}$.

(ii)   B can do $\frac{5}{8}$ of the work in 10 days

∴   B can do $\frac{5}{8} \times \frac{1}{10} = \frac{1}{16}$ of the work in one day

∴   A and B together can do $\frac{1}{40} + \frac{1}{16}$

i e , $\frac{7}{80}$ of the work in one day.

they together can do the work in $\frac{80}{7}$ or $11\frac{3}{7}$ days. **Ans**

Example 3. A and B together can finish a piece of work in $9\frac{3}{5}$ days which A alone can finish in 24 days , how long will it take B alone to do it ?

Sol.   ∵ A and B can do the work in $9\frac{3}{5}$ or $\frac{48}{5}$ days,

they can do $\frac{5}{48}$ of the work in one day.

But A alone can do $\frac{1}{24}$ of it in one day,

∴   B alone can do $\frac{5}{48} - \frac{1}{24}$ or $\frac{1}{16}$ of it in one day,

∴ B can do the whole work in 16 days. **Ans.**

Example 4. A and B can do a certain piece of work in 18 days, B and C can do it in 12 days and C and A can do it in 24 days. How long would each take separately to do it ?

Sol A and B can do $\frac{1}{18}$ of the work in one day,

   B and C ,, $\frac{1}{12}$,,   in one day,

   C and A ,, $\frac{1}{24}$,,   in one day,

A, B, C can do $\frac{1}{18} + \frac{1}{12} + \frac{1}{24}$ i e, $\frac{13}{72}$ of the work in 2 days,

   A, B, C can do $\frac{13}{72} \times \frac{1}{2}$ or $\frac{13}{144}$ of the work in 1 day,

   But A and B can do $\frac{1}{18}$ of the work in one day,

   . C can do $\frac{13}{144} - \frac{1}{18}$ or $\frac{5}{144}$ of the work in one day

   C can do the whole work in $28\frac{4}{5}$ days �

   Similarly A can do the whole work in 144 days ⎱ **Ans.**

   and B can do the whole work in $20\frac{4}{7}$ days ⎰

Example 5.   Mohan can do a piece of work in 20 days and Sohan can do it in 25 days, after working at it together for 5 days Mohan leaves off, how long will Sohan take to do the remaining work ?

   Sol   Mohan can do $\frac{1}{20}$ of the work in one day,

   Sohan can do $\frac{1}{25}$ of the work in one day,

      both can do $\frac{1}{20} + \frac{1}{25}$ or $\frac{9}{100}$ of the work in 1 day,

   , $\frac{9}{100} \times 5$ or $\frac{9}{20}$ of the work in 5 days

   Now at this stage Mohan leaves off.   Therefore the remaining work $1 - \frac{9}{20}$, i e, $\frac{11}{20}$ will be done by Sohan.

   . Sohan will do $\frac{11}{20}$ of the work in $25 \times \frac{11}{20}$ days,

                      i e, $\frac{55}{4}$ or in $13\frac{3}{4}$ days   **Ans**

Example 6 A can do a piece of work in $33\frac{1}{3}$ days and B in 20 days, 4 days before the work was actually completed A is called off   In how many days will the work be completed ?

   Sol   Since A is called off 4 days before the work is actually completed, therefore B worked alone for 4 days

      B's work for 4 days $= \frac{4}{20}$ or $\frac{1}{5}$

   . remaining work $\frac{4}{5}$ was done by A and B jointly,

   A and B jointly can do $\left(\frac{3}{100} + \frac{1}{20}\right)$ or $\frac{2}{25}$ of the

                      work in one day,

F. 20.

A and B jointly can do the whole work in $\frac{25}{2}$ days.

,,   ,,   ,, $\frac{4}{5}$ of the work in
                                                      $\frac{25}{2} \times \frac{4}{5}$ or 10 days

the work was completed in $10 + 4 = 14$ days   **Ans**

Example 7 A, B and C can do a piece of work in 12, 15, 20 days respectively, they work at it together but B and C are called off 2 and 1 day respectively before the completion of the work   In what time was the work finished ?

Sol   • A, B, C can do $\frac{1}{12} + \frac{1}{15} + \frac{1}{20} = \frac{1}{5}$ of the work
                                                                                              in one day

A, B, C can finish it in 5 days

Since B is called off 2 days before completion of the work

and C is ,,   ,, 1 day   ,,   ,,   ,,   ,,   ,,   ,,

           A worked for 2 days and C for 1 day after B was called off.

A's 2 days' work and C's 1 day's work $= \frac{1}{6} + \frac{1}{20} = \frac{13}{60}$
$i.e.$, the remaining work $1 - \frac{13}{60}$ or $\frac{47}{60}$ was completed by all the three men

Since they can complete the work in 5 days

           $\frac{47}{60}$ of the work was done in $\frac{47}{60} \times 5 = \frac{47}{12}$ days
      •   the work was completed in $2 + \frac{47}{12} = 5\frac{11}{12}$ days. **Ans.**

Example 8. A and B together can do a piece of work in 15 days, A worked for 8 days and the remaining work was completed by B in 29 days   In what time could each alone do the work ?

Sol.   Since A worked for 8 days and the remaining work was completed by B in 29 days, it is evident that if they both together work for 8 days, then the remaining work can be finished by B in 21 days

           A and B together can do $\frac{8}{15}$ of the work in 8 days,
                    the remaining work $\frac{7}{15}$ can be done by B in 21 days
      ' B can do $\frac{7}{15} \times \frac{1}{21} = \frac{1}{45}$ of the work in one day,
                    B can do the whole work in 45 days
           A can do $\frac{1}{15} - \frac{1}{45} = \frac{2}{45}$ of the work in one day,
      •• A can do the whole work in $22\frac{1}{2}$ days

                                        Hence $22\frac{1}{2}$ and 45 days   **Ans**

Example 9 *A* and *B* can do a piece of work in 12 days, *B* and *C* together in 18 days After *A* has been working at it for 4 days and *B* for 6 days, *C* finishes it in 17 days In how many days could each do the work by himself ?

Sol Since *A* alone worked for 4 days, *B* for 6 days and *C* for 17 days, therefore, let *A* and *B* together work for 4 days, *B* and *C* together for 2 days and *C* alone for 15 days Both the statements mean the same thing.

The work of *A* and *B* for 4 days $=\frac{1}{3}$
The work of *B* and *C* for 2 days $=\frac{1}{9}$
   the work of *C* alone for 15 days $= 1-(\frac{1}{3}+\frac{1}{9})=\frac{5}{9}$
   the work of *C* for 1 day $=\frac{5}{9}\times\frac{1}{15}=\frac{1}{27}$
   C can do the work in 27 days

Since *B* and *C* can do $\frac{1}{18}$ of the work in 1 day,

   B can do $\frac{1}{18}-\frac{1}{27}$ *i e* , $\frac{1}{54}$ of the work in 1 day,
. *B* can do the work in 54 days
Similarly *A* can do the work in $15\frac{3}{7}$ days
Hence $15\frac{3}{7}$, 54, 27 days   **Ans**

EXERCISE 92.

1   *A* can do a piece of work in 10 days and *B* can do it in 15 days , how long would they take to do it ?

2. *A* can do a piece of work in 8 days , *B* can do it in 10 days and *C* can do it in 20 days   In what time will they do it, all working together ?

3. *A* and *B* together can finish a piece of work in 20 days which *A* alone can finish in 30 days , what time will it take *B* alone to finish it ?

4. *A* can do a piece of work in 15 days and *B* can do $\frac{3}{5}$ of the same work in 15 days , how long will they take to do it if they work together ?

5. *A* can do $\frac{3}{5}$ of the work in 9 days and *B* can do $\frac{5}{6}$ of the work in 15 days , how long will they both working together take to do it ?

6   *A* and *B* can do a piece of work in 15 days, *B* and *C* in 20 days , *C* and *A* in 30 days , in what time can they do it, all working together ?

7. A and B can do a piece of work in $4\frac{4}{9}$ days, B and C in $5\frac{5}{11}$ days, C and A in $4\frac{4}{5}$ days, in what time can each do it working separately ?

8   A and B together can do a piece of work in 21 days, they both worked at it for 4 days and the remaining work was completed by C alone in 34 days.   In what time would C alone do the whole work ?

9. A and B can do a piece of work in 18 and 24 days respectively, they worked at it together for 8 days and then B fell ill, in what time will A alone finish the remaining work ?

10. A can do a piece of work in 20 days, he works at it for 5 days and then B finishes it in 9 days.   In what time can A and B together do it ?

11. A can do a piece of work in 20 days and B in 30 days, A worked at it for 5 days and then B finished it with the assistance of C in 15 days.   In what time can C alone finish the whole work ?

12   Paul finishes $\frac{5}{10}$ of a work in 6 days and then finishes the remaining with the assistance of Peter in 6 days In what time can Peter alone finish the same work ?

13. A and B can do a piece of work in 25 and 30 days respectively, A and B work at it together for 10 days and then C joins them and the remaining work is finished in 2 days   How long will it take C alone to do the whole work ?

14   A can do a piece of work in 24 days and B in 30 days, A worked for 6 days and then B also joined him. In how many days will the whole work be completed ?

15. A, B and C can finish a piece of work in 10, 12 and 15 days respectively   If B stops after 2 days, how long will it take A and C to finish the remaining work ?

16   A can do $\frac{4}{5}$ of the work in 15 days and B $\frac{3}{8}$ of the work in 12 days, A and B work at it together for 8 days and then C finishes it in 14 days   In what time would C alone do the whole work ?

17   A can finish a piece of work in 6 days and B in 9 days they work at it together, but A is called off 4 days

before the work is actually completed, find in what time the work was finished.

18. *A*, *B* and *C* can complete a piece of work in 8, 12 and 16 days respectively, they work at it together but *B* and *C* are called off 2 days and 3 days respectively before the work is actually completed. In what time was the work finished ?

19 *A*, *B* and *C* can do a piece of work in 12, 18 and 24 days respectively, they work at it together, *A* stops the work after 4 days and *B* is called off 2 days before the work is done In what time was the work finished ?

20. *A*, *B* and *C* can finish a piece of work in 20, 30 and 40 days respectively They work at it together but *B* and *C* are called off 4 and 5 days before the completion of the work. In what time was the work finished ?

21 *A* and *B* together can mow a field in 24 days *A* works at it for 6 days and then *B* finishes it in 33 days In what time could each mow it ?

22 *A* and *B* can finish a piece of work in 28 days, *A* works at it for $10\frac{1}{2}$ days and then *B* finishes it in $35\frac{1}{2}$ days In what time could each do it ?

23 *A* and *B* can finish a piece of work in 18 days, and *B* and *C* in 24 days *A* works at it for 6 days and then *B* for 8 days and then *C* finishes it in 23 days In what time could each do it ?

24. *A* and *B* can finish a piece of work in $11\frac{1}{5}$ days, *B* and *C* in $13\frac{7}{11}$ days, *A* works at it for 4 days and *B* for 5 days and then *C* finishes it in 18 days In what time could each do it ?

25. *A* and *B* together can do a piece of work in 12 days, *A* works as much in 10 days as *B* in 15 days in what time could each do it ?

26. *A* and *B* together can do a piece of work in 25 days, *A* does as much work in 15 days as *B* in 20 days, in what time can each do it ?

27 *A*, *B* and *C* together can do a piece of work in 10 days, *B* and *C* together work thrice as much as *A* and

A and B together work 4 times as much as C. In what time can each alone do it ? (*Hint* B + C = 3A)

28. A, B and C together can finish a piece of work in 12 days , A and C together work twice as much as B , A and B together work thrice as much as C In what time could each do it seperately ?

29 A works as much in 2 days as B in 3 days, and B works as much in 4 days as C in 5 days , in what time will A, B and C working together finish the work if A alone can finish it in 11 days ?

30. Prakash works as much in 3 days as Raj in 4 days and Raj works as much in 6 days as Paul in 5 days , in what time can Paul and Raj do it, working together if Prakash can finish it in 18 days ?

31. 20 men can complete a piece of work in 10 days, but after every 4 days 5 men are called off , in what time will the work be finished ?

32. 3 men and 4 boys can finish a piece of work in 7 days and 2 men and 3 boys in 10 days , compare the rates of working of a man and a boy

33 A can do a piece of work in 25 days, B in 20 days and C in 24 days. The three work together for 2 days and then A and B leave , but C continues, and after S_8^2 days is rejoined by A, who brings D along with him and these three finish the remainder of the work in 3 days In what time would D alone have done the whole work ?

34. A can do a piece of work in 3 days , B can do three times the work in 8 days and C 5 times the work in 12 days In what time will they do it together, supposing them to work at the rate of 9 hours a day ?

35 A and B can do a piece of work in 10 days, B and C in 15 days, and C and A in 20 days They all work at it for 6 days , then A leaves and B and C go on together for 4 days more If B then leaves, how long will C take to complete the remaining work ?

36. Four men working together all day, can finish a piece of work in 11 days , but two of them having other

engagements can work only one half time and quarter time respectively.   How long will it take them to complete the work ?

37   Six men and five boys can do a piece of work in 7 days , they work at it till they have completed $\frac{3}{4}$ of it , then two of the men leave and two more boys come in. How long will the work be in hand, if a boy does half as much work as a man ?

38   Some men and boys can do a piece of work in 20 days , if twice the number of boys more be added the work is finished in 12 days , in what time could the orginal number of boys do the work ?

39   12 men and 15 women can do a piece of work in 20 days , if 10 more women be added the work is finished 5 days before the stipulated time   In what time could a man and a woman separately do the work ?

40   If 8 men and 12 boys can finish a piece of work in 12 days, in what time will 40 men and 45 boys finish another piece of work 3 times as great, supposing that 16 men can do as much work in 8 hrs   as 12 boys in 24 hrs ?

41   Five men undertook to complete a piece of work in 48 days working 8 hours a day   One stopped working at the end of 12 days and a second at the end of 15 days The others then agreed to work 9 hours a day.   By what percentage (to the nearest unit) must they increase their rate of working to finish within the specified time ?

II   WORK AND WAGES.

§3   Consider the following examples —

Example 1 A can do a piece of work in 15 days and B in 20 days.   They finished the work with the assistance of C in 5 days and got Rs 45 as their wages ; find the share of each

Sol A did in 5 days $\frac{1}{3}$ of the work

  B did   „ „ $\frac{1}{4}$ „   „

  C „   „ „ $1-(\frac{1}{3}+\frac{1}{4})=\frac{5}{12}$ of the work

Since A, B, C, did in 5 days $\frac{1}{3}$, $\frac{1}{4}$, $\frac{5}{12}$ of the work respectively,

∴ A's share = Rs $45 \times \frac{1}{3}$ = Rs 15 ⎫
B's share = Rs $45 \times \frac{1}{4}$ = Rs $11\frac{1}{4}$ ⎬ **Ans.**
C's share = Rs. $45 \times \frac{5}{12}$ = Rs. $18\frac{3}{4}$ ⎭

Example 2. If 4 men with 5 boys earn Rs 39 in 6 days, and 5 men and 4 boys earn Rs. 49 in 7 days, in what time will 6 men and 3 boys earn Rs 60 ?

Sol Since 4 men and 5 boys earn in 6 days, Rs. 39,

(i) 4 men and 5 boys ,, ,, 1 day, Rs. $6\frac{1}{2}$.

also 5 men and 4 boys ,, ,, 7 days, Rs 49

(ii) 5 men and 4 boys ,, ,, 1 day, Rs. 7.

Multiplying (i) by 5 and (ii) by 4 we get,

20 men and 25 boys earn in 1 day, Rs. $32\frac{1}{2}$

20 men and 16 boys ,, ,, ,, Rs 28

. 9 boys earn in one day Rs. $4\frac{1}{2}$ [By subtraction]

˙ 1 boy earns in one day 8a.

Again, since 4 men and 5 boys earn in one day Rs. $6\frac{1}{2}$

4 men earn in one day Rs $6\frac{1}{2}$ − Rs. $2\frac{1}{2}$ = Rs. 4

· 1 man earns in one day Re. 1

· 6 men and 3 boys earn in one day Rs. 6 + Rs. $1\frac{1}{2}$
i.e, Rs. $7\frac{1}{2}$

∴ No of days required = $\dfrac{\text{Rs } 60}{\text{Rs. } 7\frac{1}{2}}$ = 8 **Ans.**

EXERCISE 93

1. A can do a piece of work in 10 days and B in 12 days , they finished the work with the assistance of C in 4 days and got Rs. 22. 8a. as their wages , find the share of each.

2. A and B undertook to do a piece of work for Rs 37. 8a. A alone could do it in 20 days and B in 30 days With the assistance of C they finished it in 8 days. How should the money be divided ?

3 A field can be reaped by 10 women in 4 days, or by 6 boys in 10 days, or by 2 men in 12 days. One man, three boys and three women are employed. What is the total expense, if the wages of a man, a woman, and a boy are $8a.$, $5a$, and $3a$, respectively ?

4. Three men can do as much work as five boys, the wages of three boys are equal to those of two men. A work on which 40 boys and 15 men are employed takes 8 weeks and costs £350 ; how long would it take if 20 boys and 20 men were employed and how much would it cost ?

5 If the work done by a man, a woman and a child be in the ratio of 3, 2 and 1 and there be in a factory 24 men, 20 women and 16 children, whose weekly wages amount to Rs. 204 , what will be the yearly wages of 27 men, 40 women and 15 children ?

6. If 5 men with 7 boys can earn £5 2s. in 6 days and 2 men with 3 boys can earn £1 8s in 4 days , in what time will 7 men with 6 boys earn £30 ?

7 If 3 men with 4 boys earn Rs. 58 in 8 days, and 2 men with 3 boys earn Rs 40 in the same time , in what time will 6 men and 7 boys earn Rs 210 ?

III. PIPES AND CISTERNS.

§4. Consider the following examples —

Example 1 A vessel can be filled by one pipe A in 10 minutes, by a second B in 15 minutes ; it can be emptied by a waste pipe C in 9 minutes , in what time will the vessel be filled if all the three were turned on at once ?

Sol A and B fill $\left(\frac{1}{10}+\frac{1}{15}\right)$ or $\frac{1}{6}$ of the vessel in 1 min. and C can empty $\frac{1}{9}$ of the vessel in 1 min

when all the pipes are turned on $\frac{1}{6}-\frac{1}{9}$ or $\frac{1}{18}$ of the vessel will be filled by A, B, C in 1 minute

∴ A and B will fill the vessel in 18 minutes **Ans**

Example 2 Three pipes A, B and C can fill a cistern in 15, 20 and 30 minutes respectively. They were all turned on the same time , after 5 minutes the first two

pipes were turned off. In what time will the cistern be filled ?

Sol   A, B and C can fill $\left(\frac{1}{15}+\frac{1}{20}+\frac{1}{30}\right)$ or $\frac{3}{20}$ of the cistern in 1 minute

∴ A, B and C filled $\frac{3}{20}\times5$ or $\frac{3}{4}$ of the cistern in 5 min

Now A and B are turned off ;

$1-\frac{3}{4}$ or $\frac{1}{4}$ of the cistern will be filled by C

C will fill $\frac{1}{4}$ in $30\times\frac{1}{4}$ or $7\frac{1}{2}$ minutes.

∴ the cistern will be filled in $7\frac{1}{2}+5$ or $12\frac{1}{2}$ min. **Ans**

Example 3. Two pipes A and B can fill a cistern in 6 and 8 minutes respectively If they are turned on alternately for one minute each, how long will it take the cistern to fill ?

Sol   A can fill $\frac{1}{6}$ of the cistern in one minute ,
        B can fill $\frac{1}{8}$ of the cistern in one minute.

Since A, B are turned on alternately for 1 min. each

A and B can fill $\frac{1}{6}+\frac{1}{8}$ or $\frac{7}{24}$ of the cistern in 2 min.

"   " $\frac{7}{24}\times3$ or $\frac{7}{8}$ of the cistern in 6 min

the remaining portion $=1-\frac{7}{8}$ or $\frac{1}{8}$

$\frac{1}{8}$ of the cistern will be filled by A in $6\times\frac{1}{8}$ or $\frac{3}{4}$ min.

∴ the cistern will be filled in $6+\frac{3}{4}=6\frac{3}{4}$ minutes **Ans**

Example 4 A cistern can be filled by two pipes A and B in 30 and 40 minutes respectively and a third pipe C can empty it in 20 minutes. If all the three are kept open successively for 1 minute each, how soon will the cistern be filled ?

Sol $\left(\frac{1}{30}+\frac{1}{40}-\frac{1}{20}\right)$ or $\frac{1}{120}$ of the cistern is filled in 3 min   When only $\left(\frac{1}{30}+\frac{1}{40}\right)$ or $\frac{7}{120}$ of the cistern is empty A and B can fill this portion in 2 minutes in their turn.

∴ the remaining portion $=1-\frac{7}{120}=\frac{113}{120}$.

∵ $\frac{1}{120}$ of the cistern is filled in 3 minutes

∴ $\frac{113}{120}$ of the cistern will be filled in $\dfrac{120\times3\times113}{120}$ min.

i. e., in 339 min ,

∴ the required time $=339+2=341$ minutes. **Ans.**

EXERCISE 94

1. Three taps A, B and C can fill a cistern in 12, 15 and 18 minutes respectively   They are all turned on, but after $4\frac{1}{2}$ minutes A and C are turned off   How many minutes longer will B take to fill the cistern ?

2   A cistern can be filled by two taps in 20 minutes and 30 minutes respectively and can be emptied by a third in 48 minutes   They are all turned on at once   When will the cistern be half full ?

3   A cistern can be filled by two taps A and B in 12 minutes and 14 minutes respectively and can be emptied by a third in 8 minutes   If all the taps are turned on at the same moment, what part of the cistern will remain unfilled at the end of 7 minutes ?

4. A reservoir can be filled by two taps in 6 hours and 9 hours respectively. The first tap was opened at 7 A M , and the second at 8 A.M , at what o'clock will the reservoir be full ?

5. Two taps can fill a cistern in 6 hours and 8 hours respectively and a waste pipe can empty it in 3 hours   If the cistern be full and all the three taps are turned on, what time will it take to get it empty ?

6   A cistern can be filled by two pipes in $5\frac{1}{4}$ hours and $4\frac{2}{3}$ hours respectively and emptied by an escape pipe in $2\frac{1}{3}$ hours   If all the three pipes are turned on and the cistern is half full, what time will elapse before the cistern is emptied ?

7   A cistern can be filled by 3 pipes in 30, 40 and 60 minutes respectively and emptied by an escape pipe in half an hour. The three pipes are turned on at noon but the escape pipe is at the same time accidentally left open and not closed for a quarter of an hour. At what time will the cistern be full ?

8. A cistern can be filled by two pipes in 8 and 12 hours repectively and emptied by a third in 24 hours   The first two were opened at 7 A M., and the waste pipe was

opened by a boy at 8 A.M. At what time will the cistern be full ?

9   A cistern can be filled by two pipes in 4 hours and 6 hours respectively and emptied by a waste pipe in 2 hours If the cistern be half full and the pipes were all turned on at the same moment, when will the cistern be emptied ?

10   Two pipes can fill a cistern in 3 and 4 hours respectively and a waste pipe can empty it in one hour   If the first be opened at one, the second at two and the third at three o'clock, at what time will the cistern be emptied ?

11.   Two pipes which can fill a cistern in 20 minutes and 30 minutes respectively were turned on at the same moment   After some time the first pipe was turned off and the cistern was then filled in 20 minutes more   When was the first pipe turned off ?

12   A cistern can be filled by two pipes in 30 and 40 minutes respectively   Both the pipes were turned on at the same moment, but after some time the first was turned off and the cistern was filled in 10 minutes more How long after the pipes have been turned on the first one was turned off ?

13   Two pipes can fill a cistern in 10 and 12 minutes respectively   If they are kept open successively for one minute each, how soon will the cistern be filled ?

14.   A can fill a cistern in 15 minutes and B in 20 minutes and C can empty it in 10 minutes   If they are kept open successively for one minute each, how soon will the cistern be filled ?

15.   A cistern has three pipes, A, B and C.   A and B can fill it in 12 and 15 minutes respectively and C can empty it in 10 minutes   If the three pipes are kept open alternately for one minute each, how soon will the cistern be filled ?

MISCELLANEOUS PROPOSITIONS (*Continued*)

1. TIME AND DISTANCE.

§1   Consider the following simple question —

A person walks at the rate of 5 miles an hour ; how far will he go in 3 hours ?

15 miles   **Ans.**   [5×3=15]

Here 5 miles per hour is the **speed** or **velocity** of the person , 3 hours is the **time** and 15 miles is the **distance**

Clearly therefore, **speed × time＝distance**

From this we can deduce the following conclusions —
Time＝distance−speed and speed＝distance−time

§2.   The following observations are almost self-evident but are very useful in solving questions of this class —

(*i*)   When two bodies are moving in a straight line and approaching each other from opposite directions, their *relative* speed, *i e* , the speed with which they approach each other is equal to the **sum** of their *absolute* speeds

(*ii*)   But if they are moving in the same direction, then their *relative* speed is equal to the **difference** of their *absolute* speeds

(*iii*)   If two bodies approaching each other from opposite directions meet, it means that they have together travelled the whole distance between them

(*iv*)   A moving train passes a fixed point when the train has gone over a distance equal to its own **length**

(*v*)   If two trains run on parallel lines either in opposite directions or in the same direction, one of them shall pass the other only when the former has gained over the latter a distance equal to the **sum** of the lengths of the trains.   [See §2, page 325]

(vi) When a vessel goes down a stream, the rower is *helped* by the current but if it goes up stream, his progress is *opposed* by the current   Hence in the former case the total work done by the vessel is the **sum** of works done by the rower and the current and in the latter case, it is their **difference**   [See Art 3, page 332]

Now we shall solve some examples to illustrate this method

Example 1. Two persons who live in different towns 40 miles apart started to meet each other at 10 A M   One walked at the rate of $3\frac{1}{2}$ miles an hour, and the other at $4\frac{1}{2}$ miles an hour.   When will they meet and how many miles will each walk ?

Sol _____ 40 miles _____
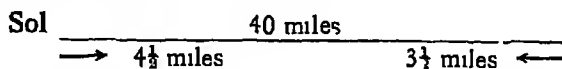
   $\longrightarrow$ $4\frac{1}{2}$ miles $3\frac{1}{2}$ miles $\longleftarrow$

Since they are walking in opposite directions their relative speed would be $3\frac{1}{2} + 4\frac{1}{2}$ $i.$ e, 8 miles per hour.

   time required to walk 40 miles $= \frac{40}{8} = 5$ hours

Hence they will meet at $10 + 5 = 15$ or 3 P M.,
One man had walked $3\frac{1}{2} \times 5$ $= 17\frac{1}{2}$ miles
and the other man had walked $4\frac{1}{2} \times 5 = 22\frac{1}{2}$ miles } **Ans**

Example 3. A policeman goes after a thief who is 176 yards before him   When and where will the policeman catch the thief when they run at the rates of $6\frac{1}{2}$ and 6 miles per hour respectively ?

Sol. Policeman   Thief
        | |
        _____
                  176 yards.
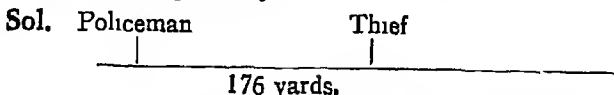
(a) Since they run in the same direction therefore, relative speed of the policeman would be $6\frac{1}{2} - 6$ i $e.,$ $\frac{1}{2}$ mile or 880 yds per hour.

        880 yds he walks in 60 minutes,

   $\therefore$ 1 yd.   ,,   ,,   ,, $\frac{60}{880}$ minutes,

   $\therefore$ 176 yds   ,,   ,,   ,, $\frac{60}{880} \times 176 = 12$ minutes

   .   the time reqd to overtake the thief $= 12$ min **Ans**

(2)   The distance from the starting point

$$= 6\tfrac{1}{2} \times \tfrac{18}{80} \text{ miles}$$

$$= \tfrac{13}{2} \times \tfrac{1}{5} = \tfrac{13}{10} \text{ miles} = 1 \text{ mile 528 yds } \mathbf{Ans.}$$

Example 3   If I walk at the rate of 4 miles an hour, I reach my destination 30 minutes too late , if I walk at the rate of 5 miles an hour I reach 30 minutes too soon.   How far is my destination ?

Sol.   Suppose I have to go one mile.

∴ it will take me 15 minutes to reach the destination at the rate of 4 miles an hour, and 12 minutes at the rate of 5 miles an hour, *i e*, I will save 3 minutes to walk one mile

But I save 30+30, *i. e.*, 60 minutes in all,
     distance required = 60—3 = 20 miles **Ans**

Example 4   A hare makes 4 leaps in the same time in which a dog makes 3, but 2 leaps of the dog cover as much distance as 3 leaps of the hare , how many leaps will the dog have to make before catching the hare, supposing the hare to have a start of 60 leaps ?

Sol   2 leaps of the dog = 3 leaps of the hare

     1 leap of the dog = $\tfrac{3}{2}$ leaps of the hare.

But the dog takes as much time in one leap as the hare takes in $\tfrac{4}{3}$ leaps.

It is clear therefore, that $\tfrac{3}{2} - \tfrac{4}{3}$ or $\tfrac{1}{6}$ leap of the hare can be made up by the dog in 1 leap

     60 leaps can be made up in 60÷$\tfrac{1}{6}$ or 360 leaps **Ans**

Example 5   I have to go to a certain place 7 miles off in 1 hour 15 minutes   If I walk some way on foot at 4 miles an hour and ride some way on horse back at 10 miles an hour I reach my destination in time   How far should I walk on foot ?

Sol   If I walk the whole way on foot I shall take 1 hour 45 min. to reach the destination, *i.e*, I shall be 30 min too late   Again if I go the whole way on horse back I shall take 42 min. only to reach the destination, *i e.*, I shall be 33 min. too soon

.   the ratio between distance travelled on foot and
horse back would be $=33\ 30$ Inverse ratio
$=11\ 10$

∴   distance I should walk on foot$=\frac{11}{21}\times 7$

$=3\frac{2}{4}$ miles. **Ans.**

Example 6. Two guns are fired from the same place
after an interval of 12 minutes, but a person approaching
the place on a motor cycle hears the report after an interval
of 11 min 15 sec Find his rate of speed, sound travelling
at the rate of 1100 ft per second

Sol The distance which the man travels in 11 min
15 sec is the same as that travelled by the report in
12 min.—11 min. 15 sec *i e.*, 45 sec

But in 45 sec the sound travels 1100×45 ft

∴ in $11\frac{1}{4}$ min the man travels$=1100\times 45$ ft

∴ in 1   ,,   ,   ,,   ,, $=1100\times 45\times \frac{4}{45}$ ft

in 1 hour ,,   ,,   ,, $=1100\times 45\times \frac{4}{45}\times 60$ ft.

$=\dfrac{1100\times 45\times 4\times 60}{45\times 3\times 1760}$ miles

$=50$ miles **Ans**

Example 7 The distance from P to Q is 24 miles,
6 miles of which is up hill, 6 miles level and 12 miles down
hill , find the difference between the time in which a person
would walk from P to Q and the time in which he will
come back again, supposing his pace up hill to be 3 miles,
down hill 6 miles and on level 4 miles per hour

Sol From P to Q the distance is 6 miles up-hill, 6
miles level and 12 miles down-hill,

whole time taken$=(\frac{6}{3}+\frac{6}{4}+\frac{12}{6})$ hrs $=5\frac{1}{2}$ hrs

Also from Q to P the distance is 12 miles up hill, 6
miles level and 6 miles down-hill,

∴ whole time taken$=(\frac{12}{3}+\frac{6}{4}+\frac{6}{6})$ hrs.$=6\frac{1}{2}$ hrs

Hence the diff $=6\frac{1}{2}-5\frac{1}{2}=1$ hr **Ans**

Example 8. A and B start at the same time from L
and M to go to M and L, a distance of 42 miles at the rates

of 4 miles and 3 miles respectively   They meet at N, then go to M and L and return immediately and meet again at D Find the distance DN.

Sol   D   N

L   M
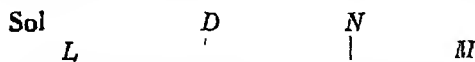
When A and B meet at N for the first time, they have together travelled 42 miles and when they meet at D for second time they have together travelled 42×3 $i.\ e$, 126 miles   The distance travelled by A when he meets at N for the first time $= LN$

$$LN = \frac{4}{4+3} \times 42 = \tfrac{4}{7} \times 42 = 24 \text{ miles}$$

Again, the distance travelled by B when he meets at D for the second time $= ML + LD$

$\therefore\ ML + LD = 126 \times \tfrac{3}{7} = 54$ miles

$\qquad LD = 54 - LM \qquad$ miles

$\qquad\quad = 54 - 42 = 12$ miles

$\qquad DN = 24 - 12 = 12$ miles   Ans.

EXERCISE 95

1. A man walks at the rate of $2\tfrac{3}{4}$ miles an hour , how long will he take to walk 605 yards ?

2 The velocity of a train is 35 miles an hour , find its velocity per second

3 Two persons living in different towns 32 miles apart started at 1 P M. to see each other at the rate of $3\tfrac{1}{3}$ and $4\tfrac{1}{3}$ miles per hour respectively   When will they meet and how many miles will each then walk ?

4 Two men started from a place P at the rate of $2\tfrac{1}{3}$ and $2\tfrac{1}{4}$ miles an hour   How many miles will they be apart after 5 hours (i) if they walk in opposite directions , (ii) if they walk in the same direction ?

5 Two gentlemen start from Lahore at the same time at the rate of $3\tfrac{1}{2}$ and $4\tfrac{1}{2}$ miles an hour   When will they be 21 miles apart, supposing (i) they walk in the same direction, (ii) in opposite directions ?

F. 21

6. Two persons A and B start from a place P A starts at 7 A. M and goes at the rate of $3\frac{3}{4}$ miles an hour and B at 8 A. M at the rate of 4 miles an hour   At what time will B overtake him and how far will he then be from the starting place ?

7. Two persons A and B start from Lahore and Amritsar respectively A goes at the rate of $3\frac{1}{2}$ miles and B at the rate of $3\frac{3}{4}$ miles an hour   If they meet each other after 4 hours, find the distance   between Lahore and Amritsar.

8. Two persons set out from Jullundhur and Ludhiana respectively   One goes at the rate of $3\frac{1}{2}$ miles and the other at the rate of $3\frac{5}{8}$ miles an hour   They meet each other after $5\frac{1}{11}$ hours.   Find the distance between the two cities.

9. A who travels $3\frac{1}{2}$ miles an hour starts $2\frac{1}{3}$ hours before B who goes the same road at $4\frac{1}{3}$ miles an hour , where will B overtake A ?

10. A policeman goes after a thief who has 100 yards' start , if the policeman runs one mile in six minutes and the thief in 10 minutes, how far will the thief have gone before he is overtaken ?

11. A thief who had escaped at 7 P. M was followed by a policeman at 9 P M. at the rate of 6 miles an hour. At what time will the policeman overtake him, supposing the thief runs at $4\frac{1}{2}$ miles an hour ?

12. A policeman runs after a thief who has $3\frac{1}{3}$ miles' start , if the policeman runs at the rate of $3\frac{3}{4}$ miles and the thief at the rate of $3\frac{1}{3}$ miles an hour, when will the policeman overtake him ?

13 A man rides at the rate of 11 miles an hour but stops 5 minutes to change horses at the end of every 7th mile , how long will he take to go a distance of 94 miles ?

14. A student walks to school at the rate of $2\frac{1}{2}$ miles an hour and reaches 6 minutes too late   Next day he increases his speed by 2 miles an hour and then reaches there 10 minutes too soon   Find the distance of the school from his home

15. I start to walk to a certain place at the rate of 4 miles an hour and reach there 15 minutes too late when I increase my speed by 1 mile per hour, I reach there 30 minutes too soon. Find the distance of the place

16 A man travelled 20 miles in 4 hours 10 minutes. He travelled some way on foot and some way on coach at 3 and 8 miles per hour respectively. How far did he travel on coach ?

17 A person walks from A to B at the rate of 3 miles an hour and after transacting some business which occupies him an hour, returns to A by the tramway at the rate of 5 miles an hour. He then finds that he has been absent for 2 hours 20 minutes Find the distance from A to B

18. A hare makes 5 leaps in the same time in which a dog makes 4, but 2 leaps of the dog cover as much distance as 3 leaps of the hare How many leaps will the dog have to make before catching the hare, supposing the hare to have a start of 100 leaps ?

19 A hare makes 4 leaps in the same time in which a dog makes 3, but 2 leaps of the dog cover as much distance as 3 of the hare How many leaps will the dog have to make before catching the hare, supposing the hare to have a start of 90 leaps ?

20 A hare sees a dog 110 yards away from her and starts off in the opposite direction at the rate of 10 miles per hour. A minute later, the dog sees the hare too and pursues him at the rate of 14 miles an hour. When will the dog overtake the hare ?

21 A person went from A to B, at the rate of 4 miles an hour and came back at the rate of 3 miles an hour. It took him 12 hours , find the distance from A to B.

22 A person goes from A to B at the rate of 5 miles an hour and comes back at the rate of 4 miles an hour. It takes him 9 hours , find the distance from A to B.

23. A, B and C can walk at the rates of 3, 4, 5 miles an hour , they start from Poona at 1, 2, 3 o'clock respectively , when B catches up A, B sends him back with a message to C , when will C get the message ?

24   A sets out from a place P to walk to a place Q
A quarter of an hour later a second person B sets out from
P to walk to Q but after walking half a mile returns to P,
where he is detained 10 minutes.   Again setting out from P
he reaches Q 5 minutes after A If A walks throughout at
the rate of 3 miles an hour and B at the rate of 4 miles an
hour, find the distance from P to Q

25.   Two persons A and B set out together on a
journey   They walked at the rate of 3 miles an hour , and
when they had proceeded for three quarters of a mile, B
returned, walking at the same rate to the place of starting
Here he was detained three quarters of an hour   Setting
out again he overtook A, who had been walking all the time,
at the end of $2\frac{1}{2}$ hours from the second time of starting.
At what rate did he walk ?

26   Sound travels at the rate of 1160 ft. per second
A shot is fired from a ship sailing at the rate of 10 miles
an hour   How far will the ship have moved when the re-
port is heard at the place $14\frac{1}{8}$ miles off ?

27   A snail creeps 1 ft 5 in. up a long pole during
12 hrs in the night but slips down 10 in during 12 hrs. in
the day   If the pole is 12 ft high, how long will it take to
reach the top ?

28   The distance P to Q is 30 miles, 8 miles of which
is up-hill, 12 miles level and 10 miles down hill , find the
difference between the time in which a person would walk
from P to Q and the time he will take to return from Q to P,
supposing his pace up hill to be 4 miles, down-hill 6 miles
and on level 5 miles per hour

29   A and B start from P and Q to go to Q and P
respectively,—the distance PQ being 60 miles A goes at
the rate of 4 and B 5 miles per hour   They meet at R,
reach their destinations, return immediately and again meet
at S Find the distance between R and S

30   A starts from P and goes towards Q at the rate
of 6 miles an hour.   Two hours afterwards, B starts from P
and going 10 miles per hour reaches Q 4 hours earlier than
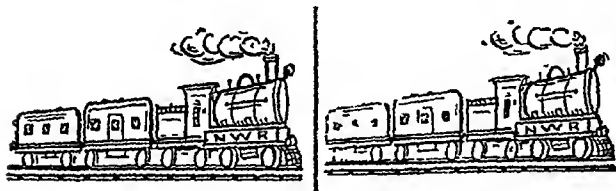A.   Find the distance between P and Q

II. TRAINS.

§2.   We shall solve some problems on Trains

Example 1   A train 176 yards long moves at the rate of 40 miles an hour , how long will it take to pass a certain telegraphic post ?

Sol.   *First position.*   *Second position*

In the first position, distance between the engine and the telegraphic post is zero, and in the second position distance between the engine and the post is equal to the length of the train It is clear, therefore, that *in passing a certain post or a standing man the train is to move its own length.*

$$\text{the required time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{176}{40 \times 1760} \text{ hrs}$$

$$= \frac{176 \times 60 \times 60}{40 \times 1760} \text{ or } 9 \text{ sec} \quad \textbf{Ans.}$$

Example 2   How long will a train 154 yards long take to cross a bridge 176 yards long at the rate of 40 miles an hour ?

Sol   *First position*   *Second position*

In the first position, distance between the engine and

the beginning of the bridge is zero   In the second position, the distance between the engine and the beginning of the bridge is equal to the sum of the lengths of the bridge and the train, *i. e.*, $176 + 154 = 330$ yards. It is clear, therefore, that *in crossing a bridge or a station the train is to move its own length plus the length of the bridge or the station.*

∴ the required time = distance ÷ speed

$$= \frac{330}{40 \times 1760} \text{ hrs.} = \frac{330 \times 60 \times 60}{40 \times 1760} \text{ or } 16\frac{7}{8} \text{ sec. Ans.}$$

Example 3   Two trains 110 yards and 88 yards long respectively are running in the same direction   The first runs at the rate of 20 miles an hour and the second at the rate of 25 miles an hour.   How long will they take to cross each other ?

Sol.   It is clear that the trains will cross each other when they have travelled a distance equal to the sum of their lengths, *i e ,* $110 + 88$ or 198 yards.   Since they are moving in the same direction, the relative speed $= 25 - 20$ or 5 miles per hour.

time reqd. = distance ÷ speed

$$= \frac{198 \times 60}{5 \times 1760} \text{ min.} = 1 \text{ min. 21 sec.   Ans.}$$

Note   Had they run in opposite directions, the relative speed would have been $25 + 20$ or 45 miles per hour and then they would have passed each other in

$$\frac{198 \times 60 \times 60}{45 \times 1760}, \text{ or 9 sec.}$$

Example 4.   Two trains starting from the same station and travelling in the opposite directions are $227\frac{1}{2}$ miles apart in 3 hours 15 min   Had they been travelling in the same direction they would have been $32\frac{1}{2}$ miles apart in the same time , find their speeds.

Sol.   The sum of their rates per hr $= \frac{455}{2} \div \frac{13}{4}$ or 70 miles

and the diff. ,,   ,,   ,, $= \frac{65}{2} \div \frac{13}{4}$ or 10 miles.

Hence the rate of the faster $= (70 + 10) \div 2$ or 40 miles⎫
and   ,,   ,,   slower $= (70 - 10) \div 2$ or 30 miles⎭ **Ans.**

Example 5. Two trains started at the same time, one from A to B and the other from B to A, at the rates of 30 and 40 miles per hour respectively. When they crossed each other, it was found that the faster train had travelled 50 miles more than the slower one. Find the distance between A and B

Sol. Ratio in the speeds of the trains $= 40.30$
$$= 4.\ 3$$
$$= \tfrac{4}{7} \cdot \ \tfrac{3}{7}$$

Diff. $= \tfrac{4}{7} - \tfrac{3}{7} = \tfrac{1}{7}$.

But the real diff. is 50 miles.

$\therefore$ distance $= 50 \times 7$ or 350 miles. **Ans.**

Aliter Difference in the speeds per hour $= 10$ miles.
But the total diff $= 50$ miles

. this difference is caused in $50 - 10$, or 5 hours

$\therefore$ the difference between A and $B = (40 + 30) \times 5$
$$= 350 \text{ miles. } \textbf{Ans}$$

Example 6 Two trains travelling in the opposite directions pass each other in 8 seconds But when they travel in the same direction at the same rates the man in the faster train passes the other in $31\frac{1}{2}$ sec. Find the lengths of the trains when their speeds are 25 and 20 miles respectively.

Sol (i) When the trains travel in opposite directions, the distance travelled in one hour $= 25 + 20 = 45$ miles.

.   „   „   „sec. $= \tfrac{45}{3600}$ mile
.   „   „   8 „ $= \tfrac{45}{3600} \times 8$ mile
$$= \tfrac{45}{3600} \times 8 \times 1760 \text{ yds.}$$
$$= 176 \text{ yds}$$

It follows, therefore, that the sum of the lengths of both the trains is 176 yds

(ii) when the trains travel in the same direction, then the distance travelled in one hour $= 25 - 20 = 5$ miles

$\because$ „   „   „ one sec. $= \tfrac{5}{3600}$ mile
$\therefore$ „   „   „ $31\frac{1}{2}$ sec. $= \tfrac{5}{3600} \times \tfrac{63}{2}$ mile
$$= \tfrac{5}{3600} \times \tfrac{63}{2} \times 1760 = 77 \text{ yds.}$$

$i.e$, the length of the slower train $= \qquad\quad 77$ yds $\Big\}$ **Ans.**
$\therefore$ „   „   „ faster train $= 176 - 77 = 99$ „

Example 7. A train starts from Lahore at 1-20 P M. and reaches Ludhiana at 5-50 P M   A second train starts from Ludhiana at 1-50 P M and reaches Lahore at 7-10 P.M. Find the ratio in the speeds of the two trains

Sol   Time taken by the first train $=4$ hrs 30 min $=\frac{9}{2}$ hrs.
and   ,,   ,,   ,, second , $=5$ hrs. 20 min. $=\frac{16}{3}$ hrs.

Ratio in their times $=\frac{9}{2}$ $\frac{16}{3}$

$\therefore$ ratio in their speeds $=\frac{16}{3}$ $\frac{9}{2}=33$ 27 **Ans**

Example 8. A train after travelling 70 miles meets with an accident and then proceeds at $\frac{3}{4}$ of its former speed and arrives at its destination 35 min too late   Had the accident happened 15 miles further on, it would have reached the destination only 23 min. late.   Find (a) the distance and (b) the speed of the train

Sol (a) Let A be the starting place, B the destination , C is the place where the accident happened and D is the place 15 miles off from C

Travelling at $\frac{3}{4}$ of its original speed means that the train would take $\frac{4}{3}$ of its usual time, i e , $\frac{1}{3}$ of the usual time more ,

$\therefore$ the excess time taken to travel $CB=35$ min.

and ,,   ,,   ,,   ,,   ,, $DB=23$ min

$\therefore$,,   ,,   ,,   ,,   ,, $CD=12$ min

12 min. is $\frac{1}{3}$ of the usual time

$\therefore$ usual time is 36 min

$\therefore$ in 36 min. the distance travelled is CD, $i.e$, 15 miles

in 1 min ,,   ,,   ,, $\frac{15}{36}$ miles

$\therefore$ in 60 min ,,   ,,   ,, $\frac{15}{36} \times 60$ or 25 miles.

i e,, the speed is 25 miles per hour   **Ans**

(b) $\therefore$ the excess time taken to travel $CD=35$ min

$\therefore$ usual time $=35 \times 3=105$ min $=\frac{7}{4}$ hrs

$\therefore$ the distance $CB=25 \times \frac{7}{4}$ $=\frac{175}{4}$ miles.

$\therefore$ whole distance $=\frac{175}{4}+70$ $=113\frac{3}{4}$ miles. **Ans**

EXERCISE 96.

1   A train runs at the rate of 15 miles an hour , find its speed per minute

2   A train moves 176 yards in 9 seconds , find its speed per hour.

3   A train running at the rate of 20 miles an hour passes a certain telegraphic post in 18 seconds , find the length of the train

4.   A train which is 110 yards long moves at the rate of 40 miles an hour what time will it take to pass a certain telegraphic post ?

5   A train 110 yards long travels at the rate of 45 miles an hour , what time will it take to pass a station 88 yards long ?

6   Two trains 195 ft and 135 ft long travel at the rates of 25 miles and 20 miles respectively   When will they pass each other when they are moving (ι) in the same direction, (ιι) in opposite directions ?

7   Two trains measuring 92 yards and 84 yards in length travel at the rates of 30 miles and 45 miles respectively in opposite directions   How long will it take them to cross each other ?

8   Two trains 88 yards and 110 yards in length respectively are running (ι) in opposite directions, (ιι) in the same direction.   The first runs at the rate of 50 miles an hour and the second at the rate of 40 miles an hour. How long will they take to cross each other ?

9.   A passenger train going 41 miles an hour and 431 ft long overtakes a goods train on a parallel line of rails The goods train is going 28 miles an hour, and is 713 ft long   How long does the passenger train take in passing the other ?

10   A train 99 yards long passes a man, coming from the opposite direction at the rate of 4 miles an hour, in 10 seconds   Find the speed of the train

11   A train 99 yards long passes a man going in the same direction at the rate of 4 miles an hour, in 10 seconds Find the speed of the train

12 A train which runs at the rate of 50 miles an hour crosses a man going in the same direction at the rate of 5 miles an hour in 5 seconds. Find the length of the train.

13. A train which runs at the rate of 50 miles an hour crosses a man walking in the opposite direction at the rate of 5 miles an hour in 3 seconds. Find the length of the train

14. A goods train moving at the rate of 20 miles an hour passes a man going in the same direction at the rate of 4 miles an hour in 7½ seconds. Find the length of the train.

15. The whole time occupied by a train 126 yds long travelling at the rate of 45 miles an hour, in crossing a bridge is 12 seconds Find the length of the bridge
[*Hindu University, Benares, 1922.*]

16 Two trains start at the same time from Lahore and Peshawar and proceed towards each other at the rates of 25 miles and 30 miles respectively When they meet it is found that one train has travelled 35 miles more than the other. Find the distance between the two stations

17 Two trains start at the same time from Mirzapur and Delhi and proceed towards each other at the rates of 16 and 21 miles per hour respectively When they meet, it is found that one train has travelled 60 miles more than the other. Find the distance between the two stations.

18 A local train which travels at the rate of 24 miles an hour, leaves Lahore at 20 minutes past 8 and reaches Amritsar at 5 minutes past 10 the same morning It stops at Moghalpura for 10 minutes and at each of the three other stations for 5 minutes. Find the distance from Lahore to Amritsar.

19. Two trains 330 ft. and 264 ft. long respectively run on parallel lines of rails. When travelling in opposite directions they are observed to pass each other in 9 seconds, but when they are running in the same direction at the same rate as before the faster train

passes the other in $27\frac{1}{3}$ seconds.   Find the speeds of the two trains in miles per hour.

20   A train running at the rate of 40 miles an hour meets a person walking along the line in the opposite direction at the rate of 4 miles an hour and passes him in $5\frac{5}{8}$ seconds, find the length of the train.   Had the person been going in the same direction as the train, in what time would it have passed him ?

21   A train 132 yards in length, travelling at a uniform speed, overtook a man walking along the line at the rate of 6 miles an hour and passed him in 12 seconds, twenty minutes later, the train overtook a second man and passed him in 11 seconds   How many hours after the train overtook the second man, would the first man also overtake him.

22.   A train moving at the rate of 45 miles an hour passes another train $1\frac{1}{2}$ times as long and moving at the rate of 27 miles an hour in the same direction in 25 seconds. How long will it take to pass a station 165 yards long ?

23.   Two trains running at the rates of 35 and 30 miles an hour respectively, on parallel rails in opposite directions are observed to pass each other in $4\frac{1}{2}$ seconds and when they are running in the same direction at the same rate as before, a person sitting in the faster train observes that he passes the other in 36 seconds, find the lengths of the trains.

24   A train starts from Lahore at 2-50 P. M. and reaches Ludhiana at 7-10 P M. and a second train starts from Ludhiana at 2-30 P M and reaches Lahore at 7-50 P M.   Find the ratio in the speeds of the two trains

25.   A man after travelling 30 miles got his cycle punctured and then started at $\frac{4}{5}$ of his former speed.   He reached his destination 30 min too late   Had it punctured 10 miles further on, he would have reached the destination only 20 min late.   Find the distance and the speed of the cycle.

III. STREAMS

§3.   Suppose a man can swim in still water at the rate of 4 miles an hour   If he were to swim in a running stream the speed of which is one mile an hour then his speed would be $4+1$, i. e , 5 miles an hour with the stream and $4-1$, i. e , 3 miles an hour against the stream

From the above explanation we can deduce the following results —

(i) **Man's speed in still water =** $\left\{ \begin{array}{l} \text{half the sum of his speeds} \\ \text{with and against the stream} \\ \text{Thus } \tfrac{1}{2}(5+3)=4 \end{array} \right.$

(ii) **Speed of the stream**   = $\left\{ \begin{array}{l} \text{half the difference of his} \\ \text{speeds with and against the} \\ \text{stream   Thus } \tfrac{1}{2}(5-3)=1 \end{array} \right.$

Example 1.   A boat is rowed down a river 20 miles in 4 hours and 44 yards up the river in 45 seconds.   Find the velocity of the river

Sol.   Speed with the river $=\frac{20}{4}$ i e., 5 miles an hour,

$$\text{Speed against the river} = \frac{44}{45} \times \frac{60 \times 60}{1760} \text{miles.}$$

$$=2 \text{ miles an hour}$$

∴ velocity of the river $=\tfrac{1}{2}(5-2)$, i e., $1\tfrac{1}{2}$ miles an hr   **Ans.**

Example 2   A man can swim with the stream at the rate of 6 miles an hour and 352 yards against the stream in 4 minutes, find his speed in still water.

Sol   Speed with the stream $=6$ miles an hour

Speed against stream $= \frac{352}{4} \times \frac{60}{1760}$

$$=3 \text{ miles an hour}$$

∴   Speed in still water $=\tfrac{1}{2}(6+3)=4\tfrac{1}{2}$ miles.   **Ans.**

Example 3.   A man rows 18 miles down a river in 4 hours with the stream and returns in 12 hours, find his speed and also the velocity of the stream

Sol   Speed with the stream $=\frac{18}{4}=4\tfrac{1}{2}$ miles an hour

Speed against the stream $=\frac{18}{12}=1\tfrac{1}{2}$ miles an hour

∴ Speed of the stream $=\tfrac{1}{2}(4\tfrac{1}{2}-1\tfrac{1}{2})$

$\left. \begin{array}{l} \qquad\qquad\qquad =1\tfrac{1}{2} \text{ miles an hour} \\ \text{and his speed} \qquad\quad =4\tfrac{1}{2}-1\tfrac{1}{2}=3 \text{ miles} \end{array} \right\}$ **Ans.**

EXERCISE 97.

1. A man can row in still water at the rate of 5 miles an hour, how long will he take to row 27 miles with the stream, when the velocity of the stream is $1\frac{1}{4}$ miles an hour?

2. A man can row a mile with the stream in 15 min and against the stream in 25 minutes   How long will it take him to row 2 miles in still water?

3 A man can swim with the stream at the rate of 3 miles an hour and against the stream at the rate of 2 miles an hour   How long will it take him to swim 7 miles in still water?

4 A man rows with the stream at the rate of $2\frac{1}{3}$ miles an hour and against the stream at the rate of $1\frac{1}{3}$ miles an hour   How long will it take him to row 12 miles in still water?

5 A boat moves a mile down the stream in 15 minutes and up the stream in 25 minutes   How long will it take the boat to move 176 yards in still water?

6 A man can swim 88 yards per minute down the stream and 55 yards per minute up the stream, find the velocity of the river

7 A person rows a mile down the stream in 20 minutes and up the stream in 25 minutes   Find the velocity of the stream.

8 A man can row on still water a distance of 4 miles in 20 minutes and 4 miles with the current in 16 minutes   How long will it take him to row the same distance against the current?

9 A man rows 15 miles down a river in 3 hours with the stream and returns in $7\frac{1}{2}$ hours, find the rate at which he rows and also the velocity of the stream.

10 A, B and C are three towns on a river which flows uniformly and B is equidistant from A and C, I can row from A to B and back in $5\frac{1}{4}$ hours and I can row from A to C in 7 hours, compare the speed of my boat in still water with that of the river

IV   RACES AND GAMES OF SKILL

§4   The following terms used in questions of races and games of skill may be noted —

(i)   A gives B a **start of 20 yards** means that B stands 20 yards ahead of A.

(ii)   **In a game of 100 A can give B 10 points** means that if A scores 100 points B scores only 90.

(iii)   **Dead heat** means a race in which no one is the winner.

Example. A can beat B by 60 yards in a mile race, B can beat C by 20 yards in a mile race   If A and C run a mile, by how much will A win?

Sol.   A can run 1760 yards while B runs 1700 yards,

and   B can run 1760 yards while C runs 1740 yards.

Now, if B runs 1760 yards C runs 1740 yards,

∴   if B runs   1 yard C runs $\frac{1740}{1760}$ yards,

∴   if B runs 1700 yards C runs $\frac{1740}{1760} \times 1700$ yards,

i.e., $\frac{29975}{22}$ or $1680\frac{15}{22}$ yards,

if A runs 1760 yards C runs $1680\frac{15}{22}$ yards,

∴   A can win by $1760 - 1680\frac{15}{22}$, i.e., $79\frac{7}{22}$ yards. **Ans.**

EXERCISE 98.

1.   In a mile race A beats B by 40 yards and C by 60 yards   By how many yards will B beat C?

2   In a mile race A gives B 40 yards' start and beats him by 44 yards.   If A runs a mile in $9\frac{1}{2}$ minutes, how long will B take to do the same?

3   A can give B 20 yards and B can give C 10 yards in a race of 200 yards   By how much could A beat C in the same race?

4   In a mile race A beats B by 110 yards and B beats C by 120 yards.   By how many yards could A beat C in the same race?

5.   A can give B 55 yards and C 80 yards in a race of 880 yards. By how much could B beat C in a mile race?

6. In a mile race Madan Lal gives Tara Chand a start of 55 yards and beats him by 105 yards   If Madan Lal runs the mile in 5 minutes, how long will Tara Chand take ?

7. A can run 320 yards in 2 minutes and B 240 yards in one minute   By how much will B beat A in a mile race ?

8 In a mile race A can give B 40 yards and B can give C 40 yards, how many yards' start should A give C so that they may end in a dead heat ?

9. A runs 20 yards while B runs 21 yds , B runs 31 yds. while C runs 30 yds , if A can run a mile in 5 min. 15 sec , what time will C take to do it ?

10 A can run 8 yards in the same time in which B can run 9   They start together.   When B has run 252 yards, how far behind is A ?

11. A works 15 sums in the same time in which B works 10 and A works 12 sums in the same time in which C works 7.   Who is the quicker worker, B or C ?

12 In a half mile race A gives B 10 yds ' start and beats him by 20 yds , B gives C 30 yds ' start in half a mile and is beaten by 60 yards , who runs faster, A or C ?

13. At a game of rackets, A can give B 8 points in 40, and B can give C 10 points in 50, how many points could A give C in 75 ?

14 A can give B 25 points, A can give C 40 points and B can give C 20 points   How many points make the game ?

15. In a game of skill, A can give B 8 points out of 38 and to C 10 points out of 95. Of B and C who is the better player and how many points can he give the other in 340 ?

16 A can give B 1 point in 5 at billiards, B can give C 3 in 20, D can give C 1 in 18. How many can A give D in 100 ?

V. TRAVELLING ROUND A CIRCLE

§5   When two or more persons start from the same place at the same time and travel round a circle in the same direction or in opposite directions, then (*i*) they will be first together again after an interval of time which is the L C M of the times in which one of the persons gains one complete round over each of the others, since each pair will be together after this time , (*ii*) they will be first together at the starting point again after an interval of time which is the L C M of the times in which each of them makes one complete round, since in that interval each of them shall make a complete number of rounds.

The following solved examples will illustrate the method of work —

Example 1   Three cyclists *A*, *B* and *C* ride round a circular course 180 miles round at the rate of 24, 30, and 42 miles a day respectively   In how many days will they all come together again supposing they all ride in the same direction ?

Sol.   *B* gains on *A* (30—24) or 6 miles a day,

  ∴ they will come together after every $\frac{180}{6}$, *i e*, 30 days, *C* gains on *B* (42—30) or 12 miles a day,

      C and *B* come together after every $\frac{180}{12}$, *i.e* , 15 days, and *C* gains on *A* (42—24) or 18 miles a day,

    ∴ *C* and *A* come together after every $\frac{180}{18}$ or 10 days.

Since *B* and *A* meet after 30 days, *C* and *B* after 15 days and *C* and *A* after 10 days,

    ∴ *A*, *B* and *C* will meet at the end of any number of days which is a common multiple of 30, 15 and 10

L C M of 30, 15 and 10 being 30,

they will meet at the end of 30 days.   **Ans.**

Example 2.   Three cyclists *A*, *B* and *C* ride round a circular course 85 miles round at the rate of 8, 12 and 20 miles an hour , *A* and *B* ride in the same direction and *C* in the opposite direction   In how many hours will they meet again ?

Sol B gains 4 miles on A in 1 hour,

they meet together after every $\frac{8\cdot5}{4}$ hours,

and B and C together pass $12+20$, i e , 32 miles an hr ,
they will meet together after every $\frac{8\cdot5}{3\cdot2}$ hrs

Also A and C together pass $8+20$ i e   28 miles an hr.
they will meet together after $\frac{8\cdot5}{2\cdot8}$ hours

A, B and C will meet together at the end of any
No of hrs. which is a common multiple of $\frac{8\cdot5}{4}$, $\frac{8\cdot5}{3\cdot2}$ and $\frac{8\cdot5}{2\cdot8}$

L C. M of $\frac{8\cdot5}{4}$, $\frac{8\cdot5}{3\cdot2}$ and $\frac{8\cdot5}{2\cdot8} = \frac{8\cdot5}{4}$

they will meet after $21\frac{1}{4}$ hours   Ans

EXERCISE 99

1   Two men A and B start together from the same
place to walk round a circular course 20 miles round A's
speed is $2\frac{1}{2}$ miles and B's $3\frac{1}{2}$ miles an hour   When will they
next meet, supposing (i) they walk in the same direction,
(ii) in opposite directions?

2   Two cyclists A and B ride round a circular course
60 miles round A's speed is 10 miles and B's 15 miles per
hour. When will they next come to the starting point,
supposing (i) they ride in the same direction, (ii) in opposite
directions ?

3   Three cyclists A, B and C ride round a circular
course 40 miles round at the rate of 8, 10 and 12 miles per
hour respectively. When will they meet together supposing
(i) they all ride in the same direction and (ii) A and B
ride in the same direction and C in the opposite direc-
tion ?

4   Three boys A, B and C run a race round a circular
path 2 miles round at the rate of 2, 3 and 4 miles per hour
respectively   When will they meet again, supposing (i) they
all run in the same direction   (ii) A and C run in the same
direction and B in the opposite direction ?

5   A, B and C start from the same point and travel in
the same direction round an island A can go round it in
12 days, B in 20 days and C in 25 days   If they start simul-
taneously from the same point, A and B in the same direc-
tion and C in the opposite direction, when will they come
together to the starting point again ?

F 22

VI CLOCKS.

§6   The dial of a clock or watch is divided into 60 minute spaces   The time in which the hands travel one space is one minute   Whilst the large hand (minute hand) moves 60 minutes, the small hand (hour hand) moves only 5 minute spaces. Clearly therefore the large hand gains 55 minute spaces on the small hand in 60 minutes   This fact is generally expressed in the words "M H gains 11 minute divisions in 12 minute divisions over H H " Or

The large hand gains 1 minute space on the small hand in $\frac{12}{11}$ minute spaces —This is the fundamental principle which works throughout in solving problems on clocks

The following points may also be remembered —

(*i*) The hands coincide across and point in opposite directions once in an hour, but they are twice at right angles when the time is between 4 and 9 and only once at other times

(*ii*) When at right angles they are 15 minute spaces apart and when they point in opposite directions, 30 minute spaces

(*iii*) At 6 the two hands are opposite each other   Hence they cannot be opposite again between 6 and 7

(*iv*) They are equidistant from a certain digit (*a*) when they coincide with, (*b*) when they are on either side of the digit

Example 1   At what time between 7 and 8 will the hands of a watch (*i*) coincide, (*ii*) be at right angles, (*iii*) point in opposite directions ?

Sol   (*i*) Observe the figure of the watch   The hour hand is 35 minute spaces ahead at 7 o'clock, hence the minute hand must gain 35 minute spaces to coincide with the hour hand.

the minute hand gains 1 min space in $\frac{12}{11}$ minutes,

the minute hand will gain 35 min spaces in $\frac{12}{11} \times 35$ minutes
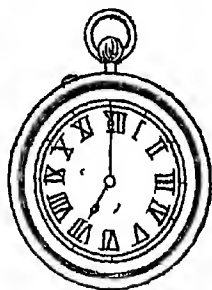
i. e , in $38\frac{3}{21}$ minutes.

Hence the hands will coincide at $38\frac{8}{11}$ min. past 7   **Ans.**

(ii)   In the figure, the hour hand is 35 minutes ahead, but to be at right angles it must be only 15 minutes ahead or back   In the first case the minute hand must gain 20 minutes and in the second case 50 minutes.

1 minute is gained in $1\frac{1}{11}$ minutes,

20 min will be gained in $20 \times 1\frac{1}{11}$ or $\frac{240}{11}$ or $21\frac{9}{11}$ min And 50 min will be gained in $50 \times 1\frac{1}{11}$ or $\frac{600}{11}$ or $54\frac{6}{11}$ min

Hence they are at right angles at $21\frac{9}{11}$ and $54\frac{6}{11}$ minutes past 7   **Ans**

(iii)   To be opposite the large hand must gain only 5 minutes since $35-5=30$

1 minute is gained in $1\frac{1}{11}$ minutes

5 minutes will be gained in $\frac{60}{11}$ or $5\frac{5}{11}$ minutes

Hence the hands will point in opposite directions at $5\frac{5}{11}$ min past 7   **Ans**

Example 2   At what time between 7 and 8 are the hands of a watch equidistant from the figure VIII ?

Sol   The position of the hands when equidistant from the figure VIII will be as when they (i) coincide or (ii) as shown in the diagram In the second case hour hand is at A and minute hand at B Thus $AS=8B$

Since distance from 7 to 8 is equal to the distance from 8 to 9, therefore $7A=9B$

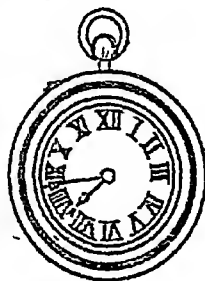It is obvious therefore that the two hands have moved 45 minute spaces between them

$12B+7A=45$ minutes

Since the minute hand moves 12 times as fast as the hour hand,

therefore, $12B=45\times\frac{13}{13}=\frac{540}{13}=41\frac{7}{13}$ minutes

Hence the time is $41\frac{7}{13}$ minutes past 7   **Ans**

Example 3   Two clocks were set correctly at noon on Sunday.   One gains 2 minutes and the other loses 3 minutes in 24 hours.   What will be the true time when (i) the first clock indicates 3 P. M. on Wednesday, (ii) the second clock indicates 8 P M on Thursday?

Sol   (i)   Time from noon on Sunday to 3 P.M. on Wednesday = 75 hours

24 hrs. 2 min of the first clock = 24 hrs. of the correct one

. 1 hour   ,,   ,,   ,, $= \dfrac{24 \times 30}{721}$,,   ,,

75 hrs. of the first clock $= \dfrac{24 \times 30 \times 75}{721}$,,   ,,

$= \dfrac{54000}{721}$ hrs   ,,   ,,

$= 74$ hrs $53\frac{547}{721}$ min

Hence the true time will be $53\frac{547}{721}$ min past 2 P.M   **Ans**

(ii)   Time from noon on Sunday to 8 P M on Thursday
$= 104$ hours

23 hrs 57 min of the 2nd clock = 24 hrs of the correct one,

. 1 hour   ,,   ,, $= 24 \times \dfrac{60}{1437}$,,   ,,   ,,

104 hrs.   ,,   ,, $= \dfrac{24 \times 60 \times 104}{1437}$,,   ,,   ,,

$= 104$ hrs $13\frac{82}{1437}$ min.

Hence the true time will be $13\frac{139}{1437}$ min past 8 P. M.   **Ans**

Example 4   In the above question find, after what interval will the one hand gain 30 minutes on the other and what time will then each show?

Sol   The first clock gains $2 + 3$, i.e., 5 min. in 24 hrs.

∴ it will gain 30 minutes in 144 hrs. or 6 days.

In 6 days the first clock will gain $6 \times 2 = 12$ minutes, and in 6 days the second clock will lose $6 \times 3 = 18$ minutes Hence the first clock will indicate 12 min past 12 P M ⎱ **Ans.** and the second clock will indicate 18 min to 12 A. M. ⎰

Example 5.   A person went out between 3 and 4 and came back between 7 and 8.   To his great surprise,

he found that the hands had exactly changed their places.
When did he go out ?

Sol. The position of the hands in
the diagram indicates the time of his
departure. As the hands now have
exactly changed their places it is clear
that the long hand has made 3
complete revolutions and one incom-
plete from B to A and the small hand
has moved from A to B, i e, both the
hands together have made full 4 revo-
lutions
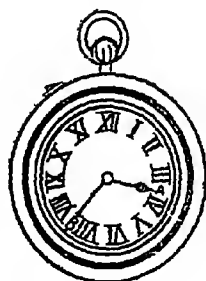
4 complete revolutions $= 4 \times 60$ or 240 minutes

Now let us find out the minute spaces moved by the
small hand in 240 minutes

. the min hand moves 12 times as fast as the hr hand

the hour hand has moved $\frac{240}{13}$ minute spaces only

Now the question becomes —

*When will the minute hand be $\frac{240}{13}$ minute spaces
ahead of the hour hand between 3 and 4 ?*

At 3 o'clock the minute hand is 15 min spaces behind
the hour hand hence to be ahead $\frac{240}{13}$ minute spaces, it will
have to gain $\frac{240}{13} + 15$, i e, $\frac{435}{13}$ minute spaces on the hour
hand

$\frac{435}{13}$ minute spaces can be gained in $\frac{435}{13} \times \frac{12}{11}$ minutes,

i e, in $36\frac{72}{143}$ minutes.

Hence he went out at $36\frac{72}{143}$ minutes past 3.   Ans.

EXERCISE 100

At what time are the hands of a clock (i) coincident,
(ii) at right angles, (iii) pointing in opposite directions ?—

1   Between 4 and 5   2   Between 5 and 6
3   Between 2 and 3   4.   Between 8 and 9.
5   Between 9 and 10.   6.   Between 6 and 7

7   At what time between 4 and 5 is the minute hand
10 minute spaces ahead of the hour hand ?

8   At what time between 7 and 8 is the hour hand
5 minute spaces ahead of the minute hand ?

9   At what time between 5 and 6 are the hands of a
clock equidistant from the figure 5 ?

10   At what time between 3 and 4 are the hands of a clock equidistant from the figure 4 ?

11.   A watch which loses 3 seconds in one hour was set correctly at noon on Monday   What time will it indicate at 4 P M on Wednesday ?

12   A watch which loses 16 minutes in 24 hours was set right at noon on Sunday , find the correct time when this watch indicates 5 hours 40 min on Monday evening

13   A clock which loses 2 seconds in one hour was set right at 8 A M on Sunday   What time will it indicate at 4 P. M on Tuesday ?

14   The school clock was set correctly at 4 P M. on Saturday.   It was found 10 minutes slow at 4 P M on Wednesday   What time will it indicate at 12 o'clock at noon on Thursday ?

15   One clock loses 10 seconds and another gains 8 sec in 24 hours   They were set correctly at noon on Tuesday   After what interval will one be 2 minutes in advance of the other and what o'clock will each of the clocks then indicate ?

16   Two watches were set correctly at 8 A M on Monday   One gains 4 minutes and the other loses 5 minutes in 24 hours   After what interval will one be half an hour in advance of the other and what time will each of the watches then indicate ?

17   A watch loses 4 minutes daily   It was set right on September 17, 1928 at 9 A M.   When will it show correct time again ?

18   A clock loses 15 minutes in 24 hours.   It was set right at noon on July 1 , when will it show correct time again ?

19.   Two clocks are set right at 10 A. M on Wednesday   One gains $2\frac{1}{2}$ minutes and the other loses 4 minutes daily   What difference will they indicate at 2 P M. on Monday and after how many days will they indicate the same time ?

20   A watch loses 3 min and another gains 4 min daily   They were set right at 3 P M.   What time will the slower watch indicate the next day when the faster shows 9 P M ?

21   Two watches were set right at 1 P. M. on Wed-

nesday. One loses $2\frac{1}{2}$ min and the other gains $2\frac{1}{2}$ min in
24 hours What time will the fast watch indicate on Saturday when the slow watch shows 10 A M ?

22 Two clocks point 2 o'clock at the same instant on the afternoon of April 25, one loses 7 seconds and the other gains 8 seconds in 24 hours When will one be half an hour in advance of the other and what time will each clock then show ?

23 Two watches, one of which gained at the rate of 1 min 5+6 sec and the other lost at the rate of 1 min. 55 8 sec daily were set correctly at noon on the 1st January 1896 When did the watches next indicate the same time and what time did each then indicate ?

24 A clock which gains 3 min 56 sec in 24 hours was set correctly at noon on January 1, 1884. Find to the nearest minute the next date at which it indicated correct time.

25 Two clocks A and B whose rates are uniform at noon yesterday indicated 11 hours 55 m'nutes A M and 0 hour 2 minutes P M respectively A indicated the correct time at 9 P M. yesterday and B at 6 A M this morning When did A and B last agree and what time did they then indicate ?

26 A man who went out between 5 and 6 and returned between 7 and 8 found that the hands of the watch had exactly changed places. When did he go out ?

27 A man who went out between 5 and 6 and returned between 6 and 7 found that the hands of the watch had exactly changed places When did he go out ?

28 A man went out between 3 and 4 and coming back noticed that the hands of the clock had exactly changed places Find the time of his departure

29 It is between 2 and 3 o'clock, but a person looking at the clock and mistaking the hour hand for the minute hand fancies that the time of the day is 57 minutes earlier than the reality What is the true time ?

30 A clock which was 16 min too slow 24 days ago, is 16 min too fast to-day at the same hour When did it last show correct time and when will it show correct time again ?

CHAPTER XX.

PERCENTAGE.

§1. **Percentage** means for every hundred.

When we say a man made a profit of 20 per cent, we mean to say that he gained Rs 20 for every hundred rupees he invested in the business, $i\ e$, $\frac{20}{100}$ for each rupee. Similarly 15 per cent means $\frac{15}{100}$ for each rupee, and so on

Percentage therefore is a fraction, the denominator of which is 100

Abbreviation of per cent. is **p c.**, and it is generally denoted by the symbol %

Example 1 What fraction is 40 per cent. ?

Sol $40\% = \frac{40}{100} = \frac{4}{10} = \frac{2}{5}$ Ans

Example 2 Write 80 per cent as a decimal

Sol $80\% = \frac{80}{100} = \frac{8}{10} = 8$. Ans.

Example 3 What percentage is equivalent to $\frac{9}{20}$?

Sol. $\frac{9}{20} = \frac{\frac{9}{20} \times 100}{100} = \frac{45}{100} = 45$ p c Ans.

Example 4 Write $\frac{3}{7}$ as percentage.

Sol. $\frac{3}{7} = \frac{\frac{3}{7} \times 100}{100} = \frac{\frac{300}{7}}{100} = \frac{42\frac{6}{7}}{100} = 42\frac{6}{7}$ p. c **Ans.**

Example 5 Express 08 as percentage

Sol. $\cdot08 = \frac{8}{100} = 8$ p. c. **Ans.**

Example 6 Find 5 per cent. of Rs 880

Sol. 5% of Rs 880 $= \frac{5}{100} \times 880 =$ Rs 44. **Ans.**

Example 7 In a school of 800 boys 45 per cent. are Hindus, 25 per cent Mohammadans, 10 per cent Sikhs and the remaining of other communities Find the number of boys of each community

344

Sol   Hindus $=\frac{45}{100}$ of $800=360$ ⎫
Mohammadans $=\frac{25}{100}$ of $800=200$ ⎬ **Ans.**
Sikhs $=\frac{10}{100}$ of $800=80$ ⎬
Others $=\frac{20}{100}$ of $800=160$ ⎭

Example 8 In Mathematics a boy got 60 marks out of 150 What percentage of marks did he get ?

Sol   Marks he obtained $=\frac{60}{150}=\frac{2}{5}$

$$\text{percentage}=\frac{\frac{2}{5}\times100}{100}=\frac{40}{100}=40\%\ \text{Ans}$$

Example 9 A man spends Rs 513 which is 9% of his income, find his income

Sol   9 per cent of the sum $=$ Rs 513
    or $\frac{9}{100}$ of the sum $=$ Rs 513.
  . .   the sum $=$ Rs $513\times\frac{100}{9}$
             $=$ Rs 5700 Ans

Example 10 A man gives 5 per cent of his income in charity, spends 75 per cent. of the remainder for the maintenance of his family, now he has Rs 228 left. What had he at first ?

Sol $100-75=25$

Since 25 per cent of the remainder $=$ Rs 228
    remainder $=$ Rs $228\times\frac{100}{25}$
                        $=$ Rs 912

Now $100-5=95$,
  Rs 912 is 95 per cent of whole income
. income $=$ Rs $\frac{100}{95}\times912=$ Rs. 960. Ans

Example 11 The population of a village is 2500, $\frac{3}{5}$ of them are males and the rest females 60 per cent of the females are married, find the percentage of married males

Sol. Population $=2500$

. males $=2500\times\frac{3}{5}=1500$

. . females $=2500-1500=1000$

married females $=1000\times\frac{60}{100}=600$

. married males are also 600,

percentage $=\frac{600}{1500}\times100=40\%$ **Ans.**

EXERCISE 101.

Express as fractions in their lowest terms —

1   15 p c   **2**   25 p c.   **3**   28 p c

4 $37\frac{1}{2}$ p c   **5** $18\frac{3}{4}$ p. c.

Express as decimals —

6 16 p. c **7** 45 p c. **8** $7\frac{1}{2}$ p c **9** $17\frac{1}{2}$ p c.

What rates per cent. do the following fractions denote ?

10 $\frac{7}{25}$ **11** $\frac{3}{8}$ **12** $\frac{9}{10}$ **13** $\frac{8}{125}$

What rate per cent do the following decimals denote ?

14 09.   **15** 33.   **16** 0125   **17** 075

Find the value of —

18 8% of Rs 1200   **19** 15% of Rs 560

20. $37\frac{1}{2}$% of Rs 728   **21** $12\frac{1}{2}$% of £ 324

22 $16\frac{2}{3}$% of £127 5s 6d

23 $3\frac{1}{3}$% of Rs 515 5a 4p

24   Find the number of which (i) 96 is 6%, (ii) $235\frac{1}{2}$ is $15\frac{7}{10}$ p c

25   What per cent (i) of Rs 35 is Rs 7 , (ii) of 625 is $\frac{1}{3}$, (iii) of 4 hrs 25 min 12 sec is 2 hrs 12 min 36 sec ?

26   In a school of 1200 students, 18% are absent, find the number of absentees

27   In a school of 760 students, 15% are Christians, 25% are Mohammadans and the remaining are Hindus , find the number of each.

28. A man saves 12% of his income which amounts to Rs 50 per month Find his annual savings

29   The population of a town increased from 4800 to 5000 in 2 years. What was the rate per cent of the increase ?

30   The population of a town decreased from 2115 to 1692 in 3 years What was the rate per cent of the decrease ?

31   A person's annual income is £ 1250, he spends 55 per cent of it   How much does he save ?

32   The population of a country is 7500000, out of them 1000000 are illiterate   Find the percentage of the illiterate people

33   In a hospital there are 1225 sick people, out of them 450 are men, 530 women and the remaining are children   What is the percentage of the children ?

34   In a school of 2100 boys, 30 per cent read in the High Department, 32 per cent in the Middle Department and the remaining in the Primary Department.   Find the number of students in the Primary Department

35   A man took from a bank Rs 125 which was 20 per cent of what he had deposited, find his balance in the bank

36   A man gives Rs 129 8a in charity which is $12\frac{1}{2}$ per cent of his income, find his income

37   19 per cent of the population of a town is 399, find the population.

38   What is the number, $66\frac{2}{3}$ per cent of which is 128 ?

39   After spending 50% of his income and then 20% of the remainder, a man has Rs 600 left, find his income

40   A man spends 5 per cent of his income on the repair of his house, 75 per cent of the remainder on the other requirements of life   Now he had Rs 1425 left, find his income.

41   A and B had equal sums of money, 33% of A's and 24% of B's money amount to Rs 543 4a. 6p   Find how much money they had.

42   The population of a village is 4500, $\frac{5}{9}$ of them are males and the rest females, 40 per cent. of the males are married, find the percentage of the married females

§2. Some Important Typical Examples.

Example 1 The population of a town increased 5 per cent every year. If the present population be 8000, find the population in 3 years' time.

Sol If the population is 100, the increase is 5

,,   ,,   is 1,   ,,   ,, is $\frac{5}{100}$

,,   ,,   is 8000 ,,   ,, $\frac{5}{100} \times 8000$

the total population in one year will be

$$8000 + (\tfrac{5}{100} \times 8000) \; i \; e \; \; 8000 \; (1 + \tfrac{5}{100}) \; \text{or} \; P\left(1 + \frac{r}{100}\right)$$

where P stands for the original population and r for the rate per cent

$\cdot$ population in one year$= P\left(1 + \dfrac{r}{100}\right)$

,,   ,, two years$= P\left(1 + \dfrac{r}{100}\right)^2$

and ,,   ,, three years$= P\left(1 + \dfrac{r}{100}\right)^3$

and so on. Hence the **formulae**

1 Population (when increasing) in n years$= P\left(1 + \dfrac{r}{100}\right)^n$

2. Population (when decreasing) in n years$= P\left(1 - \dfrac{r}{100}\right)^n$

where n stands for number of years.

Now according to the formula the population in 3 years would be $P\left(1 + \dfrac{r}{100}\right)^3$,

or $8000 \; (1 + \tfrac{5}{100})^3$ or $8000 \times \tfrac{21}{20} \times \tfrac{21}{20} \times \tfrac{21}{20} = 9261$. **Ans.**

Example 2 The population of a town was 31250 three years ago , after a year there was 5 p c. increase, after second year 4 p c decrease and in the last year 2 p c increase, find the present population.

Sol Here the formula would be

final $P = $ original $P\left(1 + \dfrac{r_1}{100}\right)\left(1 - \dfrac{r_2}{100}\right)\left(1 + \dfrac{r_3}{100}\right)$

$\therefore$ reqd. $P = 31250 \times \tfrac{105}{100} \times \tfrac{96}{100} \times \tfrac{102}{100} = 32130$ **Ans**

Example 3   The number of students in a school increases at a certain rate per cent   The number at present is 1323 and the number two years ago was 1200, find the rate per cent of the increase.

Sol   By formula we have

$$1200 \times \left(1 + \frac{r}{100}\right)^2 = 1323$$

$$\therefore \quad \left(1 + \frac{r}{100}\right)^2 = \frac{1323}{1200} = \frac{441}{400}$$

or $\quad 1 + \frac{r}{100} \quad = \frac{21}{20}$ [by sq root]

or $\quad 1 + \frac{r}{100} \quad = \frac{21 \times 5}{20 \times 5} = \frac{105}{100}$

or $\quad 1 + \frac{r}{100} \quad = 1 + \frac{5}{100}$

or $\quad\quad\quad \frac{r}{100} = \frac{5}{100} \quad \therefore \ r = 5.$ Ans

Example 4   The income of a person increases at a certain rate per cent. annually   The income at present is Rs. 625, four years ago it was Rs 576   What will it be 2 years hence ?

Sol   According to the formula, we have

$$576 \times \left(1 + \frac{r}{100}\right)^4 = 625$$

$$\left(1 + \frac{r}{100}\right)^4 = \frac{625}{576}$$

or $\quad \left(1 + \frac{r}{100}\right)^2 = \frac{25}{24}$ [by sq. root]

Now $625 \times \left(1 + \frac{r}{100}\right)^2 =$ income two years hence

or $625 \times \frac{25}{24} =$ required income,

          i e Rs 651 0a 8p   Ans

Example 5   A reduction of 20 per cent. in the price of sugar enables a man to buy 2 maunds of sugar more for Rs. 120   Find the original price of sugar

Sol   Saving in Rs. $120 = Rs \frac{20}{100} \times 120$
$$= Rs\ 24$$

For Rs 24 he can buy 2 maunds of sugar

reduced price of sugar per maund $= Rs\ 12$

the original price must be Rs $12 \times \frac{100}{80}$
$$i\ e\ Rs.\ 15\ per\ maund\quad \textbf{Ans.}$$

Example 6   A person had some eggs, 4% being worthless were thrown away, 80% of the remainder were sold and now there remained only 96   How many eggs had he at first ?

Sol $100-80$, $i\ e$ 20% of the remaining eggs $= 96$

No of eggs after being thrown away $= 96 \times \frac{100}{20} = 480$

now $480 = 100 - 4$, $i\ e$ 96% of the whole,

he had $480 \times \frac{100}{96} = 500$ eggs   **Ans**

Example 7.   If the price of sugar be raised 40%, find by how much per cent a person must reduce his consumption of it so as not to increase his expenditure

Sol   The raised price of sugar is $\frac{140}{100}$ of the former price

the person must now consume $\frac{100}{140}$ of the original amount

reduction $= 1 - \frac{100}{140}$, $i\ e\ \frac{2}{7}$ of the original amount which is equal to $\frac{2}{7} \times 100 = 28\frac{4}{7}\%$ **Ans**

Example 8   The price of coal being raised 40 p c. a person reduced his consumption of it so much that his expenses in buying the coal only increased 10 p c   How much of coal does he now consume if he at first consumed $3\frac{1}{2}$ maunds

Sol.   Suppose the price of coal at first was Re. 1 per md.

he consumed coal worth Rs $3\frac{1}{2}$,

now he consumes ,,   ,,   Rs $\frac{7}{2} \times \frac{110}{100} = \frac{77}{20}$

but now the price per md. $= Rs\ \frac{140}{100} = \frac{7}{5}$

amount of consumption $= \frac{77}{20} \times \frac{5}{7} = 2\frac{3}{4}$ mds   **Ans.**

Example 9 In an examination 75% of the candidates passed in English , 70% in Mathematics, 23% failed in both subjects and 102 passed in both subjects   Find the number of candidates

Sol   Let the number of candidates be 100 ,

75 candidates pass in English and 25 fail in it ,

70 candidates pass in Mathematics and 30 fail in it ,

and 23 candidates fail in both subjects

25 failed in English and 23 failed in Maths also, it means 2 candidates failed in English alone and

30 failed in Mathematics and 23 failed in English also, it means 7 candidates failed in Mathematics alone

the total number of failure$=2$ in English$+7$ in Mathematics$+23$ in both subjects$=32$

No of successful candidates$=100-32=68$, i e 68% now 68% of „   „ $=102$

total No of candidates$=\frac{100}{68}\times102=150$ **Ans**

EXERCISE 102

1   The population of a town decreases 5 per cent, every year, if the present population be 24000, what will it be in 3 years ?

2   The population of a town decreases 4 per cent annually , if the present population be 390625, what will it be in 4 years ?

3.   The present population of a town is 11979 , if its annual increase be 10 per cent., what was it 3 years ago ?

4   The population of a town was 25000 three years ago , after one year there was 4% increase , next year 5% decrease and last year 2% increase. Find the present population

5   The population of a town increases at a certain rate per cent annually   Now it is 5408 , if it was 5000 two years ago, find the rate per cent of the increase.

6   The number of students in a school increases at a certain rate per cent annually , the present number is

1331 , if it was 1000 three years ago, find the rate per cent. of the increase.

7. The income of a person decreases at a certain rate per cent. annually, the present income is Rs. 3610 and two years ago it was Rs 4000. Find the rate per cent. of the decrease.

8. The population of a town increases at a certain rate per cent. Four years ago it was 6400, now it is 8100 What will it be 2 years hence ?

9 The income of a person decreases at a certain rate per cent Four years ago it was 1600, now it is 900 What will it be 2 years hence ?

10. A sells his goods 10 per cent. cheaper than B and 10 per cent dearer than C , how much per cent are C's rates lower than B's ?

11 The price of rice is raised 10 per cent., by how much per cent should a man reduce his consumption of rice, so that his expenditure may be the same as before ?

12 A reduction of 5 per cent in the price of tea enables a man to buy 2 lb of tea more for Rs 23 12a Find the original price per lb

13. A person bought a quantity of tea for Rs 11. 4a On the next day the price fell down 20 per cent Had he bought the tea on the reduced price he would have got 3 lbs. more. What price did he pay per lb ?

14. A fruit vendor gives 4 per cent. of his oranges in charity and sells 90 per cent of the remainder and then 96 oranges remain , how many had he at first ?

15 Out of the eggs a man had 3 per cent were found broken By selling 80 per cent of the remainder, there remained 97 eggs only Find the number of eggs he had at first

16. If the price of coal be raised 20 p c , find by how much per cent. a person must reduce his consumption of it, so as not to increase his expenditure.

17 If the price of rice be raised 30 p c., find by how much per cent a person must reduce his consumption of it so as not to increase his expenditure

18 The price of sugar being raised 20 p. c a family reduced its consumption so much that its expenses in buying sugar increased 5 p. c. only   Find how much sugar is consumed now if at first consumption was $4\frac{1}{2}$ mds

19 In an examination 65'6 p c of the candidates took Science, 40 8 p. c took Sanskrit. If 1280 candidates took both the subjects, find the number of candidates that appeared in the examination

20 In an examination 75'8 p c. of the candidates took French and 49 4 p c took Latin, if the number of candidates that appeared in the examination be 2500, find how many took both the subjects

21 90 per cent of the boys of a school pass in English, and 85 per cent. in Mathematics   150 pass in both subjects and no boy fails in both   How many boys are there in school ?

22. In an examination 52 per cent of the candidates failed in English and 42 per cent. in Mathematics   If 17 per cent. failed both in English and Mathematics, find the percentage of those who passed in both the subjects.

23 A man is able to save $12\frac{1}{2}$ p c of his weekly wages   His wages rise 2s a week and expenses rise 10 p. c in consequence   This diminishes his savings by 17s 4d a year   Find the weekly wages   [A year$=$52 weeks]

24. 68 p. c of the persons employed in a business house are men getting an average wage of Rs. 235 a month each and the rest are women getting an average wage of Rs 118 8a. each. Find (to the nearest integer in each case) what percentage of the total wages earned is paid to (a) men, (b) women   [*Burma, 1925*]

25 The population of Rangoon in 1921 was 342000 being an increase of 15 p c on the population of 1911 , find the population of 1911.   Assuming that it continues to increase at the same rate for each period of 10 years, find what the population will be in 1951.   [*Burma, 1925*]

F. 23.

CHAPTER XXI.

AVERAGE.

§1   Suppose A has Rs 6, B Rs 9 and C Rs 15   If we collect their money and then divide it equally among them, each would get Rs $(6+9+15)\div3$ or Rs 10   Ten is said to be an average number of 6, 9 and 15

Definition.   An average number is the intermediate of the given numbers of the same kind

§To find the average number.

From the above illustration we deduce the following

Rule.   *Add together the numbers and divide the sum by the number of the given numbers*

Cor   The sum of the numbers or quantities$=$ average $\times$ their number

The following examples will illustrate the rule —

Example 1.   Find the average value of 536, 728, 832 and 620.

Sol   The sum of the numbers$=536+728+832+620$
$$=2716$$

average value $=2716\div4=679.$ **Ans.**

Example 3   In a cricket eleven, 5 boys scored 10 runs each, 3 boys scored 15 runs each, 2 boys scored 20 runs each and the eleventh boy scored 8 only.   Find the average runs per boy

Sol.   Runs of 5 boys$=10\times5=50$
,,   ,, 3 boys$=15\times3=45$
,,   ,, 2 boys$=20\times2=40$
,,   ,, 1 boy $=$ 8
Total No. of runs $=143$
average runs $=143\div11=13.$ **Ans**

Example 3 The average age of a class of 20 boys is 12 years 4 months, the average of the first 5 boys is 13 years 2 months and that of the next 13 is 11 years, find the average age of the remaining two

Sol The total age of 20 boys $=$ 12 years 4 months $\times$ 20

$= 246$ years 8 months.

,, ,, ,, of 5 boys $=$ 13 years 2 months $\times$ 5

$= 65$ years 10 months.

,, ,, ,, of 13 boys $=$ 11 years $\times$ 13

$= 143$ years

,, ,, ,, of 18 boys $=$ 65 yrs 10 mths $+$ 143 yrs,

$= 208$ years 10 months.

The age of the rem 2 boys $=$ 246 years 8 mths $- 208$

years 10 months.

$= 37$ years 10 months

average of each $= 18$ yrs 11 months Ans.

Example 4 The average temperature for Monday, Tuesday and Wednesday was $55°$, the average for Tuesday, Wednesday and Thursday was $60°$, that for Thursday being $56°$, what was the temperature on Monday ?

Sol Sum of temp. for Mon, Tues., Wed $= 55° \times 3 = 165°$

Sum of temp for Tues, Wed., Thurs $= 60° \times 3 = 180°$

temperature on Thursday $= 56°$

sum of tem for Tues, Wed $= 180° - 56° = 124°$

temperature on Monday $= 165° - 124° = 41°$. Ans

EXERCISE 103

Find the average of the numbers —

1 12, 16, 18, 20.   2. 9, 14, 21, 16.

3 $6\frac{1}{4}$, $7\frac{1}{2}$, $12\frac{1}{8}$, $6\frac{1}{8}$.   4 5 25, 9 56, 8 19,

5. Four boys are 12, 15, 18 and 20 years old, find their average age,

6. The population of three towns is 8972, 5632, 7524 respectively, what is the average population of the towns ?

7. The income of four men is Rs 35 8a, Rs 45 12a., Rs. 36 10a and Rs. 48 4a. respectively, find the average income of each.

8. In a class of 20 boys there are 5 boys of the ages of 14, 9 boys of the age of 12 and 6 boys of the age of 10. Find the average age of the boys

9. Average age of 5 boys is 17 4 years and the ages of first four boys is 15, 17, 19 and 21 years respectively. Find the age of the fifth boy

10 Average marks obtained by a boy in English, Mathematics, General Knowledge and Science are 85. He obtained 75 marks in English, 95 marks in Mathematics and 55 marks in General Knowledge, how many marks did he obtain in Science ?

11. A student obtained 300 marks in 4 subjects ; his average marks in 3 subjects are 81`5. How many marks did he obtain in the fourth subject ?

12 The average age of 4 boys is 17 years 2 months and of the 6 more boys 15 years 1 month, find the average age of the boys.

13. The average age of the boys of a class of 8 boys is 18 5 years, it is decreased to 18 years by the admission of a new boy, how old is the new boy ?

14. The average age of the boys of a class of 15 boys is 16 5 years and the average age of the same class including the teacher is 17 5 years, how old is the teacher ?

15 The average cost of 15 horses is Rs 60 5, but the average cost of 10 horses is Rs 50 5 Find the average cost of the rest of the horses

16. The average age of the boys of a class of 40 boys is 15 5 years, what will be the average age if 10 new boys are admitted whose average age is 13 8 years ?

17 Average age of 5 boys is 21 years, the average of the first 2 boys is 18 years and that of the last 2 is 22 years, find the age of the third boy

18. In a cricket eleven the average runs are 8, the average of the first 4 boys is 10 and that of the next 5 boys is 6, find the average of the remaining two boys

19   The average attendance of a class for Monday, Tuesday and Wednesday was 38 The average for Tuesday, Wednesday and Thursday was 42   The attendance on Thursday being 40, find the attendance on Monday.

20.   The average income of a man from 1918 to 1922 was Rs. 485 and from 1919 to 1923 Rs. 487 , if the income for the year 1923 be Rs. 478, find the income for the year 1918

21   The average temperature for Monday, Tuesday and Wednesday was 99° and for Tuesday, Wednesday, Thursday 100° , if the temperature on Thursday be 98°, find the temperature on Monday

22   A person bought 15 maunds of wheat at Rs 5 a maund and mixed 5 maunds of barley in it , the average cost of the mixture became Rs 4 12a.   Find the cost of barley per maund

23   A person bought 20 maunds of wheat at Rs 5. 4a. a maund, 5 maunds of grams at Rs 3 8a. a maund and 2 maunds of barley at a certain price   He mixed them together and by selling them at Rs 5$\frac{1}{16}$ a maund got a profit of 8 per cent Find the cost of barley per maund

24   There are equal number of men and women. Rs, 140 are to be divided among them.   If a man be given Re 1 5a. 4p and a woman Re 1 only, find the total number of men and women

25   The average expenditure of a man for 5 months is Rs 120 and for the next 4 months Rs. 80   Find his monthly income if he had saved Rs 25

CHAPTER XXII

PROFIT AND LOSS

§1 The price we pay for an article is called its **cost price** (C P) and the price we get by the sale of an article is called its selling price (S P.) If an article is sold for more than the cost price, it is said to be sold at a profit or gain , but if it is sold for less than the cost price, it is said to be sold at a loss

Consider the following examples —

(*i*) A man buys a watch for Rs 20 and sells it for Rs 25 , find his gain or loss

Here Rs 20 is the cost price and Rs. 25 is the selling price As it is sold for Rs 5 more than the cost price, he is said to sell it at a profit of Rs 5

<p style="text-align:center">Profit = Selling price − Cost price</p>

(*ii*) A man buys a watch for Rs 20 and sells it for Rs 15 , find his gain or loss

Here Rs. 20 is the cost price and Rs 15 is the selling price As it is sold for Rs 5 less than the cost price, he is said to sell it at a loss of Rs 5.

<p style="text-align:center">Loss = Cost price − Selling price</p>

From the above examples we also conclude that
Selling price = Cost price + Profit or Cost price − Loss
and Cost price = Selling price − Profit or Selling price + Loss

Note Cost price is also called the outlay or prime cost

§2. To find net loss or gain

The following solved examples will illustrate the theorem of profit and loss clearly —

Example 1. A man bought 60 maunds of wheat for Rs 315 and sold it for Rs 5. 8*a.* per maund , find his profit.

Sol. The profit = selling price − cost price
$$= \text{Rs. } 5 \ 8a. \times 60 - \text{Rs } 315$$
$$= \text{Rs. } 330 - \text{Rs } 315 = \text{Rs } 15 \textbf{ Ans.}$$

Example 2   A merchant bought 180 bags of wheat containing 3 maunds each at Rs 5 4a per maund and sold 100 bags at Rs 5 per maund and 80 bags at Rs 5 8a per maund, find his loss

Sol.   The loss=cost price−selling price

$$=(Rs\ 5\ 4a\ \times 180 \times 3)-(Rs\ 5\times 100\times 3$$
$$+Rs\ 5\ 8a\ \times 80\times 3)$$
$$=Rs\ 2835-Rs\ 2820=Rs\ 15.\ Ans$$

Example 3   A man buys 25 gallons of wine for £37. 8s and sells it at £1 12s per gallon, find his gain or loss.

Sol   Cost price of 25 gallons=£37 8s.

Selling price of 25 gallons $=£1\ 12s.\times 25=£40$

Since he sells it for more than it costs him, therefore he makes a profit.

The profit=selling price−cost price

$$=£40-£37\ 8s=£2\ 12s\quad Ans$$

Example 4   A merchant buys 2 cwt of tea at Re 1. 2a per lb and sells it at Rs 27. 5a per qr., find his gain or loss

Sol   Cost price of 2 cwt of tea=Re 1 2a $\times 2\times 112$
$$=Rs\ 252.$$

Selling price of 2 cwt of tea $=Rs\ 27\ 5a\ \times 2\times 4$
$$=Rs.\ 218\ 8a$$

Since he sells it for less than what it costs him, therefore he sells it at a loss.

the loss=cost price−selling price
$$=Rs\ 252-Rs\ 218\ 8a=Rs.\ 33\ 8a\ \ Ans.$$

§3   When net loss or gain is given, the cost or selling price is found as in the following examples —

Example 5   A man buys 30 chairs at Rs. 4. 12a. per chair and sells them at a profit of Rs. 9 6a, find his selling price per chair

Sol   Cost price of 30 chairs=Rs 4. 12a $\times 30$
$$=Rs\ 142\ 8a$$

Profit on 30 chairs $=Rs\ 9\ 6a$

.   selling price $=$cost price$+$profit
$$=Rs\ 142\ 8a.+Rs\ 9\ 6a$$
$$=Rs\ 151\ 14a$$

∴ selling price per chair $=Rs\ 151\ 14a.-30$
$$=Rs\ 5\ 1a.\ \ Ans$$

Or thus

∴   cost price of one chair = Rs. 4. 12a
and profit of one chair   = Rs 9 6x −30 = 5a.
∴   selling price = cost price + profit
                      = Rs. 4. 12a + 5a = Rs. 5 1a **Ans.**

Example 6   A man buys 100 sheep at Rs. 150 per score, keeps them for a month at a cost of Rs 50, at what price per sheep does he sell them, if he loses Rs. 75 ?

Sol   Cost price of 100 sheep = Rs $\frac{150}{20}$ × 100 = Rs 750
and expenses he incurred for a month   = Rs. 50
    total cost price = Rs. 750 + Rs 50   = Rs 800.
.   selling price   = cost price − loss
                      = Rs 800 − Rs. 75 = Rs 725
    selling price per sheep   = Rs 725 − 100
                                  = Rs 7. 4a. **Ans.**

Example 7.   A grocer sold 3 cwt of tea at Re 1 3a per lb gaining thereby Rs. 84, find his total cost price and also cost price per lb

Sol   Selling price of one lb = Re 1 3a

        and profit on $= \frac{Rs 84}{3 \times 112} = 4a.$

.   cost price   = selling price − profit
                  = Rs 1. 3a − 4a = 15a per lb ⎫
and total cost price   = Rs $\frac{15}{16}$ × 3 × 112 = Rs 315 ⎭ **Ans.**

Example 8   A merchant sells 40 bags of rice containing 2½ maunds each at Rs 8. 4a per maund   What is his prime cost if he loses Rs 22. 5a in this transaction ?

Sol.   Selling price = Rs 8 4a × 40 × $\frac{5}{2}$ = Rs. 825
       Loss   = Rs 22 5a.
..   cost price   = selling price + loss
                    = Rs 825 + Rs. 22. 5a.
                    = Rs 847 5a.   **Ans**

EXERCISE 104.

1   A man buys a horse for Rs 65. 10a. and keeps it for a week at the cost of Rs. 15   What does he lose if he sells it for Rs. 70. 10a ?

2   A merchant bought a flock of sheep containing 15 score at Rs. 5 4a per head.   Having kept them for 2

weeks at the cost of Re 1. 8a. per head per week, he sold the whole flock at Rs 9 per head. What did he gain or lose ?

3. A grocer buys 4 cwt of tea at Rs 175 per cwt and sells it at Re 1 12a per lb. , find his profit.

4. A man bought 500 mangoes at 5a per score and sold at 4a a dozen. What did he gain or lose ?

5 A milk-seller buys 2 maunds of milk at 3a per seer and mixes 8 seers of water with it   He then sells the mixture at 2a 9p per seer   What did he gain or lose ?

6   A merchant bought 150 quarters of wheat at £2 a quarter, the expenses of carriage, etc , being 1s 4d a quarter   He sold the whole of it at £2 10s. a quarter, find his profit

7   A merchant bought 1 cwt. of tea at 1s. 6d per lb and 2 cwt at 1s. 2d per lb   He mixed them and sold the mixture at 1s 3d. per lb , calculate his loss or gain

8   A man bought goods for Rs 250 and sold ⅓ of it the same day losing $\frac{1}{10}$ of its cost , at what price must he sell the remainder so as to gain Rs. 12 on the whole transaction ?

9. A milk seller buys 2½ maunds of milk at 2a per seer and mixes 20 seers of water with it , at what price does he sell it, if he makes a profit of Rs 2 8a on the whole ?

10   A grocer bought 1 cwt. of tea at 1s 2d. per lb. and 2 cwt at 1s 6d per lb , at what price must he sell the mixture per lb to gain £4 13s. 4d. ?

§4. To find loss or gain per cent.

Loss or gain per cent. is always reckoned on the cost price.   The following examples will illustrate the rule —

Example 1   A man bought a horse for Rs. 250 and sold it for Rs 300, find his gain per cent.
Sol   cost price   = Rs. 250,
       and selling price   = Rs 300,
       ∴ total gain   = Rs 50.
         This gain is on Rs. 250 (C P)
         gain on Rs 250 = Rs 50
         gain on Re. 1   = Rs. $\frac{50}{250}$ = Re. $\frac{1}{5}$.
         gain on Rs. 100 = Rs $\frac{1}{5}$ × 100 = 20
       ∴ 20% is the gain   Ans

Example 2. A grocer bought 15 lbs of tea at 10*a*. per lb and 10 lbs. of tea at 12*a* per lb He mixed them together and sold the mixture at 11*a* per lb, find his gain or loss per cent

Sol C P of 15 lbs of tea $= 10a \times 15 = 150a$
and C. P of 10 lbs of tea $= 12a \times 10 = 120a$
∴ total cost price $= 150a + 120a = 270a$
But selling price of mixture $= 11a. \times 25 = 275a$
∴ total gain on $270a = 275a - 270a. = 5a$
∴ gain on $270a = 5a$
∴ gain on $1a. = \frac{5}{270}a = \frac{1}{54}a.$
∴ gain on $100a = \frac{1}{54} \times 100 = \frac{50}{27} = 1\frac{23}{27}$
∴ gain per cent $= 1\frac{23}{27}$ Ans

From the above examples we learn the following

Rule. *Profit or loss per cent* $= \dfrac{total\ profit\ or\ loss \times 100}{cost\ price}$

Example 3 The cost of 12 articles is equal to the selling price of 10 articles , find the gain per cent
Sol Let cost price of 12 articles $=$ Rs 100
∴ selling price of 10 articles $=$ Rs 100.
∴ selling price of 12 articles $=$ Rs. 120
gain per cent $= 20$. **Ans.**
Note For the sake of convenience we should always suppose the cost price in such questions to be Rs 100

EXERCISE 105.

1 A man bought a horse for Rs 250 and sold it for Rs 300, find his profit per cent

2 A man buys a watch for Rs. 40 and sells it for Rs 50 , what profit per cent does he make ?

3 A table which costs Rs 7 8*a* is sold for Rs 10 , find the profit per cent

4 A horse which costs Rs. 80 is sold for Rs 90 , find the profit per cent

5. An article which costs Rs. 15 is sold for Rs 10 , find the loss per cent

6. A merchant bought some quantity of sugar at Rs 8 5*a* 4*p*. a maund and sold it at Rs. 8 10*a* 8*p*. a maund What profit per cent. did he make ?

7. A boy bought a number of pencils at $4a$ a dozen and sold them at $5a$ a score What did he gain or lose per cent ?

8   A shop-keeper bought a certain number of slates at Rs 3. $6a$ a dozen and sold at $5a$ $8p$ per slate , find his gain or loss per cent.

9   A fruit-seller bought some mangoes at $10a$ per hundred and sold them at $3a$ a score , find his gain or loss per cent

10   A fruit-seller bought 100 oranges at 4 for $1a$. and 200 oranges at 3 for $1a$ He then sold the whole lot at $4a$ $6p$ per score , calculate his loss or gain per cent

11   A man bought a horse for Rs 250 and fed it for 2 months at the cost of Rs 35 per month He then sold it for Rs 280, what did he lose per cent ?

12   The cost price of 16 articles is equal to the selling price of 20 of the same kind , find the loss per cent

13   If the cost price of 10 articles be equal to the selling price of 8, find the profit per cent ?

14   A merchant buys a quantity of sugar at £2 8s per cwt , $\frac{1}{3}$ of it is wasted and the remainder he sells at $9d$ per lb , calculate his gain or loss per cent

15   A merchant buys a quantity of sugar at £2 16s. $3d$ per cwt , $+$ lb of sugar is wasted in every cwt. , what would be his loss or gain per cent., if he sells the remainder at $6d$ per lb ?

16   By selling 66 yards of cloth I made a profit equal to the cost price of 22 yards , find my gain per cent.

17. What will be the loss p c if I sell 45 yds. of cloth and bear a loss equal to the cost price of 9 yards ?

18   A shop-keeper having purchased goods for Rs 525 sold $\frac{1}{5}$ of them at a profit of 20 per cent. and the remainder at a profit of 5 per cent , find his gain per cent on the whole transaction.

19   A merchant bought teas worth 2s. 6d. and 2s per lb respectively and mixed them together in the proportion of 2 lbs. of the former to 3 lbs. of the latter and sold the mixture at 2s 9d. per lb., find his gain per cent.

20. A market woman bought eggs at 4 an anna and an equal number at 3 an anna, she sold them at 7 for 2 annas; find her gain or loss per cent.

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§5 To find selling price.

When the cost price and gain or loss per cent are given and the selling price is required, we proceed as in the following examples —

Example 1. A horse is bought for Rs. 350, find the selling price if it is sold at a gain of 20 per cent.

Sol. If the C. P is Rs. 100, the S P. = Rs 120.
    ,,   ,,   Re 1   ,,   ,,   = Rs. $\frac{120}{100}$
    ,,   ,,   Rs. 350   ,,   ,,   = Rs $\frac{120}{100} \times 350$
∴ selling price   = Rs 420 **Ans.**

Example 2 I bought a house for Rs. 4200 and sold it at a loss of 15 per cent., find the selling price

Sol   If the C. P. is Rs 100, the S P. = Rs. 85
    ,,   ,,   Re.   1,   ,,   ,,   = Re $\frac{85}{100}$
    ,,   ,,   Rs. 4200,   ,,   ,,   = Rs. $\frac{85}{100} \times 4200$
∴ selling price   = Rs 3570. **Ans.**

From the above examples we find that the

$$\text{Selling price} = \begin{cases} 1. \dfrac{100 + \text{gain p. c.}}{100} \times \text{cost price.} \\ 2. \dfrac{100 - \text{loss p. c}}{100} \times \text{cost price.} \end{cases}$$

EXERCISE 106

1 I bought an article for Rs 60 and sold it at a gain of 20 per cent., find the selling price.

2. At what price must a horse which costs Rs. 450 be sold so as to gain $7\frac{1}{2}$ per cent.?

3. If by selling an article which costs me Rs. 66. 10a. 8p. I lose 8 per cent., find the selling price.

4 A man bought a house for Rs. 2100 and spent Rs. 133. 5a. 4p on its repair, at what price must he sell it so as to gain 20 per cent.?

5 A grocer bought 4 lbs of tea at $10a$. a lb and 8 lbs. of tea of another quality at $8a$. a lb. , at what price per lb must he sell the mixture so as to gain 25 per cent ?

6 160 maunds of rice were bought at Rs 8 per maund and sold at a loss of 10 per cent , find the total selling price and the selling price per maund

7. 180 maunds of sugar were bought at Rs. 20 per maund and sold at a profit of 20 per cent , find the selling price per seer

8. A merchant bought 80 maunds of wheat at Rs. 5 $4a$ per maund and paid Rs 10 for expenses , at what price per maund must he sell them so as to gain 20 per cent on his total outlay ?

9 A shopkeeper bought oranges at the rate of 12 for a rupee and sold them at a profit of 20 per cent How many did he sell for a rupee ?

10 If the eggs are bought at the rate of 2 for a penny how many must be sold for a shilling so as to gain 20 per cent. ?

§6   To find cost price

When the selling price and gain or loss per cent are given and the cost price is required, we proceed as in the following examples —

Example 1   By selling goods for Rs 650 a tradesman gets a profit of 30 per cent   What did he give for them ?

Sol. If the selling price is Rs 130, the cost $=$ Rs. 100

			Re. 1		$=$ Re $\frac{100}{130}$
			Rs 650		$=$ Rs. $\frac{100}{130} \times 650$
cost price					$=$ Rs. 500 **Ans**

Example 2   By selling a house for Rs. 2066 $10a$ $8p$ I lost $17\frac{1}{3}$ per cent , find the cost price.

Sol. $100 - 17\frac{1}{3} = 82\frac{2}{3} = \frac{248}{3}$, Rs. 2066 $10a$ $8p$ = Rs. $\frac{6200}{3}$. If the S P. is Rs. $\frac{248}{3}$, the C. P.$=$ Rs 100

			Re. 1		$=$ Rs. $100 \times \frac{3}{248}$
			Rs $\frac{6200}{3}$		$=$ Rs. $100 \times \frac{3}{248} \times \frac{6200}{3}$
$\therefore$ cost price					$=$ Rs. 2500. **Ans.**

Note   From the above examples we find that the

$$\text{Cost price} = \begin{cases} 1 & \dfrac{100}{100+\text{gain p c}} \times \text{selling price} \\ 2 & \dfrac{100}{100-\text{loss p c}} \times \text{selling price} \end{cases}$$

EXERCISE 107.

What is the cost price (1—5) —

1   When an article is sold for Rs 24 at a loss of 4 per cent ?

2   When an article is sold for Rs 35 at a gain of 5 per cent ?

3   When an article is sold for Rs 108 at a gain of 20 per cent. ?

4   When an article is sold for Rs. 152 at a gain of 14 per cent ?

5   When an article is sold for Rs. 156 at a loss of 22 per cent ?

6.   If a yard of cloth sold for 5a 6p gives a profit of 20 per cent., find the prime cost

7   By selling a horse for Rs 175. 2a I gain 20 per cent , find the prime cost

8.   By selling goods for Rs 77 11a a man gained 10 per cent , find the cost price

9   Find the prime cost price of a cycle which when sold for Rs. 91 gave a profit of 5 per cent

10   A merchant by selling a horse for Rs 177 3a, lost 16 per cent, , find the cost price,

11   A shop-keeper by selling a sewing machine for £5 16s gained 16 per cent , find the prime cost

12   A cow is sold for Rs 119 4a. at a gain of 12½ per cent , how much did it cost ?

13.   If apples are sold at the rate of 16 for a rupee and the gain is 12½ per cent., at what rate were they purchased ?

14   If oranges are sold at the rate of 80 for a rupee and the gain is $22\frac{1}{3}$ per cent, at what rate were they purchased ?

15. By selling a horse and a cow for Rs. 112 8a., I got a profit of 25 per cent , find the cost price of the horse if the cow cost Rs 40.

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§7   Some Important Typical Examples

The following examples of different varieties are worthy of careful notice —

(a) Goods passing through successive hands

Example 1 A sells an article to B at a profit of 25 per cent , B sells it to C at a profit of 20 per cent   If C pays Rs 60 for it, what did it cost A ?

Sol C's cost price $=$ Rs 60.

As C purchased the article from B

B's selling price also $=$ Rs. 60

and B's profit per cent $=$ Rs 20

B's cost price $=$ Rs $\frac{100}{120} \times 60 =$ Rs 50

As B purchased the article from A

A's selling price also $=$ Rs 50

and A's profit per cent $=$ Rs 25

A's cost price $=$ Rs $\frac{100}{125} \times 50 =$ Rs 40. **Ans.**

Aliter   Suppose A's cost price $=$ Rs 100

he sold to B at Rs 125 ,

and B made a profit of 20 per cent on Rs 125

his selling price $=$ Rs $\frac{120}{100} \times 125 =$ Rs 150 which is evidently the price paid by C.

If when C pays Rs 150, A's cost price $=$ Rs 100

when C pays Re 1,   „   cost price $=$ Re $\frac{100}{150}$

„ C „   Rs. 60   „   „ $=$ Rs $\frac{100}{150} \times 60$

$=$ Rs 40. **Ans**

Note   The student should understand clearly this second solution which is much simpler than the first

EXERCISE 108.

1   A sells his horse to B at a profit of 20 per cent. and B sells it to C at a profit of 5 per cent   If C pays Rs 252 for it, what did it cost A ?

2.   A sold his watch to B at a profit of 5 per cent , B sold it to C for Rs 49 14a at a loss of 5 per cent. , what did A pay for it ?

3   A sold a piece of land to B at a profit of 25 per cent , B sold it to C for £3668 10s at a loss of 13 per cent. , what did A pay for it ?

4.   A sells an article to B at a gain of 20 per cent , B sells it to C at a gain of 5 per cent. If C pays Rs. 70 for it, what did A pay for it ?

5   An article is sold by A to B at a profit of 12%   It is then sold for Rs 2 10a. by B who thereby gains 12% on what he paid for it.   What did A pay for it ?

6.   A sells goods to B at a gain of $22\frac{1}{2}$ % B sells them to C at a gain of $7\frac{1}{2}$% C gave Rs 5267. 8a. for them.   How much did A pay for them ?

7.   An article passes successively through the hands of three dealers, each of whom in selling adds as his profit 10 per cent of the price at which he bought it , if the third dealer sells the article for Rs 332. 12a , what did the first dealer pay for it ?

_____

(b) Different rates of gain or loss per cent for different selling prices

Example 2   A watch was sold for Rs 45 at a loss of 20 p. c , for what should it have been sold to gain 30 p c. ?

Sol.   selling price $=$ Rs. 45
and loss per cent. $=$ Rs 20
. cost price $=$ Rs $\frac{100}{80} \times 45 =$ Rs $\frac{225}{4}$.
Now . cost price $=$ Rs $\frac{225}{4}$
and gain per cent. $=$ Rs 30
$\therefore$ selling price $=$ Rs $\frac{130}{100} \times \frac{225}{4} =$ Rs 73 2a Ans.

Aliter. If the cost price is Rs 100, then a loss of 20 per cent means it is sold at Rs 80 , and a gain of 30 per cent means it is sold at Rs. 130.

In this question 45 corresponds to 80 and we are to find the number corresponding to 130, which is evidently

$$=\tfrac{4\,5}{8\,0}\times130=\tfrac{5\,8\,5}{8}=73\tfrac{1}{8}$$

.   the required answer$=$Rs 73. 2a. Ans

Note   The student is very strongly recommended to understand this solution which is more intelligent than the previous one

EXERCISE 109.

1   By selling an article for Rs 48 a man gains 44 per cent. , by how much must he reduce the price to gain 20 per cent. ?

2   By selling an article for Rs 66 10a. 8p. a merchant loses 20 per cent. , at what price must it be sold to gain 20 per cent ?

3   By selling an article for Rs. 85 a merchant loses 15 per cent , at what price must it be sold to gain 12$\tfrac{1}{2}$ per cent. ?

4. If a merchant, by selling an article for Rs 172. 2a gains 12$\tfrac{1}{2}$ per cent , what should he sell it for to gain 25 per cent ?

5. By selling goods for $\pounds130$ per ton, I gain 30 per cent , what should I charge per cwt to gain 7$\tfrac{1}{2}$ per cent. ?

6   If I sell an article for $\pounds87$, I gain 8$\tfrac{3}{4}$ per cent , what must I sell it for to gain 20 per cent ?

7   If by selling a table for Rs 8. 8a , I lose $\tfrac{1}{18}$ of what it cost me, find at what price I ought to sell it to gain 16$\tfrac{2}{3}$ per cent.

8   By selling a cow for $\pounds9$. 7s. I lose 6$\tfrac{1}{3}$ p. c , what must I sell it for to gain 13 per cent ?

9   By selling apples at 16 for 2a 3p a man loses 25 per cent   How many should he sell at 10 p. c. profit so that the total gain may be Rs. 3 ?

Example 3   A horse is sold for Rs. 510 at a loss of 15 per cent. , find the gain or loss per cent had it been sold for Rs 575

Sol.   When the selling price $=$ Rs   510

the loss per cent $=$ Rs   15

cost price $=$ Rs. $\frac{100}{85} \times 510 = 600$

Now   cost price $=$ Rs   600

and selling price $=$ Rs   575

loss on Rs   600 $=$ Rs   25

loss per cent $= \frac{25}{600} \times 100 = 4\frac{1}{6}$ Ans.

EXERCISE 110

1   By selling a horse for Rs 350, a merchant gains 25 per cent , what will he gain or lose per cent   by selling it for Rs   250 ?

2   If I sell an article for Rs 4, I lose $16\frac{2}{3}$ per cent , what would be the gain or loss per cent, if I sell the same article for Rs 5 $3a$ $2\frac{3}{5}p$

3   By selling a cycle for Rs 74 a person lost $7\frac{1}{2}$ per cent, find the gain or loss per cent. if he sells it for Rs. 78

4   By selling goods for Rs   248 $1a$, a person lost 19 per cent , find the gain or loss per cent if he sells it for Rs 355 $4a$

5   By selling 7 oranges for $5a$ a dealer gained 5 per cent , what per cent. would he gain by selling at 9 for $7a$. ?

6   Goods were sold for £9 7s at a loss of $6\frac{1}{4}$ per cent , what per cent   would have been gained or lost by selling them for £11. 6s. ?

7.   By selling a watch for Rs 56, I gained $\frac{1}{7}$ of the cost price , what would be the gain or loss per cent , if I sell it for Rs   63 ?

8   By selling 12 lb. of tea for £1 11s. 6d , I gain 5 per cent. , what do I gain or lose per cent. by selling 50 lb of the same kind of tea for £6 9s. 2d ?

(c) To gain a certain p. c. on the whole transaction

Example 4   A shopkeeper having purchased 40 bags of rice at Rs 5 a maund, sold ½ of them at a loss of 20 per cent , at what price per maund should the remainder be sold so as to gain 5 per cent on the whole transaction ?   A bag contains 2½ maunds of rice

Sol   Cost price of 40 bags $= $ Rs. $40 \times \frac{5}{2} \times 5 = $ Rs 500

Total selling price he wants $= $ Rs $500 \times \frac{105}{100} = $ Rs. 525

. cost price of $\frac{1}{2}$ of rice $= $ Rs. 250
. selling price of first $\frac{1}{2}$ $= $ Rs $250 \times \frac{80}{100} = $ Rs 200
∴ selling price of 2nd $\frac{1}{2}$ $= $ Rs $525 - $ Rs. $200 = $ Rs. 325
. selling price per maund $= $ Rs. $325 - 50$
$$= \text{Rs } 6 \ 8a \quad \textbf{Ans.}$$

EXERCISE 111

1   A merchant bought goods for £75 and sold $\frac{1}{7}$ of them at a loss of 20 per cent   At what price should he sell the remainder so as to gain 20 per cent on the whole transaction ?

2   A shopkeeper purchased goods for £60   He sold $\frac{1}{3}$ at a loss of 10 per cent   At what price should he sell the remainder so as to clear a profit of 10 per cent on the whole transaction ?

3   A merchant having purchased 300 maunds of rice at Rs 8 a maund, sold $\frac{1}{4}$ of them at a loss of 10 per cent , at what price per maund should he sell the remainder so as to gain 5 per cent on the whole transaction ?

4   A shopkeeper having purchased 250 maunds of wheat at Rs 8 a maund, sold $\frac{1}{2}$ at a profit of 10 per cent and the remaining half at a loss of 15 per cent.   What did he gain or lose per cent. on the whole transaction ?

5   A merchant having 100 maunds of grain sold 50 maunds at Rs 9 per maund and thereby gained $12\frac{1}{2}$ per cent.   At what rate should he sell the remaining half so that he may gain $6\frac{1}{4}$ per cent. on the whole ?

6   A merchant bought 150 maunds of wheat and sold 50 maunds of them at Rs 9 1a 1$\frac{1}{2}$p. a maund and thus cleared a profit of $7\frac{1}{2}$ per cent.   At what rate should he sell the remaining 100 maunds so as to gain 10 per cent. on the whole transaction ?

7. If goods be purchased for Rs. 600 and $\frac{1}{3}$ be sold at a loss of 10 per cent, at what gain per cent should the remainder be sold so as to gain 20 per cent. on the whole transaction ?

8 A merchant having 150 maunds of grain sold 50 maunds at Rs 5 6a per maund and thereby gained $7\frac{1}{2}$ per cent. At what gain p. c. should he sell the remainder so that he may gain 10 per cent on the whole ? .

Example 5 A merchant purchased 111 maunds of wheat He sold 26 maunds of them at a profit of 10 per cent. and the remainder at a profit of $6\frac{1}{4}$ per cent. Had he sold the whole wheat at a profit of 8 per cent, he would have gained Rs 4 13a. 4$\frac{4}{5}p$ more Find the cost per maund.

Sol   Suppose the cost price per md. $=$ Re 1
∴ gain on 26 mds $=$ Rs $26\times\frac{10}{100}$ $=$ Rs. 2 6
  „   „ 85 „ $=85\times\frac{25}{4}\times\frac{1}{100}$ $=$ Rs 5·3125
   Total gain $=$ Rs 2 6 $+$ Rs. 5 3125 $=$ Rs 7 9125
∴ gain on 111 mds. at 8% $=$ Rs. $111\times\frac{8}{100}$ $=$ Rs. 8 88
 ∴ difference in gains $=$ Rs 8 88 $-$ 7 9125 $=$ Rs. ·9675
But the actual difference $=$ Rs 4 13a 4$\frac{4}{5}p$. $=$ Rs 4·8375
   which is 5 times of Rs. ·9675
 ∴ cost price per maund $=$ Rs. 5   **Ans.**

EXERCISE 112

1. A merchant bought 124 maunds of corn. He sold 24 maunds at a gain of 25 per cent and 100 maunds at a gain of 10 per cent   Had he sold the whole at a profit of $12\frac{1}{2}$ per cent, he would have got Rs 2 less , find the cost per maund.

2. A man bought 91 sheep and sold $\frac{4}{7}$ at a profit of 15 per cent., $\frac{4}{13}$ at a profit of 10 per cent, and the remainder at a profit of 8 per cent   Had he sold all the sheep at a profit of 16 per cent, he would have got Rs. 46 3a. 2·4p more   What did he pay for a sheep ?

3. A merchant having purchased 4000 maunds of rice, sold $\frac{1}{5}$ of them at a profit of 5 per cent. $\frac{1}{4}$ of them at a profit of 10 per cent., $\frac{3}{5}$ of them at a profit of 12

per cent and the remainder at a profit of 16 per cent   Had he sold the whole at a profit of 11 per cent., he would have got Rs. 728 more, find the cost per maund

4.   I bought 50 horses and sold 15 of them at a gain of 20%, 25 at a gain of 16% and the rest at cost price. Had I sold all at a gain of 18% I should have got Rs. 560 more. Find the cost price of each horse

5.   A merchant buys 5000 maunds of rice, one-fifth of which he sells at a profit of 5 per cent , one-fourth at a profit of 10 per cent and the remainder at a profit of 16 per cent   If he had sold the whole at a profit of 15 per cent he would have made Rs. 438. 12a. more   What was the cost of rice per maund ?

6   A merchant bought 630 maunds of ghee, sold $\frac{1}{2}$ at a gain of 5 per cent. and $\frac{1}{3}$ at a gain of 8 per cent   and the remainder at a gain of 12 per cent   Had he sold the whole at a gain of 10 per cent. he would have gained Rs. 115. 8a. more.   Find the cost price per maund.

(d) Mixture.

Example 6.   If 34 lbs. of tea at Re. 1. 2a a lb. be mixed with 29 lbs of tea at 12a. a lb , at what price per lb must the mixture be sold to gain 5% on the whole outlay ?

Sol   The total cost $= 34 \times 18a + 29 \times 12a.$
$$= 612a + 348a = 960a.$$

Add to it 5% gain, i e., $\frac{5}{100} \times 960 = 48a$

(34 + 29), i e., 63 lbs. are sold at 960 + 48 or 1008a.

. 1 lb. is sold at $\frac{1008}{63}$ or 16a , i.e , Re 1   Ans

EXERCISE 113.

1   A man buys 100 lbs. of tea at 5s per lb and 40 lbs. at 3s 6d. per lb , he mixes them and sells the mixture at a loss of $12\frac{1}{2}$ per cent , at how much per lb does he sell the tea ?

2.   A man buys 100 lbs of tea at Rs 2 per lb , 200 lbs at Re 1. 8a a lb and 300 lbs at Re. 1 per lb , he mixes them and sells the mixture at Re 1. 10a per lb. How much per cent. is his profit ?

3   Chicory is mixed with coffee bought at Re 1 4*a*
per lb in the proportion of 2 lbs of chicory to 5 lbs of
coffee   The mixture is sold at Re 1 2*a*   per lb. which
gives a loss of 6⅖ per cent.   Find the cost price per lb. of
the chicory

4   A grocer bought 50 mds of sugar at Rs 11 4*a*.
a md and mixed with it 70 mds. of sugar at a certain price.
If by selling the mixture at 5*a* a seer, he gains 17³⁄₁₅%, find
the price per md of the latter kind of sugar

5   A shopkeeper buys ½ cwt of tea at 4*s* 2*d* per lb.
and mixes it with tea at 7*s* 11*d* per lb   How much of the
latter must he mix with the former that he might sell the
mixture at 3*s*. 8*d* per lb and gain 20% on his outlay ?

6   How much water must be added to a maund of
milk so that by selling the mixture at cost price, there may
be 12⅓% profit ?

7   A man buys milk at 2½*d* per quart, dilutes it
with water and sells the mixture at 3*d* per quart   How
much water is added to each quart of milk if his profit is
60 p c ?

8   A man purchased milk at Rs 9 8*a*. a maund and
mixed some water in it , by selling the mixture at Rs. 10 a
maund, he gained 25%   How much water did he mix with
each maund of milk ?

(*e*) Different kinds of articles bought and sold at different rates

Example 7.   A man buys eggs at 5*a*. a dozen and an
equal number at 8*a* a score , he sells them at Rs 2 8*a* a
hundred and thus loses 4*a*.   Find the number of eggs he
had bought

Note   We can suppose any number of eggs he bought, but to
avoid fractions we should suppose the number of eggs equal to the
L C M of 12 and 20 which is equal to 60

Sol.   Cost of 60 eggs at 5*a* a dozen $=25a$

    „   60 „   at 8*a* a score $=24a.$

    total cost of 120 eggs $=49a.$

Selling price of 120 eggs at Rs. 2 8*a* a hundred$=48a.$

. loss on 120 eggs=$49a - 48a$. $=1a$

If the loss is $1a$ the number of eggs $= 120$

  „   „ $4a.$ „   „   „ $=120 \times 4$ $= 480$

  ∴   the required number $=480$ Ans

EXERCISE 114.

1 I bought a certain number of eggs at $4a$ a dozen and sold at $7a.$ a score   How many eggs did I buy, if I made a profit of Re 1 ?

2 I bought some mangoes at $4a.$ a dozen and an equal number at $5a$ a score. I then sold them at Re 1 $14a$ a hundred and thus cleared a profit of Re. 1, how many mangoes did I buy ?

3. A man buys apples at $5a$ a dozen and an equal number at $8a$ a score, he sells them at Rs. 2. $8a$ a hundred and thus loses $2a$, how many apples did he buy ?

4. I buy a number of mangoes at 2 for a pice and half the same number at 4 for 3 pice , at what price must I sell them to gain 20 per cent on the cost ? If my total profit is $14a$, how many did I buy ?

(*f*) Buying at a smaller price and selling at a greater price and *vice versa.*

Example 8   I sold a table at a profit of 10 per cent , had I sold it for Rs 2 more, 25 per cent. would have been gained   Find the cost price

Sol   Let the cost price of the table be$=$Rs 100

. selling price in the first case $=$Rs 110

and „   „   „   „ second case $=$Rs 125

            difference$=$Rs. $125 - $Rs. $110=$Rs   15

If the diff $=$Rs 15, the cost is supposed to be Rs 100

If the diff $=$Re 1 „   „   would be Rs $\frac{100}{15}$

  if the diff $=$Rs. 2 „   „   „   Rs $\frac{100}{15} \times 2=$Rs. $\frac{40}{3}$

  cost price$=$Rs 13 $5a$ $4p$ Ans

Example 9   A man sold an article at a profit of 10 per cent   If he had bought it at 10 per cent less and sold it for $\frac{1}{2}a$ more he would have got a profit of 25 per cent. Find the cost price.

Sol   Suppose the cost price $=100a$
∴ selling price in the first case $=110a$
Cost price in the second case $=100-10=90a.$

selling price in the second case $=\frac{125}{100}\times 90=\frac{2\cdot 5}{3}a.$
Diff between the two selling prices $=\frac{225}{2}-110=\frac{5}{2}a.$
But the actual difference $=\frac{1}{2}a$

∴ actual cost price $=\frac{2}{5}\times 100\times\frac{1}{2}=20a$

$=\text{Re. } 1 \ 4a.$ **Ans.**

EXERCISE 115.

1. I sold an article at a profit of 25 per cent , had I sold it for Rs 2 more, $37\frac{1}{2}$ per cent would have been gained. Find the cost price.

2 Alfred sold his watch at a profit of 20 per cent , had he sold it for Rs. 3 more, 32 per cent would have been gained   Find the cost price

3 Thomas sold his books at a profit of 5 per cent., had he sold them for Re. 1 $8a$ less, he would have lost 4 per cent.   Find his cost price

4 Bashir Ahmed sold his chairs at a loss of 15 per cent., had he sold them for Rs. 8 more he would have cleared a profit of 35 per cent., find the cost price.

5. $12\frac{1}{2}$ per cent more is gained by selling mangoes at $2a$. a dozen than $3a$ a score.   Find the cost price of mangoes.

6 A person sold an article at a profit of 5 per cent. If he had bought it at 5 per cent less than he did and sold it for Re 1 less he would have cleared a profit of 10 per cent   Find the cost price

7. A merchant sells a horse at a profit of 25 per cent If he had bought it at 4 per cent. less than he did and sold it for Rs. 11. $8a$ less he would have gained the same percentage of profit.   What did it cost him ?

8. Baldev sells his watch at a profit of 5 per cent. If he had bought it for 5 per cent. less than he did and sold it for 1s. less he would have gained 10 per cent.   Find his cost price.

9 A watch was sold at a profit of $12\frac{1}{2}$ per cent, had it been bought at 5 per cent. more and sold for 16s more, the seller would have gained $16\frac{2}{3}$ per cent What was the cost price ?

10 A man bought a watch and sold it at a loss of 10 per cent., if he had received Rs 15 more he would have gained 20 per cent What did the watch cost ?

(g) **Change in the total cost of articles owing to a change in their rates**

Example 10. A lb of tea and 4 lbs of sugar cost Rs. 3 $2a$, but if sugar rose 50 per cent. and tea 10 per cent. they would cost Rs. 3 $11a$, find the prices per lb. of tea and sugar.

Sol If both tea and sugar rose 50 per cent, the cost of 1 lb of tea and 4 lbs. of sugar would be

$$Rs 3. 2a. \times \tfrac{150}{100} = 75a.$$

But tea rose only 10 per cent

.. 40 per cent of the cost of 1 lb of tea $= 75a$ — Rs 3. $11a$.
$$= Re. 1 ,$$

. cost of 1 lb. of tea $= Re 1 \times \tfrac{100}{40} = Rs 2$ $8a$ **Ans.**

. cost of 4 lbs. of sugar $= Rs. 3$ $2a$ — Rs 2. $8a. = 10a$

.. cost of 1 lb. of sugar $= 2\frac{1}{2}a$ **Ans**

EXERCISE 116.

1. 2 lbs of tea and 5 lbs of sugar cost 7s 6d, but if sugar were to rise $33\frac{1}{3}\%$ and tea 20% they would cost 9s 4d, find the cost of the tea and sugar per lb

2 12 lbs of tea and 25 lbs. of coffee together cost Rs 43 $5a$. $4b$, but if the tea were to rise $2\frac{1}{2}\%$ and the coffee to fall $4\frac{1}{2}\%$ the same quantities would cost Rs. 42. $15a$ $4b$., find the price of the tea and the coffee per lb

3 One lb. of tea and 3 lbs of sugar cost Rs. 3, but if sugar were to rise 50 p. c and tea 10 p c they would cost Rs. 3. $8a$, find the prices per lb. of tea and sugar.

4   2 lbs. of tea and 5 lbs of sugar cost Rs. 2. 13a. but if sugar were to rise 25% and tea 10% they would cost Rs. 3 2a 9p , find the prices per lb. of tea and sugar

5.   10 mds of rice and 15 mds. of wheat cost Rs. 102 8a , but if rice were to rise 5% and wheat to fall 10% they would cost Rs. 99. 12a , find the prices per md of rice and wheat

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(h) Discount

Example 11   A tradesman marks his goods at 25 per cent above cost price and allows discount of $12\frac{1}{2}$ per cent. for cash payment. What profit per cent. does he make ?

Sol If the cost price is 100, his marked price = Rs 125.

But discount to the cash purchaser $\Big\}$ $\frac{12\frac{1}{2}}{100} \times 125 =$ Rs $15\frac{5}{8}$
= $12\frac{1}{2}$% on Rs 125

the reduced price he gets from the cash purchaser
= Rs 125 − Rs. $15\frac{5}{8}$ = Rs $109\frac{3}{8}$,

i. e , his gain per cent. = $109\frac{3}{8} - 100 = 9\frac{3}{8}$ **Ans**

Example 12. I buy goods for Rs. 39. 9a 4p. What price must I charge in order to make $10\frac{1}{2}$ p c on my capital, after allowing 15 p c. discount to the customers ?

Sol   Suppose I fix the price = Rs. 100

I receive Rs 100 − Rs. 15 = Rs 85

cost price = Rs $85 \times \frac{100}{110\frac{1}{2}} = \frac{85 \times 100 \times 2}{221} = \frac{1000}{13}$.

Now, if the C. P. is Rs $\frac{1000}{13}$ the fixed price is Rs. 100
∴ if the C P is Rs 39 9a 4p then the fixed price should be Rs. $\frac{13}{1000} \times 100$ of Rs 39 9a. 4p = Rs. 51 7a 4p **Ans.**

EXERCISE 117

1.   A publisher sells books to a retail dealer at Rs. 5 a copy, but allows 25 copies to count as 24   If the retailer sells each of the 25 copies at Rs. 6. 12a., what profit per cent does he make ?

2. A tradesman marks his goods 30 p c above the cost price, but he allows his customers 10 p. c off his bill and thus gains Rs. 3 3a on the goods , find the amount of the bill

3 I buy goods for Rs 5040 and incur 10 per cent. expenses , what must I charge in order to make 10 per cent. profit on my capital after allowing 10 per cent discount ?

4 A trader allows a discount of 5 per cent to his customers What price should he mark on an article, the cost price of which is Rs 712 8a so as to make a clear profit of 33$\frac{1}{3}$ per cent on his outlay ?

5. A manufacturer marks an article for sale at 50 per cent over its cost price, but gives the retail dealer 13 articles to the dozen and allows a discount of 12$\frac{1}{2}$ per cent. for cash What does he gain per cent. on the cost of the article, when he gets cash price ?

(i) To sell a number of articles at a loss or gain equal to the selling price of a part of them

Example 13. I bought 20 maunds of rice for Rs 62 8a. and sold them at a loss equal to the selling price of 5 maunds Find the selling price per maund

Sol   Suppose S P of 20 mds. $=$ Rs 100
.   Loss $=$ Rs $\frac{100}{20} \times 5$ $=$ Rs 25
and cost price $=$ Rs 100 + 25 $=$ Rs 125
But the actual cost price $=$ Rs 62. 8a.
If C. P is Rs 125, the S P. $=$ Rs 100
    „   „ Re. 1   „ „ $=$ Re $\frac{100}{125}$
    „   „ Rs. $\frac{125}{2}$ „ „ $=$ Rs $\frac{100}{125} \times \frac{125}{2} =$ Rs 50
.   selling price per maund $=$ Rs 50 $\div$ 20 $=$ Rs. 2$\frac{1}{2}$. Ans.

EXERCISE 118.

1 I bought 50 maunds of sugar for Rs 662. 8a. and sold it at a loss equal to the selling price of 3 maunds Find the selling price per maund.

2. I bought 80 chairs for ~~Rs.~~ 690 and sold them at a loss equal to the selling price of 12 chairs , find the selling price per chair

3 A merchant bought 56 cows for Rs 2320. 8*a*. and sold them at a gain equal to the selling price of 5 cows. Find the selling price per cow

4. I bought 50 chairs for Rs. 385 and sold them at a loss of as much money as I got for 5 chairs. Find the selling price per chair.

5 I bought 30 maunds of rice for Rs 180 7*a*. 6*p*. and sold them at a loss of as much money as I got for 3 maunds , find the selling price per maund

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(*j*) To get a percentage of profit equal to the cost price

Example 14. A person sold his watch for Rs 24 and got a percentage of profit equal to the cost price , find the cost price.

Sol. . cost price $\times \dfrac{\text{cost price} + 100}{100} =$ selling price

.. cost price $\times \dfrac{\text{cost price} + 100}{100} =$ Rs 24.

or $(\text{cost price})^2 + 100 \text{ cost price} =$ Rs. 2400.

Completing the square we have

$(\text{cost price})^2 + 100 \text{ cost price} +$ Rs. 2500 = Rs. 4900

Or cost price + Rs 50 = Rs. 70   [by square root.

. cost price = Rs. 20. **Ans.**

It follows therefore that when a percentage of profit equal to the cost price is given we have the

Rule. $\sqrt{\text{S. P.} \times 100 + (50)^2} - 50 =$ cost price.

EXERCISE 119.

1. An article when sold for Rs 96 gives a certain profit per cent. which is equal to the cost price , find the cost price.

2. A person sold his watch for Rs 85. 4. *a*. It yielded a certain profit per cent which is equal to the cost price , find the cost price.

3. A horse when sold for Rs 39 gives a certain gain p c which is equal to the cost price , find the cost price

4. An article when sold for Rs. 75 gives a percentage of profit equal to the cost price , find the cost price

5 A table when sold for Rs. 17 4*a*. yields a percentage of profit equal to the cost price , find its prime cost.

(*k*) Miscellaneous types

Example 15 A merchant sold a quantity of wheat at Rs 5. 8*a* 2⅖*p* a maund and cleared a profit of 2 per cent. , what is the quantity of wheat sold, if the total profit is Rs. 63 ?

Sol Selling price of one md = Rs 5 8*a* 2⅖*p* = Rs.$\frac{441}{80}$
and gain = 5 per cent
∴ cost price ,, ,, = Rs $\frac{100}{105} \times \frac{441}{80}$ = Rs $\frac{21}{4}$
∴ gain on one maund = Rs $\frac{441}{80}$ — Rs $\frac{21}{4}$ = Rs $\frac{21}{80}$
But the total gain = Rs 63
quantity sold = 63 ÷ $\frac{21}{80}$ = 240 mds Ans

EXERCISE 120.

1. A merchant sold a quantity of wheat at Rs. 13. 4*a* a maund and thus cleared a profit of 6 per cent. What is the quantity of sugar sold if the profit is Rs. 16 8*a* ?

2. A merchant by selling a quantity of rice at Rs 7. 8*a*. a maund sustained a loss of $7\frac{9}{13}$ p c find the quantity sold if his total loss be Rs 73 2*a*

3. A man sold a number of sheep at Rs. 15 each with a profit of 25 per cent. and a total gain of Rs 180. Find how many sheep did he sell

4. A merchant buys a certain number of yards of cloth at 5*s* a yard , he sells half of them at 6*s*. 4*d*. per yard, but the remainder, being damaged, he can only dispose of at a loss of 10 per cent under the invoiced price. His gain on the transaction is £14. 3*s*. 4*d*. Find how many yards were purchased.

Example 16   A person bought two horses for Rs. 550, sold one of them at a profit of 10 per cent and tne other at a loss of $8\frac{1}{3}$ per cent. He neither gained nor lost in this business , find the cost price of each horse

Sol.   gain on one horse   = loss on the other,

$$\frac{\text{cost price of one}}{\text{cost price of the other}} = \frac{8\frac{1}{3}}{10}$$

or C. P. of one C P of the other $= 8\frac{1}{3}$ 10

$$= \tfrac{25}{3} \quad 10$$

$$= 5 \quad 6$$

∴ cost price of the first horse $= \frac{5}{11} \times 550 = $ Rs 250}
and „   „   „ second horse $= \frac{6}{11} \times 550 = $ Rs. 300} **Ans.**

Example 17   A person bought two articles for Rs 270   He sold one of them at a gain of $12\frac{1}{2}$ p c and the other at a loss of 10 per cent   It was found that both the articles had fetched the same price   Find the cost price of each article

Sol   Suppose Rs 100 is the cost price of one article, then the S P of the article $=$ Rs. $112\frac{1}{2}$

the S P of the second article is also $=$ Rs $112\frac{1}{2}$
and the loss on it $= 10$ p c

C P of the second article $=$ Rs $\frac{100}{90} \times \frac{225}{2} = $ Rs 125
ratio between the prices $= 100 \quad 125 = 4 \quad 5$

∴ C P of the first article $=$ Rs $\frac{4}{9} \times 270 = $ Rs 120}
and C. P. „ „ second „ $=$ Rs $\frac{5}{9} \times 270 = $ Rs. 150} **Ans**

EXERCISE 121.

1. A person bought two horses for Rs   560, sold one of them at a loss of 5% and the second at a gain of $6\frac{1}{2}$ % He neither gained nor lost in this transaction.   Find the cost price of each horse

2   A person bought two horses for Rs 3600. The large one he sold at a loss of 5 % and the smaller one at a profit of 7 %   He neither gained nor lost in this transac tion   Find the cost price of each horse.

3. The cost price of two clocks is Rs 135   One was sold at a profit of $12\frac{1}{2}\%$ and the other at a loss of 10%, but both the clocks fetched the same price , find the cost price of each clock.

4 A merchant has a horse and a cow together worth Rs 225   He sells the horse at a loss of 20% and the cow at a gain of 60%   It was found that both the animals had fetched the same price , find the price of each

5 A person bought two articles for Rs 325   He sold the one at a gain of 5 p c. and the other at a loss of 10 p c He found that both of them had fetched the same price Find the price of each article

6 A merchant bought two cows for Rs 520   He sold one of them at a loss of 15 p. c and the other at a gain of 36 p c   It was found that both the cows had fetched the same price , find the price of each cow.

Example 18. A person has goods worth £300 , he sells $\frac{2}{3}$ of them so as to lose 10 p c. By how much per cent should he raise the selling price in order to gain 10 per cent. on the whole ?

Sol   S P of $\frac{1}{3} = £\frac{90}{100} \times 100 = £90$

but total S P. $= £\frac{110}{100} \times 300 = £330$

remaining S P $= £330 - £90 = £240.$

In case he sells the remaining goods also at a loss, the remaining S P $= £\frac{90}{100} \times 200 = £180$

diff in the rem. selling prices $= £240 - £180 = £60$

∴ excess per cent $= \frac{60}{180} \times 100 = 33\frac{1}{3}$ **Ans**

EXERCISE 122

1 A person has goods worth Rs 75 , he sells $\frac{1}{3}$ of them at a loss of 4 p c   By how much per cent should he raise that selling price in order to gain 4 p c. on the whole ?

2. A person has goods worth Rs. 450   He sells $\frac{2}{3}$ of them at a loss of 4 p c. By how much per cent should he raise that selling price in order to gain 10 p c. on the whole ?

3   A person pays 10 p c expenses on the goods and allows 10 p. c. to his customers on cash payment   By how much should he raise the selling price from the cost price in order to get a profit of 8 p c on his outlay ?

4.   A man having bought a quantity of goods for Rs 1500, sells $\frac{1}{3}$ at a loss of 4 p. c , by how much per cent. must he raise that selling price in order that by selling the remainder at the increased rate, he may gain 4 p. c. on the whole transaction ?

Example 19   A person bought some paper at 8*a*. per 5 quires and by selling it got on every 32 quires, a profit equal to the selling price of 8 quires , find the selling price per quire.

Sol   Since by selling 32 quires the profit $=$ the selling price of 8 quires,

. it is evident that the selling price of 24 quires $=$ the cost price of 32 quires

$\therefore$ the cost price of 32 quires $= \frac{8}{5} \times 32 = \frac{2 5 6}{5} a.$

$\therefore$ the selling price of 24 ,, $= \frac{2 5 6}{5} a.$

,,   ,,   ,, of 1 quire $= \frac{2 5 6}{5} \times \frac{1}{2 4} a.$

$= 2 \frac{2}{1 5} a$ **Ans.**

EXERCISE 123.

1.   A person bought some knives at the rate of 4 for 15*a*. and by selling them got on every 40 knives a profit equal to the selling price of 10 knives. Find the selling price of each knife.

2   A person bought some books at the rate of Rs 5 10*a*. per dozen and got a profit equal to the selling price of 5 books on the sale of every 30 books   Find the selling price of each book.

3.   A person bought some chairs at Rs $3\frac{1}{2}$ per chair and sold 20 of them at a loss equal to the selling price of 8 chairs.   Find the selling price of each chair

4.   A person bought some bottles at 40*s* per dozen. How should he dispose them of in order to get, on a capital of every £100 invested, a profit equal to the selling price of 5 dozen bottles ?

Example 20. A person bought two horses at the same price He sold one of them at a profit of 12 p. c. and for the other he received Rs 15 more than he received for the first By this bargain he got 15 p c profit on the whole. What did each horse cost him ?

Sol.　　　15 p. c —12 p. c.＝3 p c

　　　　　　　　　　　＝Rs $\frac{3}{100}$ of the real price,

　　　$\frac{3}{100}$ of the real price ＝Rs 15,

　∴ real price＝$\frac{100}{3}\times 15$ ＝Rs 500,

　∴ cost of each horse is Rs 250. **Ans**

Example 21 A merchant bought two horses. The cost price of one of them is $\frac{4}{3}$ times that of the other. He sold the former at a gain of 15 p c. and the latter at a loss of 5 per cent He got Rs 149 in all by this transaction. What did each horse cost him ?

Sol Suppose he bought the second horse for Re. 1

.. cost of the first horse　　　＝Rs $\frac{4}{3}$

. cost of both the horses＝$1+\frac{4}{3}$＝Rs. $\frac{7}{3}$

Now the S P of the second　　＝Re. $\frac{95}{100}\times 1$＝Re $\frac{19}{20}$

and　　,,　　,,　　,, first　　　＝Rs $\frac{115}{100}\times\frac{4}{3}$＝Rs $\frac{23}{15}$

selling price of both the horses＝$\frac{19}{20}+\frac{23}{15}$　＝Rs $\frac{149}{60}$.

Now if the S P is Rs $\frac{149}{60}$, the C P.＝Rs $\frac{7}{3}$

if　　,,　　S P is Re. 1　　　　,,　　＝Rs $\frac{7}{3}\times\frac{60}{149}$

,,　　,,　　S P. is Rs 149　　　,,　　＝Rs. $\frac{7}{3}\times\frac{60}{149}\times 149$

　　　　　　　　　　　　　　　　　　＝Rs 140.

Now divide Rs. 140 in the ratio 4 3,

　　the cost of the horses＝Rs. 80 and Rs. 60　　**Ans**

EXERCISE 124

1　　A person bought two motor cars, each for the same price He sold one of them at a profit of 10 p. c and the other for Rs 480 more than the first He thus made a profit of $17\frac{1}{2}$ % , find the price of the motor cars

2　　A person having bought two houses at the same price, sold one of them at a profit of 15 % and for the other he received Rs 320 more than what he received for the first He thus got a profit of 18 % on the whole. Find the price of each house

3　　A merchant bought two horses. The price of one

F 25.

of them exceeds by $\frac{1}{4}$ the price of the other. He sold the horse of the higher price at a loss of 12½% and the other at a gain of 5%. Thus he received Rs. 428. 12a in all, find the price of each horse

4. A person bought two houses. The price of one of them was $\frac{3}{8}$ of that of the other. By selling the larger house at a gain of 5% and the other at a gain of 12 p. c., he got Rs. 3773 in all. Find the price of each house.

5. I bought 200 books and sold 80 of them at a profit of Rs. 20 and the remaining for Rs 1000 gaining thereby 42 p c on the whole, find the price of a book.

6 A person bought two clocks The cost price of one of them exceeds by $\frac{1}{4}$ the cost of the other. He sold the better one at a gain of 10 p c and the other at a gain of 7½ p c and thus got Rs 98 in all, find the cost price of each.

REVISION

§8. The examples given in the preceding exercises sum up the whole chapter and now we close it with questions of various types.

EXERCISE 125

1. A tradesman by selling an article for Rs. 35 1a. gained 10 per cent, find the cost price

2. A merchant buys sugar at 7 annas 6 pies per seer and sells it at 6p. per chk Find the gain per cent

3 By selling eggs at 3 for an anna I gain 5 per cent. on my outlay, what do I gain or lose per cent. by selling them at 7a. a score?

4. A wholesale dealer sells an article at a gain of 20 per cent to a retail dealer, who selling it for Rs 12 gains 20 per cent., what did the wholesale dealer pay for it?

5 A tradesman gains 6 per cent by selling an article for Rs 795, how much per cent. does he lose by selling the same for Rs 690?

6. A merchant buys some cloth at such a price that by selling it at Rs 4 6a per yard he will gain 5 per cent. on his outlay, what percentage will he gain or lose, if the cloth be sold at Rs. 3 14a per yard?

7 I bought 50 maunds of wheat for Rs 262. 8a. and was obliged to sell it at a loss of as much money

as I received for $2\frac{1}{2}$ maunds of wheat. Find the sale price per maund

8. A person sold his watch for Rs 50 instead of Rs 35 Thus he got 45 p c more profit , find the cost of the watch.

9. A person bought two watches for Rs 106 He sold one of them at a profit of 25 p. c and the other at a loss of $4\frac{1}{6}$ p c Both the watches fetched the same price ; find the cost price of each watch

10 Eggs were bought at $4a$ a dozen and sold at $7a$ a score , find the number of eggs bought if the gain is Re 1

11 By selling goods for Rs 144, the profit per cent. is equal to the cost price , find the cost price.

12 A person has goods worth £275, he sells $\frac{1}{5}$ of them so as to lose 20 p c By how much per cent should he raise the selling price in order to gain 20 per cent on the whole ?

13 A tradesman was making a profit of 26 per cent. on his outlay when he was selling an article at 1s $3\frac{3}{4}d$ The cost price has since been reduced by a penny and he has taken a penny off the selling price What percentage of profit is he now making ?

14. A grocer sold a quantity of tea at Re. 1 $2a$ a lb. with a profit of $12\frac{1}{2}$ per cent and a total gain of Rs 6. Find how many lbs of tea did he sell

15 How much should I have to pay for a watch which costs £3 4s to make, if the maker sold it to a dealer at a profit of 25 p c and the dealer sold it to me at a profit of $12\frac{1}{2}$ per cent ?

16 If a reduction of 20 p c. were made in the price of eggs, it would enable a person to obtain 54 more for 21 shillings Find the present price.

17 A trader allows a discount of 5 per cent. for cash payment , how much per cent above the cost price must he mark his goods to make a profit of 10 per cent ?

18. A coal merchant had 150 tons of coal, of which he sold 50 tons at Rs 27 per ton, and found that he was gaining $12\frac{1}{2}$ per cent At what rate must he sell the remainder so that he may gain 10 per cent. on the whole ?

19   A man buys 27 sheep for Rs 90 and sells 12 at a loss of 5 per cent , at how much price must he sell the remainder, in order that he may gain $2\frac{1}{2}$ per cent. on the whole ?

20.   Find the cost price of rice when an additional profit of $2\frac{1}{2}$ per cent raises its price by $1a.$ $7p$ per maund.

21   A man bought a number of oranges at 3 for an anna and an equal number at 2 for an anna.   At what price per dozen should he sell them to make a profit of 20% ?

22   By selling 5 yards of cloth for Re. 1, a man gains a profit of 15 per cent. , what will be the gain or loss per cent , if he sells the same cloth at 6 yards for Re 1 ?

23   A quantity of tea is sold at Re 1 $2a.$ per lb. at a profit of $12\frac{1}{2}$ per cent. and a total gain of Rs. 10   Find how many pounds of tea were sold.

24.   An article when sold at a gain of 20 per cent. yields Rs 7. $8a$ more than when sold at a loss of 4 per cent , find its prime cost

25   A piece of cloth is sold for Rs. 54 at a profit of $12\frac{1}{2}$ per cent   If it had been sold at Rs 2 $6a.$ a yard, the profit would have been Rs. 9 , find how many yards are there in the piece.

26.   A merchant sells some goods to a retailer at 50 per cent profit , but the latter failing he only receives $8a.$ a rupee , find the merchant's gain or loss per cent.

27   A tradesman by means of a false balance defrauds 10 per cent in buying goods and 10 per cent. in selling. What per cent. does he gain on his outlay by his dishonesty ?

28.   A merchant having purchased two horses for Rs 1350, sold one at a gain of $7\frac{1}{2}$ per cent and the other at a loss of 6 per cent. , he found that he had neither gained nor lost in this transaction   Find the cost price of each horse.

29   A horse was sold at a loss of 10 per cent , if it had been sold for Rs 35 more 4 per cent would have been gained.   Find the cost price.

30   A man sells two horses for Rs 1955 each   On one he gains 15 per cent. and on the other he loses 15 p. c. Find his total gain or loss

31   A merchant sells sugar to a customer, using false weights and thereby gains $11\frac{1}{9}$ per cent. on his outlay. What weight does he substitute for a seer ?

32   By selling a watch at a gain of 10 per cent a man got Rs 15 more than half its cost price. What is the price of the watch ?

33   By selling 4 dozen mangoes for Rs. 13, it was found that $\frac{8}{10}$ths of the outlay was gained, what ought to have been the retail price per mango in order to have gained 60 per cent ?

34   Two articles are sold for Rs. 396 each, one at a gain of 10 per cent. and the other at a loss of 10 per cent Find the gain or loss per cent on the whole

35.   A person bought some slates at $3a$ per slate Out of them 3 dozen were found broken and the remaining he sold at $4a$ per slate and thus cleared a profit of Rs 3 How many slates did he buy ?

36   I bought two horses for Rs 385 and sold one of them at a gain of $12\frac{1}{2}$ per cent and the other at a loss of Rs $13\frac{1}{3}$ per cent I neither gained nor lost in this transaction, find the cost price of each ?

37   A grocer bought 30 lbs of tea at $12a$. per lb and 50 lbs at Re. 1 $2a$. per lb At what rate per lb. must he sell the whole to gain 10 per cent at least of the returns ?

38   By selling a horse and a cow for Rs 310, a person gained 15 per cent. on the horse and 10 per cent on the cow. Had he sold the cow at a gain of 15 per cent and the horse at a gain of 10 per cent he would have realized Rs 308 $12a$ Find the cost price of each

39   Rs 61 $4a$. was spent in buying apples at Re 1. $7a$ $4p$ a score When they came to be sold, part of them was worthless, but the rest on being sold at a profit of 30 per cent realized Rs 68. $4a$; how many scores were there of worthless ones ?

40   If I sell a horse for Rs. 132 and a cow for Rs. 88 I gain 10 per cent on the whole But if I sell the horse for Rs 150 and the cow on original price, I gain 15 per cent Find the original cost price of each.

CHAPTER XXIII

PROPORTIONAL PARTS AND PARTNERSHIP.

I. PROPORTIONAL PARTS

§1. To divide a given quantity into *proportional parts* is to divide it into parts proportional to the given numbers

Note *Again* means a second time. *As much again* means as much once and as much a second time. *i e*, twice as much. *Half as much again* means as much once and half as much a second time, *i. e*, $1\frac{1}{2}$ times *as much*

The following examples will illustrate the process —

Example 1 Divide Rs 355 4a among A, B and C so that their shares may be proportional to 3, 4 and 5.

Sol. The sum of the parts $= 3 + 4 + 5 = 12$

Therefore the whole sum Rs 356 4a is to be divided into 12 equal parts of which A shall have 3 parts, B 4 and C 5

∴ 12 parts $=$ Rs 356 4a.

∴ 1 part $=$ Rs. 29 11a.

∴ A's share $=$ Rs 29 11a $\times$ 3 $=$ Rs. 89 1 a.
B's share $=$ Rs. 29. 11a $\times$ 4 $=$ Rs. 118 12 a. } Ans
C's share $=$ Rs 29. 11a $\times$ 5 $=$ Rs 148 7 a.

Example 2 Divide Rs. 279 among A, B, C in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$ respectively.

Sol A B C

or $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{5}$ } Multiply each term by 30, the L C. M.
or   15   10   6 } of the denominators 2, 3, 5.

$15 + 10 + 6$, *i e*, 31 parts $=$ Rs. 279

∴   1 part $=$ Rs. 9

∴ A's share $=$ Rs $9 \times 15 =$ Rs 135
B's share $=$ Rs. $9 \times 10 =$ Rs 90 } Ans.
C's share $=$ Rs. $9 \times 6 =$ Rs 54

Example 3 Divide Rs. 420 among A, B and C so that A may receive twice as much as B and B twice as much as C.

Sol. Let C's share be=1.

. B's share=2 [since B gets twice as much as C]

and A's share=4 [since A gets twice as much as B]

hence the sum of the parts=$1+2+4=7$

∴ 7 parts   =Rs 420
∴ 1 part   =Rs. 60

. A's share   =Rs. 60×4=Rs 240 ⎫
  B's share   =Rs. 60×2=Rs 120 ⎬ **Ans**
  C's share   =Rs 60×1=Rs. 60 ⎭

Example 4. Divide Rs. 381 3a among A, B and C so that A's share ˙ B's share as 5 6 and B's share C's share as 3 4

Sol We can put the conditions given above at once as under —

A   B . C   Now find out either C's share
5   6   or A's share by the unitary method
     3   4

If B gets 3, C gets 4.

If B gets 1, C gets $\frac{4}{3}$

If B gets 6, C gets $\frac{4}{3}\times6$=8,

hence A B C=5 6 8 ,

now proceed as in previous examples.

Example 5 Divide Rs 375 among A, B and C so that if Rs 4, Rs 5, Rs 6 be subtracted from their respective shares, the remainder, may be in the ratio 3 , 4 5.

Sol. $4+5+6=15$, Rs $375-$Rs. 15=Rs 360.

Now divide Rs. 360 in the ratio 3 4 5 and then add Rs 4, 5, 6, respectively to the shares

Example 6 Divide Rs 1680 among A, B and C so that A may receive $\frac{1}{3}$ as much as B and C together and B $\frac{2}{3}$ as much as A and C together

Sol *First condition*	**Second condition**
If B and C get Re. 1,	If A and C get Re 1,
A gets Re $\frac{1}{3}$.	B gets Re $\frac{2}{3}$.
Total in this case=$1\frac{1}{3}$=$\frac{4}{3}$	Total in this case=$1\frac{2}{3}$=$\frac{5}{3}$.

Now find the share of A in terms of the 2nd condition.

If the total is $\frac{9}{8}$, A gets $\frac{1}{8}$

,, ,, is 1, A gets $\frac{1}{8} \times \frac{8}{9} = \frac{1}{9}$

,, ,, is $\frac{5}{3}$, A gets $\frac{1}{9} \times \frac{5}{3} = \frac{5}{18}$,

$\therefore$ C will get $1 - \frac{5}{18} = \frac{13}{18}$

Now the proportional parts are $\frac{5}{18}$, $\frac{2}{3}$ and $\frac{13}{18}$,

<div align="center">or 5, 12 and 13</div>

Now the sum of the parts $= 5 + 12 + 13 = 30$

    30 parts = Rs 1680,

     1 part = Rs 56.

$\therefore$ A's share = Rs. 56×5 = Rs 280 ⎞

   B's share = Rs $56 \times 12 =$ Rs 672 ⎬ **Ans.**

   C's share = Rs $56 \times 13 =$ Rs 728 ⎠

Aliter A's share $(B + C)$'s share = 1   5

               B's share $(A + C)$'s share = 2 · 3

Now divide Rs 1680 (i) in the ratio 1 5

                    (ii) in the ratio 2 3

$\therefore$ A's share = $\frac{1}{6}$ of Rs 1680   = Rs 280 ⎞

   B's share = $\frac{2}{5}$ of Rs. 1680   = Rs 672 ⎬ **Ans**

$\therefore$ C's share = Rs 1680 $-$ (Rs 280 + Rs 672) = Rs 728 ⎠

Example 7   Divide 184 into three parts such that 4 times the first $= 5$ times the second $= 8$ times the third

Sol. Let 4 times the first $= 5$ times the second $= 8$ times the third $= 1$, then

first part $= \frac{1}{4}$, 2nd part $= \frac{1}{5}$ and 3rd part $= \frac{1}{8}$

$\therefore$ first part 2nd part · 3rd part $= \frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{8}$

                            $= 10$ 8 5.

Now proceed as in previous examples.

Example 8   Divide Rs 82 among 5 men, 8 women and 10 boys in such a way that a woman is to receive twice as much as a boy and a man as much as a woman and a boy together ; what do the women receive ?

Sol   Since one woman receives as much as 2 boys,

. 8 women receive as much as 16 boys.

Again one man receives as much as one woman and one boy,

∴ 5 men receive as much as 5 women and 5 boys,

　　　i. e as much as 10 boys and 5 boys,

　　　i. e , as much as 15 boys

∴ 5 men = 15 boys and 8 women = 16 boys

　men's share ' women's share boys' = 15　16　10

Now proceed as in previous examples.

Example 9　685 coins consist of guineas, half-sovereigns and half-crowns , the *values* of guineas, half-sovereigns and half-crowns are in the ratio of 15　8　6 , find the number of each coin

Sol.　Ratio in the values = 15 guineas half-sovereigns worth 8 guineas half-crowns worth 6 guineas.

$$\text{No of coins} = 15 \quad \frac{8 \times 21}{10} \quad \frac{6 \times 21}{2\frac{1}{2}}$$

$$= 15 \quad \tfrac{84}{5} \quad \tfrac{252}{5}$$
$$= 75 \quad 84 \quad 252$$
$$= 25 \quad 28 \quad 84$$

Now $25 + 28 + 84 = 137$ and $685 \div 137 = 5$

$$\left.\begin{array}{l}\text{guineas} \qquad = 5 \times 25 = 125 \\ \text{half sovereigns} = 5 \times 28 = 140 \\ \text{half-crowns} \quad = 5 \times 84 = 420\end{array}\right\} \quad \text{Ans}$$

Example 10　How many eight-anna pieces, four-anna pieces and two-anna pieces the *numbers* of which are in the ratio of 3　5 : 4 are together worth Rs 156 ?

Sol　*Values* of three groups of coins are as

3 eight-anna pieces : 5 four-anna pieces . 4 two-anna pieces or as 12 two-anna pieces 10 two-anna pieces . 4 two-anna pieces or as 12 10 4 = 6　5 : 2

　the amount in 8 anna bits = Rs $\frac{156}{13} \times 6 =$ Rs　72
　the amount in 4-anna bits = Rs $\frac{156}{13} \times 5 =$ Rs　60
　the amount in 2 anna bits = Rs $\frac{156}{13} \times 2 =$ Rs. 24.

Therefore there are $72 \times 2 = 144$ eight-anna, $60 \times 4 = 240$ four-anna and $24 \times 8 = 192$ two anna pieces　**Ans.**

Example 11. Total salary of A, B and C is Rs. 350 , if they spend 75%, 80% and 56% of their salaries respectively, their savings are as 10   12   33 , find their respective salaries

Sol A's saving $= 100 - 75 = 25$% of his salary

       Bs' saving $= 100 - 80 = 20$% „   „   „

       C's saving $= 100 - 56 = 44$% „   „   „

    $\frac{25}{100}$ of A's salary $\frac{20}{100}$ of B's $\frac{44}{100}$ of C's

                              $= 10$ 12   33

$i. e.$, (1) $\frac{1}{4}$ of A's salary $\frac{1}{5}$ of B's salary $= 10$ 12

and   (2) $\frac{1}{5}$ of B's salary $\frac{11}{25}$ of C's salary $= 12$ 33

From (1) we get $\frac{1}{4} \times 12$ of A's salary $= \frac{1}{5} \times 10$ of B's salary

         or 3 times A's salary $= 2$ times B's salary

           A's salary B's salary $= 2$ 3

From (2) we get $\frac{1}{5} \times 33$ of B's salary $= \frac{11}{25} \times 12$ of C's salary

      or $\frac{33}{5}$ times B's salary $= \frac{132}{25}$ times of C's salary

       B's salary C's salary $= \frac{132}{25}$ $\frac{33}{5}$

      or 132×5 33×25 $= 4$ 5.

The ratios are therefore as follows —

         A B C

         2   3

              4   5

We can now proceed by the method already explained in Ex 4

Example 12. A's present age is to B's as 8   5 and 20 years ago it was as 12   5   Find the present age of each.

Sol A's present age to B's present age is as 8   5 , and 8 is $2\frac{2}{3}$ times $(8-5)$ Similarly

A's former age was to B's former age as 12   5 , here 12 is $1\frac{5}{7}$ times $(12-5)$.

It follows therefore that

A's present age $= 2\frac{2}{3}$ $(A$'s $- B$'s present ages$)$

and A's former age $= 1\frac{5}{7}$ $(A$'s $- B$'s former ages$)$

Since the difference of the ages of two persons is always the same though the ratio of the ages is always varying,

．A's present age　A's former age$=2\frac{2}{3}$　$1\frac{5}{7}$

or $2\frac{2}{3}$ times A's former age$=1\frac{5}{7}$ times A's present age

or A's former age$=(1\frac{5}{7}-2\frac{2}{3})$ or $\frac{9}{14}$ of his present age

but A's former age $=$A's present age-20 years

．A's present age-20 years$=\frac{9}{14}$ of A's present age

$(1-\frac{9}{14})$ of A's present age$=20$ years

Or　$\frac{5}{14}$　,,　　,,　　,,　$=20$ years

$$\left.\begin{array}{ll} \text{A's present age} & =20\times\frac{14}{5}=56 \text{ yrs.} \\ \text{A's age}\quad B\text{'s age} & =8\quad 5 \\ B\text{'s present age} & =\frac{5}{8}\text{ of }56=35\text{ yrs.} \end{array}\right\}\text{Ans.}$$

Example 13　Four years ago the ages of A and B were as 13　9 and eight years hence they will be as 4　3 , find their present ages.

Sol.　In $4+8$, $i\,e$　12 years the ratio would be changed from 13　9 to 4　3.　Note　$13-9=4$ and $4-3=1$

Now reduce the second ratio to its equivalent ratio so that the difference between the terms are the same as that between the terms of the first $i\,e$, 4.

$4\quad 3=16\quad 12.$

If A's age 4 years before had been　13　years it would have taken only $16-13$, $i\,e$, 3 years to become 16

$3\quad 12\qquad 13\quad A$'s age 4 years ago

or A's age 4 years ago $=\dfrac{12\times 13}{3}=52$ years.

$$\left.\begin{array}{ll} \text{A's age}=52+4 & =56 \text{ years.} \\ \text{and }B\text{'s age}=\frac{9}{13}\times 52+4 & =40 \text{ years} \end{array}\right\}\quad\text{Ans.}$$

EXERCISE 126

1.　Divide Rs 526. 8a. into parts proportional to 2, 3 and 7.

2　Divide Rs　487　8a　into parts proportional to $\frac{1}{3}$ and $\frac{1}{4}$

3　Divide Rs　626　9a　into 4 parts in the proportion of $\frac{1}{4}$, $1\frac{1}{2}$, ˙75, ˙625.

4   The magnitude of the three angles of a triangle are in the ratio of 3   5   1.   Find the number of degrees in each angle

5   A number is divided into 3 parts in the proportion of 3 , 5 and 7 , if the first part is equal to 24, find the number.

6   A sum of money was divided into parts proportional to 2 1, 2 5 and 3 2 , the smallest part was Rs 10. 8*a*., what was the sum divided ?

7   Divide Rs. 738 among *A*,*B* and *C* so that if their shares be diminished by Rs 7, Rs 5 and Rs 6, respectively the remainders may be in the ratio of 5   6   7

8   Divide Rs 586 among *A*, *B* and *C* so that if Rs. 15, Rs 20 and Rs 49 respectively be added to their shares the total may be in the ratio of 4   5   7.

9   Divide Rs 517 among *A*, *B* and *C* so that *A*'s share   *B*'s share is equal to 4   5 and *B*'s share   *C*'s share is equal to 3   4

10.   Divide Rs 126   6*a*. among *A*, *B* and *C* so that *A* may have $1\frac{1}{2}$ of *B*'s share and *B* double of *C*'s share.

11   Divide Rs 450 among three persons   so that first man's share , second man's share $=4$ 5 and second man's share   third man's share $=5$ 6

12   Divide Rs 252 8*a*. among *A*, *B* and *C* so that *A* may get twice as much as *B* and *B* thrice as much as *C*.

13   Divide £250 among *A*, *B* and *C* so that *A*'s share   *B*'s share $=1$ $\frac{3}{4}$ and B's share   C's share$=\frac{3}{4}$ $\frac{5}{6}$

14.   Divide Rs 1125 among *A*, *B*, *C*, and *D* so that *A* may receive twice as much as *B*, thrice as much as *C* and 4 times as much as *D*

15.   Rs 350 is divided among *A*, *B* and *C* , *B*'s share is equal to *A*'s share and Rs 40 more, *C*'s share is equal to *A*'s share and Rs 70 more , find each one's share.

16.   Rs 625 is to be divided among *A*, B and *C* , if *A* receives Rs. 130 less than *C* and *C* receives Rs 25 less than B, how much will each receive ?

17. Divide Rs. 395 among A, B and C so that B may get 25 per cent more than A and 20 per cent more than C.

18. Divide Rs 1525 among A, B and C so that B may get 20 per cent less than A and C may get 20 per cent less than B

19 The sum of the ages of three men is 108 years Twelve years ago their ages were in the proportion 5 4 3, find their ages

20. Divide a guinea between A, B, C and D so that B's share is $\frac{1}{3}$ more than A's, C's $\frac{1}{3}$ more than B's and D's $\frac{1}{3}$ more than C's

21 Divide Rs 551 in three parts so that 4 times the first, five times the second and twice the third part may all be equal

22 A purse contains rupees, eight-anna pieces and four-anna pieces, their numbers are in the proportion of 2, 3 and 4, find the number of each coin if the amount of money in the purse be Rs. 81

23. How many sovereigns, crowns and florins whose numbers are proportional to 2 5, 3, and 4 are together worth £365 ?

24 10880 coins consist of pounds, shillings and pence, the *values* of pounds, shillings, and pence are as 4 3 2, find the number of each coin.

25 Divide Rs 2440 among A, B, C and D so that B's share may be $\frac{5}{3}$ of A's and C's share $\frac{7}{10}$ of B's share and D's share $\frac{1}{3}$ of B's and C's together

26 Divide Rs 6270 among A, B and C so that A shall receive $\frac{3}{7}$ of as much as B and C together and B shall receive $\frac{2}{3}$ of as much as A and C together

27. Divide Rs 3080 among A, B and C so that A shall get $\frac{2}{7}$ of as much as B and C together and B shall get $\frac{3}{11}$ of as much as A and C together

28. Divide Rs 12540 among A, B and C so that A may receive $\frac{3}{7}$ of as much as B and C together and B may receive $\frac{2}{5}$ of as much as A and C together

29 The sum of £177 is to be divided among 15 men, 20 women and 30 children in such a manner that a man

and a child may together receive as much as two women
and all the women may together receive £60. What will
they respectively receive ?

30. The sum of Rs 2840 1a is to be divided between
7 men, 11 women, 5 boys and 6 girls, so that for every
Rs 3 12a. a man receives, a woman may get Rs 2 3a
and for every Rs 2 10a. a woman receives, a boy may get
Re 1 14a and a girl Re. 1. 2a. Find how much each
person receives

31 A five storied building brought in a rent of
Rs. 1250 The ratios of the rents yielded by the succe-
sive stories, i e , the first to the second , the second to the
third and so on are 3 2, 1 2 , 16 9 , 9 5 Find what rent
each story yielded

32. The total of the salaries of A, B, C is Rs, 444, if
they spend 80 p c , 85 p c, 75 p c of their salaries respec-
tively, their savings are as 7 6 9 , find their respective
salaries.

33. A, B, C's total income is Rs 1440 If they spend
80 p c , 85 p c , 75 p c of their incomes, their savings are
as 8 9 20 , find how much each earns

34 Divide Rs 950 among A, B, C so that Rs. 70
more than $\frac{4}{5}$ of A's share, Rs. 45 more than $\frac{8}{4}$ of B's share
and Rs. 30 more than $\frac{3}{8}$ of C's share may be all equal.

35. A person died leaving Rs 3800 with direc-
tions to his pregnant wife that she should keep Rs 1520 for
herself and give Rs 2280 to the child if a son is born and
in case a daughter is born to give Rs 1520 to her and keep
Rs 2280 for herself. But as God would have it twins,
i.e , a daughter and a son, were born How is the money
to be divided now ?

36 A's present age is to B's as 10 9 and 12 years
ago it was as 7 6. Find the present age of each.

37 A's present age is to B's as 13 10 and 20 years
ago it was as 8 5 Find the present age of each

38. Five years ago the ages of A and B were as
4 3 and ten years hence they will be as 11 9 Find their
present ages.

39. Six years ago the ages of A and B were as 10.7 and nine years hence they will be as 13 10   Find their present ages.

40   Divide 312 into three parts such that if they are divided by 5, 6, 7 respectively, the quotients shall be in the proportion of 4, 3, 2

II PARTNERSHIP.

§2   When some persons begin to trade together with a joint stock and the gain or loss is ascertained in proportion to the money contributed by each person, this system of business is called **partnership**

Partnership is of two kinds—*simple and compound*

Simple partnership   When the capital contributed by each partner is supposed to remain for the same period of time, it is called simple partnership

Compound partnership   When it remains for different times it is called compound partnership

We give below examples of each kind.

Example 1 A, B and C are partners in a business. A contributes Rs 1200, B Rs 900 and C Rs 800 and they gain Rs 580 , how should the profit be divided ?   [Simple partnership]

Sol.   We shall proceed as in art 1 of this Chapter, *i e*, Rs 580 will be divided in the ratio of 1200.900 800 or 12 9 8

Example 2   Three men A, B and C enter into partnership A puts in £300 for 4 months, B puts in £400 for 5 months and C puts in £200 for 8 months   They gain £240 , what should each man receive as his share of gain ? (Compound partnership.)

Sol.   £300 for 4 months will produce the same gain as £300 × 4 or £1200 in one month Likewise £400 of 5 months and £200 for 8 months will yield the same gain as £2000 and £1600 for one month, respectively.

Now proceed as in simple fellowship.

The profit will be divided in the ratios 1200, 2000 1600
or 3 , 5 , 4.

Now $3 + 5 + 4 = 12$

A's share $= \frac{3}{12}$ of £240 $=$ £60   ⎫
B's share $= \frac{5}{12}$ of £240 $=$ £100   ⎬ **Ans**
C's share $= \frac{4}{12}$ of £240 $=$ £80   ⎭

Example 3   A and B rent a field for Rs 60, A puts in 50 cows for 6 months and B puts in 40 cows for 5 months, how much should each pay for the rent?

Sol.   50 cows grazing for 6 months must eat as much as 50×6 or 300 cows in one month, also 40 cows for 5 months must eat as much as 40×5 or 200 cows in one month

the rent must be divided proportional to 300 and 200 or 3 and 2

$3 + 2 = 5$,

A must pay $\frac{3}{5}$ of Rs. 60 $=$ Rs 36 ⎫
                                   ⎬ **Ans**
B must pay $\frac{2}{5}$ of Rs 60 $=$ Rs 24 ⎭

EXERCISE 127.

1   Two persons enter into partnership, one contributes Rs 900 and the other Rs 1200 and they gain Rs. 280, what is each person's share of the profit?

2   A, B and C enter into partnership. A puts in Rs 2000, B Rs 5000 and C Rs 11000, what would be the share of each in Rs 2179 2a profit?

3   A, B and C form a joint stock of Rs 75000, of which Rs. 36000 are contributed by A, Rs 30000 by B and the remainder by C   At the end of the year, the profit is found to be Rs 16750, find the share of each, Rs 800 a month being allowed as salary to C as acting partner.

4   A and B are partners in a business, A puts in Rs 833. 5a 4p and B Rs. 966 10a. 8p. At the end of the year the profit is found to be Rs 300 only, find the share of each, Rs 2 8a a month being allowed as an allowance to A as acting partner.

5   A contributes a certain capital for 4 months and B contributes Rs 400 for 5 months   If their profits be in the ratio of 3 4 how much did A contribute?

6 Four merchants trading with a capital of Rs. 11900 find after a year their respective shares increased by Rs 265. 13a. 4p., Rs 372 2a 8b., Rs 531· 10a 8p and Rs 638 Find how much they subscribed to the original capital

7. A starts business with a capital of Rs 4000, after 3 months he admits a partner B with a capital of Rs 6000 and after further 4 months another partner C was admitted with a capital of Rs 9000 At the end of a year after A started the business, the profit was found to be Rs. 2450. What is each person's share of the profit?

8. A, B and C entered into a partnership. A advanced Rs 1200, B Rs. 1500 and C Rs 2000 After 2 months, C withdrew Rs 200 and after 5 months, B put in Rs 300 more. At the end of a year, the profits of the concern were Rs. 1130 Find each one's share in the profit

9. A and B advanced Rs 1500 and Rs 1800 respectively in a business and took Rs 2000 from the Punjab National Bank on the condition that they must pay Rs 120 a year to the bank as interest, at the end of a year the profit is found to be Rs 1110, find the share of each in the profit.

10 A started business with a capital of Rs 2100, after 4 months he admitted another partner B What amount should be put in so that the profit may be divided equally at the end of the year?

11 A, B and C are partners in a business and their shares are in the proportion of $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4}$. A withdraws half his capital at the end of 4 months and after 8 months more a profit of Rs 2024 is declared What is A's share?

12. A employs Rs. 2000 in a business for 8 months, B who joins later keeps his capital for 6 months and receives Rs 150 out of a total profit of Rs. 400. How much does B subscribe to the business?

13 A and B are partners in a business in which A has Rs 4900 and B Rs. 1400, B is the working partner and hence receives 6% of the total profit, the rest being divided in proportion to the capital If the profits are Rs 450 what does each receive?

F 26

14. A starts a business with a capital of Rs. 1500 and admits two partners B and C after 3 months and 6 months respectively After a year the profits are divided in the ratio of 5 + 3   What amount did B and C each contribute ?

15. A starts a business with a capital of Rs 1700 and admits two partners B and C after 3 months and 6 months respectively.   After a year the profits are divided in the ratio of 2 3 5.   What amount of money did B and C each contribute ?

16. A is a working and B a sleeping partner in a business A possesses £1200 of the capital, B £2000   A receives 10% of all profits for managing and the rest is proportionally divided.   The total profits being £800, find the share of each

17   A is a working and B a sleeping partner in a business.   The capital of A is £2400 and that of B £4000. At the end of one year, the profits amounted to £1600. Being the manager, A receives 10% of the profits.   How should the remainder of the profits be divided ?

18   Kirpal Singh and Nihal Singh hire a meadow Kirpal Singh puts in 120 cows and Nihal Singh 150 After 3 months Nihal Singh sold 100 of his cows   if the rent of the meadow be Rs 585 for a year, how much of the rent will each pay ?

19   Two persons rented some fields for Rs. 2430 for 10 months.   One of them put in 27 oxen for 3 months and the other 270 sheep for 7 months If 3 oxen eat as much as 11 sheep, find how much of the rent each ought to pay

20   A and B are partners in a business in which A subscribed Rs 5000 and B Rs. 7500   The gross receipts for a year are Rs 3200, ⅓ of this is spent in salaries, Rs 30 in insurance of the premises   A is to receive 8 p. c. on his capital and B 4 p. c. on his, the remainder of the profits is to be divided proportionally to the capital.   Find the net rent receipts of A and B

21   A and B began business with capitals as 4:7, at the end of 7 months A withdraws, if they received profits in the ratio of 2 5, how long was B's capital in the business ?

CHAPTER XXIV

MISCELLANEOUS PROPOSITIONS

I. ALLIGATION OR MIXTURE

§1. Alligation is the method of mixing two or more things of the same kind but of different qualities It consists of two kinds—*medial* and *alternate* .

§2 Alligation Medial is that in which the qualities and prices of several things which compose the mixture are given and we are to find the mean price of that mixture.

Example 1 A grocer mixes 3 lb of tea at $10a$ per lb , 4 lb of tea at $8a$ per lb , 5 lb of tea at $9a$ per lb and 6 lb of tea at Re 1 $2a$ $2p$. per lb. , find the price of the mixture per lb

Sol   Price of 3 lb. at $10a$. per lb $= 30a.$
  „   4 lb. at $8a$ per lb $= 32a$
  „   5 lb at $9a$ per lb $= 45a.$
  „   6 lb at Re 1 $2a$ $2p$ per lb $= 109a.$
  price of 18 lb $= 216a$
  price of   1 lb $= 216 - 18$ $= 12a$ **Ans.**

Rule. *Multiply the number of each quantity by the value of a unit of the same quantity and divide the sum of the products by the sum of the numbers*

§3 Alligation Alternate is hat in which the prices of several things are given and we are to find in which ratio those things are to be mixed so that the mixture may be of a given price

Two ingredients

Example 2 How must a grocer mix teas at Re 1. $2a$ a lb and Re. 1 $7a$. a lb so as to make a mixture worth Re 1. $5a$ a lb ?

Sol Re 1 $2a.=18a.$, Re 1 $7a.=23a$, Re 1 $5a=21a$

To make the mixture worth $21a$ per lb., the grocer will gain $21-18=3a$ per lb in the first case and will lose $23-21=2a$ per lb. in the second case It is clear, therefore, that the gain in using the cheaper tea must be equal to the loss in using the dearer one. Thus

No of lbs. of cheaper tea $\times 3a$ = No of lbs of dearer tea $\times 2a$

$$\therefore \quad \frac{\text{No of lbs of cheaper tea}}{\text{No. of lbs. of dearer tea}} = \tfrac{2}{3}$$

Hence the teas must be mixed in the ratio of 2 3, that is to say, *in the inverse ratio of the differences of the two prices and the mean price*

We can now arrange the method of work thus —

$\left.\begin{array}{l} 21-18=3a \text{ gain} \\ 23-21=2a \text{ loss} \\ \text{Ratio in the gain and loss} = 3\ 2 \\ \text{f teas must be mixed} \\ \text{in the inverse ratio } 2\ 3. \text{ Ans.} \end{array}\right\}$

$$\begin{array}{c c} & 21 \\ \underline{18 \qquad\qquad 23} \\ 23-21 \quad 21-18 \\ = \quad 2 \ \cdot \ 3 \ \text{ Ans.} \end{array}$$

Rule *Two ingredients are mixed in the inverse ratio of the differences of the two prices and the mean price.*

Three ingredients.

Example 3 In what proportion should teas at $9a.$, $10a$ and $13a$ a lb. be mixed to make a mixture worth $12a.$ a lb ?

Note When two ingredients are mixed the mean price of the mixture is always greater than the price of one ingredient and less than the price of the other Assuming this principle, let us find out which number out of 9, 10, 13 can be linked with another number when the mean price is supposed to be $13a$ in all cases

Sol (i) First and third pairs can be linked because one is less than 12 and the other is greater than 12.

(ii) Second and 3rd pairs can also be linked because the principle stated holds good for this pair also

In the (i) case the ratio of the profit and loss $= 3\ 1$

the ratio in the quantities $= 1\ 3$.

In the (u) case the ratio of the profit and loss$=2.1$

the ratio in the quantities$=1\cdot2$

Now put these ratios as under and then add ·—

(1)	(2)	(3)
1	..	3
	1	 2
1	1	5. Ans

Test of correctness. Price of 1 lb. at $9a$. $= 9a$.

„   1 lb. at $10a$ $=10a$.

„   5 lb. at $13a$. $=65a$.

.. „   7 lb. of mixture $=84a$.

„   1 lb „   „ $=12a$.

Note   We may note that after having obtained one answer, we can obtain as many more as we please, by multiplying or dividing each of the quantities found by 2, 3, 4 5, etc.   For if two ingredients when mixed together make loss and gain equal their halves, third parts or doubles, triples, etc , when mixed together will make no difference

Four ingredients

Example 4. To mix water with spirits worth Rs 7, Rs. 5, Rs 2 per gallon for making a mixture worth Rs. 4 per gallon, how much quantity of each is to be taken ?

Note   Water having no price 0 must be put in its place and considered as the first quantity

Sol   The following pairs can be linked —

(i)   first and 2nd, $i\,e$, 0 and 7

(ii)   first and 3rd,   „   0 and 5

(iii)   second and fourth,   „   7 and 2

(iv)   third and fourth,   „   5 and 2

In the (i) case the ratio of the profit and loss$=4\colon3$

∴ the ratio of the quantities$=3$ 4   . (1)

In the (ii) case the ratio of the profit and loss$=4$ 1

∴ the ratio of the quantities$=1$ 4   (2)

In the (iii) case the ratio of the loss and profit$=3\cdot2$.

∴ the ratio of the quantities$=2$ 3   ..   (3)

In the (iv) case the ratio of the loss and profit $=1.2$

$\therefore$ the ratio of the quantities $=2.1$ . .   (4)

Now put these ratios as under and then add —

(1)	(2)	(3)	(4)
3	4		
1		. 4	
	2	..	..3
		2	1
4	6	6	4

Halves of the quantities found above can be taken.
(See Note above)

$\left.\begin{array}{l} \text{2 gallons of water} \\ \text{2 gallons at Rs 2 per gallon} \\ \text{3 gallons at Rs. 5 ,,} \\ \text{3 gallons at Rs. 7 ,,} \end{array}\right\}$ **Ans**

The student can verify this result as in Ex. 4

§4. An important result.

If a vessel contains x gallons of wine and if y gallons are drawn and replaced by water, then if y gallons of the mixture are drawn and replaced by water and if this operation is repeated n times in all, then

$$\frac{\text{Wine left in vessel after } n\text{th operation}}{\text{Whole quantity of wine in vessel}} = \left(\frac{x-y}{x}\right)^n.$$

Proof After the first operation $x-y$ gallons of wine are left in the vessel, $i.e.$,

quantity of wine after the 1st operation $= \dfrac{x-y}{x}$ of whole

   ,,   ,, ,, ,, ,, 2nd ,, $= \left(\dfrac{x-y}{x}\right)^2$,,

$i.e.,$ $\dfrac{\text{wine left in vessel after 2nd operation}}{\text{whole quantity of wine}} = \left(\dfrac{x-y}{x}\right)$,,

and hence after n operations this ratio will be $= \left(\dfrac{x-y}{x}\right)^n$.

Example 5. Four gallons are drawn from a cask full of wine, it is then filled with water. Four gallons of the mixture are drawn and the cask is again filled with water. The quantity of wine now left in the cask is to that of the water in it as 36 : 13. How much does the cask hold?

Sol. Here, $\dfrac{\text{wine left}}{\text{water}} = \dfrac{36}{13}$

$\therefore \dfrac{\text{wine}}{\text{wine left} + \text{water}} = \dfrac{36}{36+13} = \dfrac{36}{49}$.

$\therefore$ by the result proved above

$\left(\dfrac{\text{total quantity of wine} - 4}{\text{total quantity of wine}}\right)^2 = \dfrac{36}{49} = \left(\dfrac{6}{7}\right)^2$

$\therefore \dfrac{\text{total quantity of wine} - 4}{\text{total quantity of wine}} = \dfrac{6}{7} = \dfrac{7-1}{7} = \dfrac{28-4}{28}$

i. e., total quantity of wine $= 28$ gallons. **Ans.**

EXERCISE 128

1. How must milk at $3a$. per seer be mixed with milk at $5a$ per seer, to make a mixture worth $3a$. $6p$ per seer?

2. How must tea at $12a$ per lb be mixed with tea at $15a$. per lb. to make a mixture worth $14a$. $6p$ per lb.?

3. In what ratio must sugar worth Rs. 10 per maund be mixed with sugar worth Rs. 13 per maund to make a mixture worth Rs 11 $4a$ per maund?

4. In what ratio must milk at $2a$ per seer be mixed with milk worth $3a$ $6p$. per seer, to make a mixture worth $2a$ $6p$ per seer?

5. How must wheat at Rs 4 $10a$ per maund be mixed with wheat at Rs 5 $4a$ per maund to make a mixture worth Rs. 4 $12a$. per maund?

6. A merchant buys sugar at Rs 13 a maund and some more at Rs 17 a maund. How must he mix them so as to make the mixture worth Rs 15 $4a$ a maund?

7. How should I mix tea worth $15a$ per seer with tea worth $9a$ per seer so that by selling the mixture at $13a$ $9p$ per seer I may gain 10 per cent ?

8. In what ratio must a shopkeeper mix sugar worth Rs 5 per maund with sugar worth Rs 7 per maund so that by selling the mixture at Rs. 7 $3a$ per maund he may gain 15 per cent. on his outlay ?

9 How must ghee at Rs 75 per maund be mixed with vegetarian ghee at Rs 40 per maund, so that by selling the mixture at Rs 60 a maund the profit may be 20 per cent ?

10 To mix some wheat at Rs 5 per maund, with some at Rs 6 per maund, and with some at Rs $7\frac{1}{2}$ per maund, so as to make a compound worth Rs 7 per maund how much of each quantity is to be taken in the composition ?

11 How should a grocer mix some tea at $5a$. per seer with some at $7a$. per seer and with some at $10a$ per seer so as to make a mixture worth $8a$ per seer ?

12 A merchant buys sugars at Rs 7, Rs 9' and Rs. 11 per maund respectively how must he mix them so that by selling the mixture at Rs. 11 a maund, he may gain 10 per cent ?

13 A grocer buys teas at $9a.$, $11a$ and $13a$ per lb. respectively, how must he mix them that by selling the mixture at $15a$. per lb he may gain 25 per cent ?

14 A shopkeeper buys 32 seers of milk at $3a$. per seer, how much water should he mix so that by selling the mixture at $2a$ $6p$ per seer he may gain $6a$ $6p$?

15 A shopkeeper buys 25 seers of milk at $4a$. per seer, how much water should be mixed with it so that by selling the mixture at $3a$ per seer he may lose only $1a$. ?

16 A man buys 40 seers of milk at $3a$ per seer, how much water should he mix with it so that by selling the mixture at $2a$ $6p$ per seer the profit may be $5a$?

17 A person buys some quantity of milk at $3a$ per seer and drinks a quarter of it himself With the remainder he mixes 6 seers of water and sells it at $2a$ $6p$ per seer,

his gain is $3a$.   How much milk does he buy ?

18   A wine merchant has four kinds of brandy,   some
at Rs 9 per gallon, some at Rs 8, some at Rs 6 and some
at Rs 4 per gallon   He wants to make a mixture of
84 gallons worth Rs. 7 per gallon   How much of each sort
should be taken to make the mixture ?

19   To mix spirits worth Rs 8, Rs. 6 and Rs 3 per
gallon for making a mixture worth Rs 5 per gallon, how
much of each quantity is to be taken ?

20   A grocer wishes to mix teas at $2s$, $3s$, $3s$ $6d$
and $4s$ per lb respectively , how must he mix them
(using the first two kinds in the proportion of 2   3 and
the last two in the proportion of 3   4) so that by selling
the mixture at $3s$ $4d$. per lb $\frac{1}{16}$ of the receipts may be clear
profit ?

21   Nine gallons are drawn from a cask full of wine,
it is then filled with water   Nine gallons of the mixture
are drawn and the cask is again filled with water   The
quantity of wine now left in the cask is to that of the water
in it as 16   9   How much does the cask hold ?

22   Five gallons are drawn from a cask full of wine,
it is then filled with water   Five gallons of the mixture
are drawn and the cask is again filled with water   The
quantity of wine now left in the cask is to that of the water
in it as 25   11   How much does the cask hold ?

23   There are four vessels of equal capacity, $\frac{1}{5}$ of the
first, $\frac{1}{3}$ of the second, $\frac{1}{4}$ of the third and $\frac{1}{3}$ of the last is filled
with spirit   The first is then filled with water and from
this mixture the second is filled up, again from this second
mixture the third is filled up and likewise the fourth from
the third   What proportion of spirit to water is there in
the fourth vessel ?

---

§5   Interchange of ingredients.

Suppose there is a lump of two ingredients , now
if an equal quantity of another lump, in which the
proportion of the ingredients is interchanged is added
to the first lump, then evidently the quantities of the two
ingredients become equal in the new compound   For
instance, suppose there are two vessels , in the first of them

4 gallons of wine are mixed with 9 gallons of water and in the second, 9 gallons of wine are mixed with 4 gallons of water. If we mix up the contents of the two vessels, then in the new mixture, there will be evidently 13 gallons of wine and 13 gallons of water.

Example 6. The price of gold is £3. 17s. 10½d per oz. A lump of gold and silver weighing 18 lb is worth £637. 7s. 0d., but if the ratio of gold and silver be interchanged it would be worth only £259. 1s 0d Find the ratio of gold and silver in the lump and the price of silver per oz.

Sol. If the two lumps are added together, there would be 18 lbs of gold and 18 lbs of silver and the price of the two lumps together would be £637 7s 0d. + £259 1s 0d or £896. 8s. 0d

Now 18 lbs. of gold + 18 lbs of silver = £896 8s 0d but 18 lbs of gold = £3. 17s 10½d. × 18 × 12 = £841 1s 0d.

∴ 18 lbs of silver = £55. 7s

$$1 \text{ oz of silver} = \frac{£55 \ 7s.}{18 \times 12} = 5s \ 1\tfrac{1}{3}d \text{ Ans}$$

$$\text{Again cost of 1 oz of the first lump} = \frac{£637 \ 7s.}{18 \times 12}$$

$$= £2 \ 19s. \tfrac{1}{6}d = 59\tfrac{1}{72}s$$

Cost of 1 oz. of gold = £3 17s 10½d. = 77⅞s. and cost of 1 oz of silver = 5s 1½d. = 5⅛s

```
                 59 1/72
    77⅞                      5⅛
   59 1/72 − 5⅛  ·  77⅞ − 59 1/72
        53⅘  .  18 31/36
```

Hence gold   silver = 53⅘   18 31/36
                    = 485 × 4   679
                  ∴ = 20 · 7. Ans

EXERCISE 128—A.

1. The price of gold is Rs. 38 15a per oz A lump of gold and silver weighing 18 lbs is worth Rs 7246 8a but if the weights of gold and silver are interchanged, it would be worth only Rs. 1717 8a Find the ratio of gold and silver in the lump and the price of silver per oz.

2   A person bought apples and pears for Rs 2 3*a*. If the numbers of apples and pears be interchanged it would have cost him only Re 1. 9*a*. If apples cost 9*a*. and pears 3*a* a dozen, how many of each did he buy ?

3   Two alloys are composed of silver and copper in the following ratios by weight —98 2 in the first and 95 5 in the second   These two are melted together in the ratio of 5 3. Find the ratio of silver to copper in the resulting mixture

4   Two vessels contain mixtures of wine and water, in one there is twice as much wine as water and in the other 3 times as much water as wine   Nine gallons from the first and six gallons from the second are drawn off to fill a third vessel   Find the ratio of wine to water in the third vessel

5.   From a vessel filled with 20 gallons of spirit, 2 gallons are removed and the vessel is then filled with water What amount of spirit is left after this has been done twice ?

6.   Gold is worth £3, 17*s*. 11*d*. an ounce, an alloy containing 7 times as much gold as silver is worth 3 times as much as another alloy which contains 3 times as much silver as gold   In what ratios must these alloys be mixed to form one worth £2 10*s* an ounce?   [*Burma 1924*]

7   Two tins contain originally 20 pints of milk and 10 pints of water respectively. Four pints of the liquid are now drawn from each tin and placed in the other, the liquid being thoroughly mixed   The same process is repeated a second time. Find the percentage of milk in each tin in the final mixtures.   [*Burma 1923.*]

§6   Application of the Alligation Rule

Example 1. Rs 38. 12*a* were divided among 120 children, each girl had 8*a* and each boy 4*a*, how many boys were there ?

$$\text{Sol}\quad \text{The mean amount} \left.\begin{array}{l}\\ \text{each child had}\end{array}\right\} = \frac{\text{Rs. 38 } 12a}{120} = \tfrac{920}{120}a. = \tfrac{31}{6}a.$$

$$\therefore \text{ No. of boys} = \tfrac{17}{24} \times 120 = 85. \text{ Ans.}$$

girls boys

8*a*   4*a*.

$\tfrac{31}{6}a.$

$\tfrac{31}{6} - 4\quad 8 - \tfrac{31}{6}$

or $\tfrac{7}{6} \cdot \tfrac{17}{6}$

or 7·17

Example 2   A person had 250 lbs of tea, some of it he sold at 10 p c. profit, some at $12\frac{1}{2}$ p. c. p ofit and the rest at 15 p c. profit.   He gains $12\frac{9}{10}$ p. c on the whole. Find how much of each quantity he s old on different rates.

Sol.   The mean S P | (1)   (2)   (3)
       = $112\frac{9}{10}$ | 110 $112\frac{1}{2}$ 115

Now we can link (1), (3) and (2), (3)
ratio in the (1) and (3) qua $= 115 - 112\frac{9}{10} \cdot 112\frac{9}{10} - 110$
                           $= \frac{21}{10}\ \frac{29}{10} = 21\ 29$
also ratio in the (2) and (3) qua $= 115 - 112\frac{9}{10}\ \ 112\frac{9}{10} - 112\frac{1}{2}$
                           $= \frac{21}{10}\ \frac{4}{10} = 21\ 4$

Now write these ratios and add —

	(1)	(2)	(3)
Since $7 + 7 + 11 = 25$	21	.	29
$250 \div 25 = 10$		21	4
	21	21	. 33
	= 7	7	11

∴ quantity at 10   p c. $= 10 \times 7 = 70$ lbs. ⎫
      „   at $12\frac{1}{2}$ p c $= 10 \times 7 = 70$ lbs. ⎬ **Ans**
and   „   at 15   p. c $= 10 \times 11 = 110$ lbs. ⎭

Note   Other answers are also possible   The students can check this answer supposing the cost price to be Re 1 per lb

How to get other answers.

Ratio between (1) and (3)   Ratio between (2) and (3)
(1) = 21 29   (1) = 21   4
(2) = 42 58   (2) = 42   8
(5) = 105 145 and so on   (3) = 84   16 and so on

Take any pair e g.,   (Again

(1)	(2)	(3)		(1)	(2)	(3)
42		..58		105		145
	21	.4			84	16
. 42	21	62		105	84	161
				= 15 .	12	23

will produce a different | will produce another answer
answer. |and so on

Note   The student should note carefully that if he will link the first pair with the first, the second with the second and so on, he will get the same answer but if he will link different pairs the answers will be different but all correct

EXERCISE 129

1   Rs 78. 12a were divided among 150 children, each boy got 12a. and each girl 6a   Find the number of each

2   Divide Rs 1580 among 180 claimants, so that some may receive Rs 8. 5a 4p. and others Rs 9. 5a. 4p., find the number of each kind of claimants.

3.   A sum of £5 10s is made up of 75 coins which are either florins or shillings, how many are there of each coin ?

4   A person has 150 lbs of sugar, part of which he sells at 10 p c. profit and the rest at $12\frac{3}{4}$ p. c   profit.   He gains $11\frac{1}{8}$ p c. on the whole.   Find how much is sold at 10 p c. profit

5   A cloth merchant sold 55 yards of cloth for Rs 88 12a, some at Re 1 8a per yard and the rest at Re 1. 12a. per yard, how much is sold at Re 1 12a per yard ?

6   A person had 200 lbs. of tea, some of it he sold at 10 p c. profit, some at 15 p c profit and the rest at 20 p c profit   He gained $14\frac{1}{4}$ p c. on the whole.   How much did he sell on different rates ?

II   EQUATION OF PAYMENT.

§7   Two or more debts incurred by a person at *different* times may be discharged by one payment, the time of payment of the total debts is called the *equated time* and the process by which we find this time is known as the *equation of payments*.   In such cases, we *assume* that the sum of the interests of *all* the debts for their respective periods is equal to the interest of their sum for the *equated time*   Hence, to find the equated time, we have the following simple

Rule.   *Multiply each debt by the corresponding time and then divide the sum of the products by the sum of the debts*

Example   If Rs. 75 be due in 4 months, Rs 125 in 5 months and Rs. 150 in 7 months, what is the equated time ?

Sol   The required time $= \dfrac{75 \times 4 + 125 \times 5 + 150 \times 7}{75 + 125 + 150}$

$$= \dfrac{300 + 625 + 1050}{350}$$

$= \tfrac{1975}{350} = \tfrac{79}{14} = 5\tfrac{9}{14}$ months. **Ans.**

EXERCISE 130.

1.   Rs. 100 is due in three months, Rs 210 in 2 months and Rs 90 in 7 months   Find the equated time

2   What is the equated time of payment when $\frac{1}{2}$ of a debt is due in 3 months, $\frac{1}{5}$ in 8 months and the remainder in 15 months ?

3   A owes B £4600, of which £1000 is to be paid in 50 days, £1300 in 40 days and the remainder in 140 days Find the equated time

4   A owes B Rs 7300 to be paid in $5\frac{44}{73}$ months , he pays Rs 1500 at the end of three months and Rs 2100 at the end of five months, when will the remainder be due?

5   Out of a debt due 15 months hence, $\frac{1}{2}$ was paid at 4 months, $\frac{1}{4}$ at 6 months and $\frac{3}{6}$ at 12 months   How many months may the payment of the remaining portion of the debt be deferred ?

6   A man owes four debts, the equated time of payment being 6 months. The first debt of £500 is due in 3 months, the second of £450 in 4 months, the third of £300 in 5 months When is the fourth debt of £1000 due ?

III. PASTURE WITH GROWING GRASS.

§8   Examples on this subject can best be done by means of simultaneous equations   The following solutions illustrate the method ,—

Example 1 If 29 sheep can feed on a field of uniformly growing grass in 7 days or 25 sheep can feed on the same field in 9 days, how many sheep will feed in 6 days ?

Sol.   Denote amount of original grass by a units and each day's growth by b units

Now 29 sheep feeding for 7 days is equivalent to 29×7 $= 203$ sheep for 1 day and similarly 25 sheep for 9 days is equivalent to 225 sheep for 2 day

. by the question

$a + 7b$ units of grass feed 203 sheep for one day, $i e$, equal to 203 units

and $a + 9b$ units of grass feed 225 sheep for 1 day, $i.e$, equal to 225 units

˙ omitting the common unit we have

$$\left. \begin{array}{l} a + 7b = 203 \\ \text{and } a + 9b = 225 \end{array} \right\} \qquad \begin{array}{l} \text{whence } b = 11 \text{ units} \\ \text{and } a = 126 \text{ units.} \end{array}$$

Now supposing the required number of sheep to be x and remembering that x sheep being fed for 6 days is equivalent to $6x$ sheep for 1 day, we have, from the question (as before),

$$a + 6b = 6x$$

Substituting values of a and b, we have

$$x = 32 \text{ sheep.}   \text{Ans}$$

Example 2.   If 40 oxen eat the grass of a field growing uniformly in 12 days and if 25 oxen eat the grass of the same field in 20 days, find how long 30 oxen will take to consume it

Sol.   With the same symbols and with the same explanation as in Example 1, we have,

$$\left. \begin{array}{l} a + 12b = 480 \\ \text{and } a + 20b = 500 \end{array} \right\} \qquad \begin{array}{l} \text{whence } b = \tfrac{5}{2} \\ \text{and } a = 450 \end{array}$$

Now suppose that 30 oxen require x days, then

$$a + xb = 30x,$$

or $450 + \tfrac{5}{2}x = 30x,$

whence $x = \tfrac{900}{55} = \tfrac{180}{11} = 16\tfrac{4}{11}$ days.   **Ans,**

EXERCISE 131.

1. A field of 15 acres grass growing uniformly, is consumed by 20 sheep in 96 days, but by 30 sheep in 60 days How many sheep will consume it in 24 days ?

2 A meadow of 50 acres with uniformly growing grass is consumed by 80 oxen in 60 days but by 60 oxen in 90 days. How many oxen will eat half of it in 30 days ?

3 If 133 horses eat the grass of a meadow in 13 days and 112 horses could eat the grass of the same meadow in 16 days (the grass growing uniformly), in what time could 125 horses do it ?

4. If 25 sheep consume 10 acres of pasture (grass growing uniformly) in 30 days, and 30 sheep consume 8 acres of it in 18 days, how many acres will be consumed by 80 sheep in 60 days ?

5. A field of uniformly growing pasture is consumed by 120 oxen in 20 days or by 150 oxen in 15 days. How many oxen will consume it in 12 days, if at the end of 8 days, 30 of them are removed ?

6. A cistern constantly flowing is emptied by a number of waste pipes If 10 of these are opened, the cistern is emptied in 15 minutes but if 25 be opened, it is emptied in 5 minutes How much time will it take to be emptied if 15 are opened ?

7 If 23 sheep eat 8 acres of pasture in 26 days and 25 sheep consume 7 acres of the same in 20 days (the grass growing uniformly), how many acres of it would 33 sheep consume in $5\frac{7}{9}$ days ?

8 A leaky cistern is filled in 5 hrs. with 30 pails of 3 gallons each but in 3 hrs with 20 pails of 4 gallons each, the pails being poured in at intervals. Find the capacity of the cistern and the time in which the water would flow out

I

1. Simplify $\dfrac{5\frac{1}{2} \times 1\frac{2}{9\frac{2}{4}} - 1\frac{3}{4} - \frac{5}{13} - 1\frac{1}{16}}{1\frac{1}{3} \times \frac{5}{8} - \frac{22}{87} \div \frac{1}{4} - 2\frac{1}{4}}$

2. A woman has a certain number of eggs, she sells ·3 of the number and one more to one person, ·3 of the remainder to a second person, and ·5 of the remainder to a third person, after these sales she has 15 eggs left. How many had she at first?

3. (a) The total expenses of a family when wheat is 16 srs a rupee are Rs 40½, when wheat is 12 srs a rupee they are Rs. 42½, find its total expenses when wheat is 10 srs a rupee, supposing other expenses remain the same

(b) Find the square root of $1 \div \frac{1}{3}$ (03+5)^3 correct to 5 places of decimals

4. A garrison of 2000 men has provisions for 42 days How long will the provisions last, if the garrison be increased by 800 men?

5. Find the value of 55 mds. 22 srs 11 chk. of ghee at Rs 25. 13a 4p per maund.

6. A can do in 9 days as much work as C can do in 6 days, and B in 15 days as much as C in 12 days, what time would B take to finish a piece of work which A can do in 36 days?

7. Find the least number which being divided by 8, 12 and 16 leaves in each case a remainder 3 but when divided by 15 leaves no remainder

8. A room is 37 ft 3 in long, 18 ft. 9 in. wide and 14 ft high Find the cost of papering its walls with paper 2 ft wide at 6d per yard

9. One boy runs 300 yards and another 285 yards in 1 minute How many yards' start must the second have so that they may run a dead heat in a mile race?

10. The average age of the boys in a school of 650 boys is 15 6 years 50 boys leave, thereby diminishing the average of the school to 15 2, find the average age of those who leave

417

F 27

II

1   What is the greatest number consisting of 5 digits which can be added to 8321, so that the sum may be exactly divisible by 15, 20, 24, 27, 32 or 36 ?

2   Simplify $\left\{ \dfrac{2}{3-\dfrac{1}{1-\frac{1}{2}}} - \left(5 - \dfrac{2}{\frac{3}{2}-\frac{1}{6}}\right) \text{ of } \frac{1}{3} \right\} - \dfrac{\frac{1}{2}+\frac{5}{4}}{1\frac{1}{2}}.$

3   The cost of matting a room 16 ft. broad and 12 ft high at $3a$ per sq yard is Rs 7. $9a$ $4p$ What will be the cost of papering its walls at the same rate, allowing for six doors, each 6 ft. by 3 ft. ?

4   A man buys an article and sells it at a gain of 10 per cent.   If he had bought it at 10 per cent less and sold for Re. 1 more, he would have gained 25 per cent. Find the cost price.

5   A man bought two heaps of mangoes, one for Rs. 10. $5a$ and the other for Rs. 18. $0a$ $9p$ If the price of each mango be the same, and not less than two and not more than three annas, find the total number of mangoes he bought.

6 A can do a piece of work in 25 days, B in 20 days and C in 24 days.   They three work together for 6 days and then B leaves   In what time would A and C finish the remaining work ?

7.   Find the value of $\dfrac{12+\sqrt{009}}{1-\sqrt{4}}$ correct to 3 places of decimals

8   If 5 compositors working 8 hours a day can compose a book of 240 pages in 21 days, in what time would 3 compositors working 10 hours a day compose a book of 360 pages ?

9   A merchant buys two kinds of tea at $1s$ $11\frac{1}{4}d$. and $1s$ $5d$. per lb. respectively.   In what proportion must he mix them so as to gain $37\frac{1}{2}$ per cent. by selling the mixture at $2s$ $3\frac{1}{4}d$ per lb ?

10.   Find the least sum of money that must be added to Rs. 25316 $12a$. $8p$ to make the sum divisible by 221.

III

1.   Reduce $'0416 \times \dfrac{\dot{1}42857}{(\frac{6}{13}+\frac{3}{8})\times 40}$ of Rs. 5 8a. to the fraction of 1a.

2   What least number must be added to $7\frac{1}{5}$. that the result being divided by $1\frac{1}{4}$, the quotient shall be an integer ?

3   A and B complete a piece of work in 8 days, B and C do the same in 12 days, and A, B and C finish it in 6 days   In how many days will A and C complete the work ?

4   If by selling sugar at 24s a cwt a grocer would lose 4 per cent, at what price per lb. must he sell it to gain 12 per cent ?

5.   The population of a town is 50000   If the number of males were increased by 9 per cent, and the number of females by 4 per cent, the population would become 53200   Find the number of males and females in the town

6   Make out a bill for the following articles supplied by Messrs Mool Chand & Co to L Gujar Mal —

10 lbs of tea at Re 1 3a per lb , 6 seers of sugar at Rs 2 3a per bag of 5 seers , 4 tins of coffee at Re 1a. per tin , 8 silk handkerchiefs at Rs 3. 8a per dozen , 3 mds 37 seers of Portland cement at 8 seers per rupee, a child's perambulator, price Rs 30   Subtract 10 per cent discount for cash

7   The area of a rectangular field is $\frac{3}{5}$ of an acre , and its length is twice its breadth , determine the lengths of its sides approximately.

8   A man has £5 17s consisting of sovereigns, half-crowns and shillings in the proportion of 2, 3, 11   How many has he of each coin ?

9   Find the number of complete kilometres in 850 miles taking a centimetre to be 3937 inches   [*Burma 1923*]

10   A clock which gains $2\frac{1}{4}$ minutes in 24 hours is 3 minutes slow at noon on Sunday when will it indicate correct time and what will it indicate at 6 o'clock on Monday evening ?

IV

1. Find the value of $\frac{49}{21}$ of $\frac{(3\frac{1}{3}-2\frac{1}{2})-\frac{5}{6} \text{ of } \frac{3}{8}}{2\frac{2}{5}-(\frac{1}{2}+\frac{1}{4})}$ of £46.

2. Find the greatest number of 4 digits and the least number of 5 digits that have 135 for their G. C M.

3. Multiply 675348 by 2739113 in three lines.

4. Find the value of 5 tons 15 cwt. 3 qr. 7 lb of coal at £4 13s. 4d per ton

5. A candidate who gets 40 marks fails by 12½ marks, the minimum to be secured being 35 per cent   Find the maximum

6. If 100 horses consume a stack of hay 24 ft long, 10 ft. high and 8 ft. broad, in 8 days, for how many days will a stack 20 ft long, 15 ft high and 6 ft broad, supply 150 horses ?

7. A cyclist starts for a ride at 8-55 A.M, By 10-34 A M. he has ridden 16 miles 154 yards.   How far will he have gone altogether by 1-16 P.M, his speed being the same throughout ?

8. Find the cost price of an article which if sold at 7 per cent. profit brings 5s more than if it were sold at 17 per cent. loss

9. A multiplication sum having been worked is partially rubbed out, the figures that remain are the entire multiplicand 999 and the last three digits 193 in the product   Restore the complete work.

10. A person buys 80 tons of coal and after selling them again at 1s 6d per sack finds that he has gained £4, had he sold them for 1s 4d per sack he would have lost £6.   Find the weight of each sack and the cost price per ton.

V

1. Find the missing digits indicated by the mark+ in 46*3b9* if it is divisible by 45

2. A and B have between them 132 horses, '25 of A's· 142857 of B's   How many has each of them ?

3   A box made of board an inch thick, measures on the outside 20 in long, 14 in wide and 8 in deep, find the cubic contents of the interior and the cost of painting the outside at $9p$ per sq ft.

4   A tradesman demanded a price for a watch which was 40 per cent above cost and gave the purchaser 10 per cent. discount on the price asked, gaining thereby Rs 8 $2a.$, what was the cost price ?

5   A ship with 1200 men on board had sufficient provisions to last 17 weeks The survivors of a wreck having been taken aboard, the provisions were consumed in 15 days How many men were taken aboard ?

6   Find the square root of $1-(0678)^3$ to 4 places of decimals.

7   A man bought 4 sorts of wheat at an average price of Rs 6 a maund. If the prices are such that they increase by a common difference of $5a$ per maund, find the cost of each sort per maund

8   Find, by practice, the price of $521\frac{2}{3}$ maunds of sugar at Rs 22. 11a. 11p per md. (*Use two aliquot parts only*)

9.   A school of boys and girls consists of 453 children, the number of boys is $5\frac{2}{3}$ of the number of girls. How many boys are there ?

10.   The breadth of a room is two-thirds of its length and three-halves of its height and the contents are 5832 cubic ft. Find the dimensions of the room

VI

1.   Simplify $\dfrac{5}{7}+\cfrac{3}{5\frac{1}{2}+\cfrac{+\frac{1}{3}}{1-\cfrac{3}{25\frac{3}{7}+\cfrac{8}{7\frac{2}{3}-5\frac{11}{14}}}}}$

2.   Express $\frac{3}{8}$ of 7s 6$d.$ + 1 25 of 5s −·545 of 9s 2d as a decimal of £10

3.   In a game at fives, out of 15 points, A can give B 3, also A can give C 7 points, how many points can B give C so as to make an even match ?

4   A cistern contains $2+3\frac{3}{4}$ cubic ft of water. Find the length of the side of a second cistern 4 ft. 4 in deep with a square base which contains four times as much water as the first.

5   What profit per cent is made by selling an article at a certain price, if by selling at two-thirds of that price there would be a loss of 20 per cent ?

6   A can do a certain piece of work in 72 days, B in 96, C in 120 days C begins alone for the first day, then A and B join in and all three work for 12 days, after which A leaves off, in how many more days will B and C finish it ?

7   Find the square root of $\dfrac{1000\ 20001}{1000}$

8   A rectangular courtyard the sides of which are 5 11. costs Rs 1.14 $6a$ for paving at $10a$ $6p$ per sq yard. Find the lengths of its sides

9.   Twelve bullocks can plough a square field, the length of a side of which is 90 yds in $2\frac{1}{3}$ days   In how many days will 20 bullocks plough a square field, the length of a side being 150 yds.?   [*Burma 1923*]

10   The average weight of the 8 oars-men in a boat is increased by $2\frac{1}{2}$ lbs, when one of the crew who weighs 11 stones 12 lbs is replaced by a new man. What is the weight of the new man ?

VII

1   (a) Simplify $\dfrac{\frac{1}{2}+\frac{1}{4}\ \text{of}\ \frac{1}{2}+\frac{5}{6}}{\frac{1}{13}\ \text{of}\ (1+5\frac{1}{2})+\frac{5}{6}\ \text{of}\ \frac{1}{23}\ \text{of}\ (7-2\frac{2}{5})-\frac{1}{3}}$.

(b) Express $\frac{3}{5}$ of Re 1 $5a$ as the decimal of Re 1 $4a$.

2   What is the greatest length which is contained whole number of times exactly in both $25\frac{15}{20}$ ft. and $21\frac{9}{30}$ ft.?

3   A man left $\frac{1}{2}$ of his property to his eldest son, $\frac{1}{3}$ to his youngest, $\frac{1}{12}$ to his daughter and the remainder which was Rs 120 to his wife, what was his whole property worth ?

4 A policeman goes after a thief who has 528 yards' start, if the policeman goes at the rate of a mile in 7 minutes and the thief at the rate of a mile in 10 minutes, how far will the thief have gone when he is overtaken ?

5. A hollow circular cylinder stands on a solid cubic pedestal of the same material whose edge is 6 ft , the internal and external diameters of the cylinder are 4 and 5 ft , the weight of the cylinder is the same as that of the pedestal, find its height

6 A number of men can be formed either into a solid square, or into a hollow square 9 deep, having 970 men in the front rank of each side, how many men are there in each side of the solid square ?

7. A spirit merchant buys 80 gallons of whisky at 18s per gallon and 180 gallons at 15s per gallon and mixes them. At what price must he sell the mixture to gain $8\frac{1}{3}$ per cent upon his outlay ?

8 Find the sq root of $7\,4538 - 6\,8 - 8\,5 - 2\,'03 \times 1\,'17$

9 A rectangular field 50 ft long and 30 ft. wide has a path of uniform width running *outside* all round the sides. The area of the path is 425 sq ft , find its width.

10 Find two least integers such that $\frac{4}{5}$ of the first shall be equal to $\frac{2\frac{4}{5}}{1}$ of the second.

VIII

1 Multiply Rs 2. 1a by $\dfrac{\frac{4}{5} + \frac{6}{7} + \frac{13}{15} + \frac{10}{-1}}{\frac{1}{5} + \frac{5}{7} + \frac{7}{15} + \frac{13}{-1}}$

2 Find the G C M. and L C M. of 49 383$_4$ and 142569

3. Find by Practice, the time of building a wall 27 yds long, 1 yd. thick and 6 ft. high of which one cubic yd is built in 3 hrs, 18 min, 45 sec.

4 A's rate of working is to B's as 4 to 3 and B's is to C's as 2 to 1   How long will it take C to do what A would do in 6 days ?

5. Find by how much the square root of $9 + \cfrac{1}{1 + \cfrac{1}{7 + \frac{1}{9}}}$

differs from $\frac{585}{116}$, which of these comes nearest to $3 + \frac{1}{10}\sqrt{2}$?

6 In what proportion must a grocer mix one kind of tea at 12a. per lb with another at Re 1. 5a per lb in order that by selling the mixture at Re 1 8a per lb., he may make a profit of 20 per cent ?

7 In an examination, A gets 10 p c. less than the minimum number of marks required for passing, B obtains $11\frac{1}{9}$ p c. less than A and C $41\frac{3}{17}$ p c less than the number of marks obtained by A and B together Does C pass or fail ?

8 A dealer buys 80 baskets of rice at Rs 5. 12a per basket, he mixes it with 32 baskets of another kind and sells the mixture at Rs 6 6a. per basket making a profit of 19% How much did he pay for the second kind of rice ?
[*Burma 1933*].

9 A can beat B by 5 yds. in a 100 yards' race and B can beat C by 10 yds in a 200 yards' race, by how much can A beat C in a 400 yards' race?

10 I went for a walk between 4 and 5 and returned between 7 and 8 The hands had exactly changed places, at what time did I go for the walk ?

IX

1. Simplify —
$$\frac{3\frac{7}{8} \times 1\frac{1}{17} + 4\frac{11}{13} - 4\frac{3}{16}}{5\frac{1}{9} - 7\frac{7}{8} - 28\frac{7}{20} + \frac{1}{8}} + \frac{3\frac{7}{8}}{4\frac{1}{7}} \times 3\frac{5}{8} \times 5\frac{6}{7} - 17\frac{1}{2} + \frac{5}{8}.$$

2 Eight bells which toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds respectively, begin tolling all simultaneously with the clock striking. How many hours must elapse before they all toll simultaneously again with the clock striking? (The clock is supposed to strike at the hour only)

3 By how much does the difference of $1\frac{1}{26}$ and $\frac{9}{13}$ fall short of their sum ? Express the defect as a decimal.

4. How long will it take to walk along the four sides of a square field which contains 16 acres 401 sq. yds. at 3 miles an hour ?

5   The length of a hall is 3 times the breadth   The cost of whitewashing the ceiling at $5\frac{1}{2}d$ per sq yd is £4 12s 7'1d and the cost of papering the walls at 1s 9d. per sq yd. is £35   Find the height of the hall.

6   In mixing tea, 1 lb in every 100 lbs is wasted. In what proportion must a dealer mix teas which cost him 1s 9d and 1s 4d per lb respectively, so as to gain 10 per cent. by selling the mixture at 1s 8d per lb. ?

7   A hare is 500 leaps before a greyhound and takes 4 leaps to his 3, but 2 of the greyhound's leaps cover as much ground as 3 of the hare's , in how many leaps will the greyhound catch the hare ?

8   I spent £5 in buying eggs at 2 a penny and the same sum in buying eggs at 3 a penny. I sold them all at the rate of 5 for two pence. How much did I gain or lose ?

9.   If the four-penny loaf weighs 3 lbs. 9 oz when wheat is at 9s 4d per bushel, what ought the six-penny loaf to weigh when wheat is 11s 1d. per bushel ?

10.   (a) Extract the square roots of $5\frac{1}{16}$ and 76'195441.

(b) A man bought a piece of land for Rs 140, and intending to sell it, fixed such a price, that by selling it at $12\frac{1}{2}\%$ under the intended price, he would still have a gain of $12\frac{1}{2}\%$ on the prime cost   At what price did he intend to sell the land ?

X

1.   How many times is $\frac{1}{7}$ of $13\frac{1}{3}$ of 2s $2\frac{1}{4}d$ contained in $2\frac{1}{4}$ of 3s. $4d + 4\frac{4}{11}$ of 1s $1\frac{3}{4}d + \left(\dfrac{27\frac{5}{6} \text{ of } 28\frac{1}{4}}{20\frac{3}{4} \text{ of } 75\frac{1}{3}} + \dfrac{3\frac{6}{7} \text{ of } 1\frac{1}{3}}{7\frac{5}{7} \text{ of } 2\frac{2}{3}}\right)$ of 16s. 8d

2   On measuring a distance of 32 yds with a rod of a certain length, it was found that the rod was contained 41 times with $\frac{1}{2}$ an inch over. How many inches will there be over in measuring 44 yds. with the same rod ?

3   (a) A who travels $3\frac{1}{2}$ miles an hour starts $2\frac{1}{3}$ hrs before B who goes the same road at $3\frac{1}{2}$ miles an hour, where will he overtake A ?

(b)   A man having to walk 36 miles finds that in 3 hours 20 mins he has walked five-sevenths of the remaining distance.   Find his speed

4   If 3 fowls and 4 pigeons cost Rs 2 3a 6p and 5 fowls and 2 pigeons cost Rs 2 12a, find what must be paid for 4 fowls and 3 pigeons.

5.   A horse was sold for Rs 60, at a certain loss. Had it been sold for Rs 81. the gain would have been $\frac{3}{4}$ of the former loss.   Find the cost of the horse

6   Four men undertake to complete a piece of work in 12 days, at the end of 5 days only $\frac{2}{9}$ of the work is accomplished, how many extra men (working at the same rate) must be employed to complete the work in the given time ?

7.   (a) The sides of a rectangle are 3 4 and the area is 1452 sq ft.   Find its length and breadth.

(b)   A packet of matches which cost 4a. to manufacture was sold at 9a per packet   The cost of manufacture has risen $8\frac{1}{2}\%$, the matches are still sold at 9a per packet, but each packet contains now only 10 boxes instead of 12   Compare the percentage profit made now by the manufacturer with what he made formerly

[*Burma 1923.*]

8   One clock gains 2 minutes in 3 days, another loses 6 minutes in 6 days, if they are set right at 12 o'clock to-day, when will their times differ by a quarter of an hour ?

9.   The subscription to a certain memorial fund amounted to Rs 976 9a and each person subscribed as many annas as there were subscribers   Find the number of subscribers.

10   A contractor engaged to finish six miles of railway in 200 days, but after employing 140 men for 60 days he found that only one and a half miles were completed. How many additional men must be engaged that the work may be finished within the given time ?

CHAPTER XXV.

SIMPLE INTEREST.

§1   If I borrow a certain sum of money for a certain period from a money lender, I am expected to pay a certain extra sum of money at a fixed rate for the use of the money borrowed   The consideration thus paid is called the **Interest**, the money borrowed is named the **Principal** and the sum of interest and principal together is termed the **Amount.**

§2   The **Rate** is the money paid for the use of a certain sum for a certain time   Thus if a certain sum is borrowed on the condition that for the use of every rupee in the loan for a month $3p$ will be paid, it is said to have been borrowed *at the rate of $3p$ per rupee per month*   Again, if it is borrowed on the condition that for the use of every Rs. 100 in the loan for one year, Rs 6 will be paid, it is said to have been borrowed *at the rate of 6 per cent per annum*

§3.   Per annum means for a year.   The words *per annum* are generally omitted   When we say "at the rate of 5 per cent ," we mean 5 per cent per annum.

§4   If the interest is paid as it falls due, it is called **Simple interest**   *The word interest is always to be understood as simple interest.*

§5   To find interest when the rate per rupee or £ per month is given.

Example 1.   Find the simple interest on Rs. 120 for 8 months at $3p$. per rupee per month.

Sol   Int. on Re 1 for 1 month $=3p.=$ Re $\frac{1}{84}$

    ,,   ,, 1 for 8 months $=$ Re. $\frac{1}{84} \times 8$

    ,,   ,,120 ,,   ,, $=$ Rs. $\frac{1}{84} \times 8 \times 120$

               $=$ Rs. 15. **Ans.**

Hence when rate per rupee or £ per month is given we have the following

Rule. *Int = Principal × rate per Re. × time in months.*

Example 2 Find the simple interest on £320 8s. 6d for 1½ years at 4d per £ per month.

Sol. Principal = £320. 8s. 6d

Rate   = 4d = £$\frac{1}{80}$ per £ per month.

Time   = 18 months.

∴   Interest = (£320 8s. 6d) × $\frac{1}{80}$ × 18

$= \dfrac{(£320.\ 8s\ 6d\ ) \times 3}{10}$

$= \dfrac{£961.\ 5s\ 6d}{10} = £96\ 2s.\ 6\frac{3}{5}d.$ **Ans.**

EXERCISE 132.

Find the interest on —

1.   Rs. 192 for 4 months at 4p. per rupee per month.
2.   Rs 96 for 1½ years at 2p per rupee per month.
3   Rs 384 for 8 months at 5p. per rupee per month.
4   Rs. 800 for 2 months at 1p. per rupee per month.
5   Rs. 128 for 8 months at 3p. per rupee per month
6   Rs. 50 for 8 months at 3p. per rupee per month.
7   Rs. 125 for 6 months at ½a. per rupee per month.
8.   Rs. 175 for 9 months at 2p. per rupee per month.
9   Rs. 192 for 1½ years at 4p per rupee per month.
10. Rs 384 for 1¼ years at 5p per rupee per month.
11. Rs. 320 for 1⅓ years at 1p. per rupee per month.
12   £260. 13s 4d. for 1½ yrs. at 6d. per £ per month.
13. £350. 6s. 8d. for 2¼ yrs at 5d. per £ per month.
14. Rs. 521. 10a 8p for 1¾yrs. at 4p. per rupee per mo.
15. Rs 624. 8a. 4p for 1½yrs at 6p. per rupee per mo.

§6. To find interest when the rate per cent. per month is given

Example 3. Find the simple interest on Rs. 520 for one year 3 months at 12a. per cent. per month.

Sol   Int. on Rs 100 for 1 month $=$ Re. $\frac{3}{4}$

$\therefore$,,   ,,   ,, for 15 months $=$ Rs. $\frac{3}{4} \times 15$

$\therefore$,,   Re 1 for ,,   ,, $=$ Rs $\frac{3}{4} \times 15 \times \frac{1}{100}$.

$\therefore$,,   Rs 520 for ,,   ,, $=$ Rs. $\frac{3}{4} \times 15 \times \frac{1}{100} \times 520$

$$= \text{Rs. } \frac{117}{2}$$

$$= \text{Rs } 58 \; 8a. \text{ Ans.}$$

Hence when the rate per cent. per month is given we have the following

Rule *Interest* $= \dfrac{Principal \times rate \, p.c \times time \, in \, months.}{100}$

Thus in the above question

$$\text{Interest} = \text{Rs. } \frac{520 \times 12 \times 15}{16 \times 100} = \text{Rs } \frac{117}{3}$$

$$= \text{Rs } 58. \; 8a. \quad \text{Ans}$$

Example 4. Find the simple interest on Rs 721. 5a. 8p for 1 year 4 months at 8a per cent per month.

     Sol.   Principal $=$ Rs. 721 5a 8p

         Rate $=$ Re. $\frac{1}{2}$ per cent per month.

         Time $=$ 16 months

     $\therefore$ interest $=$ (Rs 721 5a. 8p) $\times \frac{1}{2} \times 16 \times \frac{1}{100}$

                   $=$ (Rs 721 5a. 8p) $\times \frac{2}{25}$.

	Rs.	a	p
$=$	721	5	8
			2
5)	1442	11	4
5)	288	8	8
	57	11	4 Ans

EXERCISE 133

Find the interest on —

1. Rs 525 for 2 years 8 months at 12a. p. c. per month.
2. Rs 630 for 1 year 8 months at 8a. p. c per month
3. Rs 780 for 2 years 1 month at 12a p c. per month
4. Rs. 625 for $2\frac{1}{4}$ years   at   5a 4p p. c per month.
5. Rs 675 for $2\frac{2}{3}$ years   at   10a. 8p. p c. per month.

6. £ 720 for $1\frac{1}{2}$ years at 15s.   p c. per month.

7. Rs 960 for $1\frac{3}{4}$ years at 13a 4p. p. c. per month

8   Rs 560 7a 6p for 10 months at 12a p c per month.

9   Rs. 995 11a. 8p. for 1 year 3 months at Re. 1. 2a. per cent per month.

10   Rs 1531 13a 4p. for 2 years 1 month at 14a. per cent per month.

---

§7   To find interest when the rate per cent. per annum is given.

Example 5   Find the simple interest on Rs. 566. 10a 8p for 5 years at 3 per cent

Sol.   Rs. 566. 10a 8p. = Rs $\frac{1700}{3}$

· int on Rs. 100 for 1 year = Rs 3

· „ „ Re. 1 for 1   „ = Re $\frac{3}{100}$

· „ „ Re 1 for 5 years = Re $\frac{3}{100} \times 5$

· „ „ Rs $\frac{1700}{3}$ for 5 „ = Rs $\frac{3}{100} \times 5 \times \frac{1700}{3}$

  = Rs 85   Ans.

Hence when the rate per cent. per annum is given we have the following

Rule. $Interest = \dfrac{Principal \times rate \times time}{100}$

Sometimes it is convenient not to reduce the principal to its highest denomination. If the fraction occurs in the rate % or in number of years it is still more convenient to multiply together first

Example 6   Find the simple interest and the amount on Rs 829. 4a 6p for $2\frac{2}{3}$ years at $6\frac{1}{4}$ per cent.

Sol. Principal = Rs. 829. 4a 6p.

  Time = $2\frac{2}{3}$ years.

  Rate = $6\frac{1}{4}$ p c.

∴ interest = (Rs 829 4a 6p) $\times \frac{8}{3} \times \frac{25}{4} \times \frac{1}{100}$

  = $\dfrac{Rs\ 829\ 4a\ 6p}{6}$

  = Rs 138 3a. 5p Ans.

Note The amount may be obtained by adding the interest to the principal

EXERCISE 134.

Find the simple interest on

1   Rs 750   for 1½ years at 4 per cent.
2   Rs 800   for 2½ years at 4 per cent.
3. Rs 350   for 3 years at 4 per cent.
4   Rs 825   for 2½ years at 5 per cent.
5   Rs. 728   for 3½ years at 6¼ per cent.
6. Rs. 533 5a 4p.   for 3⅓ years at 2¼ per cent.
7   Rs. 724 2a. 8p   for 3⅓ years at 5 per cent
8. Rs. 536. 10a. 8p   for 4⅓ years at 10 per cent.
9   Rs 833. 5a 4   for 1²⁄₇ years at 3⅓ per cent
10 Rs. 531. 12a 4p   for 2½ years at 8 per cent.
11 Rs 728 13a 4p. for 4 years at 6¼ per cent.
12. Rs 525 12a 6p   for 4¾ years at 8 per cent.
13 Rs 712   6a. 2p   for 7½ years at 6⅔ per cent.
14 Rs 631. 3a. 7p   for 5⅛ years at 3¼ per cent
15. Rs 749   7a 8p   for 2⅔ years at 2¼ per cent

---

§8 To find interest when time is given in months and days

When the time is given in months and days, 12 months are reckoned to the year and 30 days to the month

Example 7 Find the simple interest on £553 2s. 6d for 2 years 3 months 12 days at 8 per cent

Sol. Principal = £553 2s 6d

   Rate = 8%

   Time = 2 yrs 3 months 12 days

   $= 27\frac{2}{5}$ months $= \frac{137}{5}$ months $= \frac{137}{60}$ yrs

.. interest $= (£553. 2s 6d) \times 8 \times \frac{137}{60} \times \frac{1}{100}$

   $= £553. 2s 6d = \frac{137}{750}$

$$
\begin{array}{rccl}
 & £ & s. & d. \\
 & 553 & 2 & 6 \\
 & & & 17 \\ \hline
 & 9403 & 2 & 6 \\
 & & & 8 \\ \hline
 & 75225 & 2 & 0 \\
 & 553 & 0 & 6 \\ \hline
10)& 75778 & 2 & 6 \\
15)& 7577 & 16 & 3 \\
5)& 505 & 3 & 9 \\ \hline
 & 101 & 0 & 9
\end{array}
$$

$\therefore 137 = 17 \times 8 + 1$

750 (brace for the last three divisions)

$\therefore$ interest $= £101$ $9d.$ **Ans.**

Note   We can solve such questions by means of Aliquot parts also, $e\ g$, thus —

	£.	s.	d.	
	553	2	6	
			8	Multiply by rate
	4425	0	0	
			2	„ by time.
	8850	0	0 = int. of 2 yrs
3 mon $= \frac{1}{4}$ of a year	1106	5	0 = „ of 3 mo
10 days $= \frac{1}{9}$ of 3 mos	122	18	4 = „ of 10 days
2 days $= \frac{1}{5}$ of 10 $days.$	24	11	8 = „ of 2 days.
10	10103	15	0 Divide by 100
10	1010	7	6
	101	0	9 Ans

Aliter   Working in decimals we shall proceed thus :—

Principal $= £553$ $2s$ $6d.$

$= £553\ 125$

8 p. c. of $£553\cdot125 = £44\ 25$

2

$£88\ 50\quad=$ int. of 2 yrs

$£11\ 0625=$ „   „ 3 months

$£\quad 7375=$ „   „ 6 days.

$£\quad \cdot7375=$ „   „ 6 days.

3 months $= \frac{1}{4}$ of a yr.

6 days $= \frac{1}{15}$ of 3 months

6 days $= \frac{1}{15}$ of 3 months

$£101\cdot0375$

20

$\cdot7500 s.$

12

$9\ 00 d.$

$\therefore$ $£101.9d$ Ans.

EXERCISE 135

Find the simple interest on —

1　Rs 2100 at 3p per rupee per mo for 5 mos. 20 days

2　Rs. 1752 at 12a p c. per mo for 6 months 18 days

3　Rs. 1520 at 8a. per cent per month for 2 years 3 months 15 days.

4.　Rs. 3500 at 2¾ per cent per annum for 1 year 2 months 15 days.

5.　Rs 1236. 13a. 7p. at 3¾ per cent for 3 years 2 months 12 days.

6　Rs 1605 7a. 6p. at 6 per cent. for 2 years 4 months 8 days。

Find the amount of —

7.　Rs 1533 5a 4p for 2 years 7 months 15 days at 4 per cent.

8　Rs 1256 10a 8p for 1 year 4 months 15 days at 6¼ per cent

9　Rs. 1666. 10a 3p for 3 years 6 months 20 days at 7½ per cent

10　Rs. 1526. 11a 1p for 1 year 2 months 12 days at 3⅓ per cent

§9.　To find interest when the time is given in days or years and days

When the time is given in days or in years and days, the year is taken to consist of 365 days It will be convenient if the student commits to memory the following multiplication table —

$$73 \times 1 = 73$$
$$73 \times 2 = 146$$
$$73 \times 3 = 219$$
$$73 \times 4 = 292$$
$$73 \times 5 = 365$$

Example 8　Find the simple interest on Rs. 356 5a. 8p for 1 year 219 days at 6¼ per cent,

F28.

Sol Principal=Rs. 356. 5a. 8p.
    Rate=6¼%
    Time=$1\frac{219}{365}$ or $\frac{8}{5}$ years.

∴ Interest=Rs 356 5a. 8p $\times (\frac{25}{4} \times \frac{8}{5} \times \frac{1}{100})$
       =Rs 356. 5a 8p.$\times \frac{1}{10}$
       =Rs 35 10a. 2p Ans.

§10 When simple interest is to be calculated from one date to another, the first day is not taken into consideration Thus —

Example 9 Find the simple interest on £151 6s 8d. from July 15, 1928 to December 8, 1928 at 12½ per cent per annum.

<table>
<tr><td></td><td>July</td><td>16</td></tr>
<tr><td>Sol Principal=£151 6s. 8d</td><td>Aug</td><td>31</td></tr>
<tr><td>Time =$\frac{146}{365}=\frac{2}{5}$ year</td><td>Sep</td><td>30</td></tr>
<tr><td>Rate =$\frac{25}{2}$ p c</td><td>Oct</td><td>31</td></tr>
<tr><td>∴ Interest=£151 6s.8d $\times(\frac{2}{5} \times \frac{25}{2} \times \frac{1}{100})$</td><td>Nov.</td><td>30</td></tr>
<tr><td>=£151. 6s 8d.$\times \frac{1}{20}$</td><td>Dec.</td><td>8</td></tr>
<tr><td>=£7. 11s 4d. Ans</td><td colspan="2">————</td></tr>
<tr><td></td><td colspan="2">146 days.</td></tr>
</table>

§11 The student should be on guard to count 29 days for February if the year be a leap one, for instance the time from 1st January 1896 to 19th October 1896= 30+29+31+30+31+30+31+31+30+19=292 days or $\frac{292}{365}=\frac{4}{5}$ year.

EXERCISE 136.

Find the simple interest on—

1. Rs 365   for 125 days at 4 p. c.
2. Rs. 730   for 130 days at 5 p. c.
3. Rs. 625   for 73 days at 4½ p. c.
4   Rs 835   for 146 days at 6¼ p. c.
5. Rs. 730   for 192 days at 10 p. c.
6   Rs. 533. 5a. 4p for 219 days at 2¼ p. c.
7   Rs 666 10a 8p. for 292 days at 3¾ p. c.
8   £720 from 15th January to 29th March at 3 p.c.

9.　£520 from July 1 to September 12 at 5 p c.

10　£625 from August 1 to October 13 at $3\frac{1}{8}$ p c

11　£525 from November 15 to June 22 at 8 p c

12　£675 from August 4 to October 16 at $6\frac{1}{4}$ p c.

13.　£716 13s. 4d from July 15 to Dec. 8 at 5 p c

14　£728 6s 6d. from Dec 16, 1929 to July 23, 1930 at $6\frac{1}{4}$ per cent

15　Rs 1666 10a 8p. from June 15, 1928 to Aug 27, 1930 at 4 per cent

16.　A person borrowed Rs 7300 on November 15, 1923 at 4 per cent. What amount should he repay on July 6, 1924 to clear off the account ?

17　A borrowed Rs 4500 on August 4, 1925 at $3\frac{1}{3}$ per cent and paid back the sum with interest on Oct 16, 1925　B also borrowed Rs 3500 on the same day and paid the sum together with interest on Dec. 28, 1925 at 3 per cent Who paid the interest more and by how much ?

18　A borrowed Rs. 5000 on Sept. 15, 1912 at 4 p c and paid back Rs 3160 on July 4, 1913　Find what amount should he pay on Nov 27, 1913 to clear off the account

19　Find to the nearest penny the interest on £169. 15s 11d from 9th Nov 1867 to 3rd March 1868 at $5\frac{3}{4}$ p. c.

20.　Rs. 8545 were lent to a person for 2 years 3 months at $3\frac{3}{4}$ p c , after the expiry of that period he gave a horse and Rs 172 14a $8\frac{9}{5}p$ in cash as interest , find the price of the horse.

INVERSE CASES OF INTEREST

§12　To find Principal

Consider the following examples when *Time, Rate per cent. and Interest or Amount are given.*

Case 1　When the Interest is given

Example 1　What sum of money will produce Rs, 99 interest in 3 years at 4 per cent. ?

Sol　Let us suppose the sum = Rs 100

.. Interest on Rs. $100 = \dfrac{100 \times 3 \times 4}{100} = $ Rs 3×4

Now by unitary method we shall proceed thus —

If the interest is Rs 3×4, the sum $=$ Rs 100

If   „   is Re. 1,   „ $=$ Rs $\dfrac{100}{3 \times 4}$

If   „   is Rs 99,   „ $=$ Rs $\dfrac{100 \times 99}{4 \times 3}$

$=$ Rs. 825 **Ans.**

From the asterisked line we conclude that when time, rate per cent and interest are given we have the following

Rule. $Principal = \dfrac{Interest \times 100}{Time \times Rate}$.

Case 2.   When the amount is given

Example 2   What sum of money, if put out on interest at 4 p c will amount to Rs 560 in 3 years ?

Sol.   Let us suppose the sum $=$ Rs. 100

Interest on Rs 100 $= \dfrac{100 \times 4 \times 3}{100} =$ Rs 4×3

Amount $=$ Rs $(4 \times 3 + 100)$.

Now by unitary method we shall proceed thus —

If the amount is Rs. $(4 \times 3 + 100)$ the sum $=$ Rs 100,

$\therefore$ „   Re 1   „ $=$ Rs $\dfrac{100}{4 \times 3 + 100}$

„   Rs 560   „ $=$ Rs $\dfrac{100 \times 560}{4 \times 3 + 100}$

$=$ Rs 500 **Ans**

From the asterisked line we conclude that when time, rate p c and amount are given we shall have the following

Rule $Principal = \dfrac{Amount \times 100}{Time \times Rate + 100}$.

Example 3   What principal will produce Rs. 172 8a interest in 5 years at $3\frac{3}{4}$ p c ?

Sol. $\because$ Principal $= \dfrac{Interest \times 100}{Time \times Rate}$

$\therefore$ reqd. principal $=$ Rs. $\dfrac{172 \ 8a. \times 100}{\frac{15}{4} \times 5}$

$=$ Rs. $\dfrac{345 \times 100 \times 4}{2 \times 75} =$ Rs. 920 **Ans.**

Example 4 What principal will amount to Rs. 1702 8a in 1½ years at 9 % ?

Sol   Principal = $\dfrac{\text{Amount} \times 100}{\text{Time} \times \text{Rate} + 100}$

∴ reqd principal = Rs $\dfrac{1702\ 8a\ \times 100}{\frac{3}{2}\times 9 + 100}$ or $\dfrac{\frac{3405}{2}\times 100}{\frac{7}{1}7}$

= Rs $\frac{3405}{2}\times 100 \times \frac{2}{117}$ = Rs 1500 **Ans**

EXERCISE 137

1 What sum of money will produce Rs 52 as interest in 4 years at 2⅓ per cent ?

2 What sum must be put out at simple interest for 3 years at 2½% so as to produce Rs 62 8a interest ?

3. What sum will earn Rs 189 in 1½ years at 12% ?

4 What sum will earn Rs 48 in 5 years at 1½% ?

5 On what sum will the simple interest amount to Rs. 180 in 2¼ years at 15% ?

6. On what sum will the simple interest amount to Rs. 33. 4a 6p in 2½ years at 5% ?

7 What principal will amount to Rs. 672 in 3 years at 4% ?

8 What principal will amount to Rs 840 in 5 years at 8 per cent ?

9 What principal in 3 years 6 months will become Rs 1175 at 5 p c ?

10 What sum of money, if put out at simple interest at 3¾ p c will amount to Rs. 2805 in 4½ years ?

11. What sum will amount to Rs. 8414 4a in 2½ years at 3½ p c simple interest ?

12 How much must I put in the Bank so that in 7 years at 2½ p. c S. I, I may withdraw £8812 10s. altogether ?

13 What sum should Tom deposit in the savings account, so that in 2⅓ years at 7¼ p c he may withdraw £199 10s altogether ?

14 What sum of money if put in a bank for 2¾ years at 3¾ p c simple interest will amount to Rs 1765 ?

15 What sum of money in 2⅓ years at 5 p c. simple interest will amount to Rs 883 2a ?

§13   To find rate per cent

When Principal, Amount or Interest and Time are given, rate per cent is found as shown in the following examples —

Example 1   At what rate per cent will the simple interest on Rs 520 be Rs 104 in 4 years ?

Sol   By unitary method we proceed thus

∴   Interest on Rs 520 for 4 years = Rs 104

,,   ,,   Re   1   for 4 years = Rs. $\frac{104}{520}$

,,   ,,   Re.   1   for 1 year   = Rs $\frac{104}{520 \times 4}$

∴   ,,   ,,   Rs 100 for 1 year   = *Rs $\frac{104 \times 100}{520 \times 4}$

= Rs 5   Ans.

Hence from the asterisked line we have the following

Rule   *Rate per cent.* = $\dfrac{Interest \times 100}{Principal \times Time}$.

Example 2.   At what rate per cent will Rs. 720 amount to Rs 774 in 3 years ?

Sol   Interest = Rs. 774 − Rs 720 = Rs 54.

∴   rate per cent. = $\dfrac{\text{Interest} \times 100}{\text{Principal} \times \text{Time}}$

∴   reqd. rate   = $\dfrac{54 \times 100}{720 \times 3} = \frac{5}{2} = 2\frac{1}{2}\%$.   Ans

EXERCISE 138.

1   At what rate per cent. will Rs. 550 gain Rs. 66 in 3 years ?

2.   The interest of Rs 350 amounted to Rs. 42 in 3 years ; find the rate per cent

3   At what rate per cent. will the interest on Rs 750 be Rs. 150 in 4 years ?

4.   At what rate per cent. will Rs. 600 gain Rs 120 in 5 years ?

5 At what rate per cent. will Rs. 400 amount to Rs 428 in 2 years ?

6 At what rate per cent will Rs. 2000 amount to Rs. 2135 in 3 years ?

7. Rs 800 amounted to Rs 980 in 3 years , find the rate per cent.

8 Rs 600 amounted to Rs 654 in $2\frac{1}{4}$ years , find the rate per cent.

9 At what rate per cent. will Rs. 1225 amount to Rs. 1439 $6a.$ in $2\frac{1}{2}$ years ?

10 The interest on £716 amounted to £134. 5s. in $3\frac{3}{4}$ years , find the rate per cent

11. At what rate per cent will Rs. 1275 $13a$ $4p$ amount to Rs 1371 $8a.$ $4p$ in $2\frac{1}{2}$ years ?

12 At what rate per cent. will Rs. 1327 $10a$ gain Rs. 398 $4a.$ $7\frac{1}{5}p.$ in 6 years ?

13 At what rate per cent will Rs. 522 $1a$ $4p$ gain Rs 130. $8a.$ $4p.$ in 5 years ?

14 At what rate per cent. will Rs. 533 $5a.$ $4p.$ amount to Rs. 595. $13a.$ $4p$ in $2\frac{1}{3}$ years ?

15 Find the rate per cent if Rs 266. $4a.$ amount to Rs. 299. $8a.$ $6p.$ in 5 years

§14 To find Time,

The method of finding Time when Principal, Rate per cent. and Amount or Interest are given will be understood from the following examples —

Example 1 In what time will the interest on Rs. 800 amount to Rs. 160 at 5 per cent per annum ?

Sol Interest on Rs. 800 at 5 p. c for 1 year $=$ Rs $\dfrac{800 \times 5}{100}$

Now by unitary method we proceed as follows —

∙ Rs. $\dfrac{800 \times 5}{100}$ is the interest for 1 year,

∴ Re. 1 is the interest for $\dfrac{100}{800 \times 5}$ years,

∙ Rs. 160 is the interest for $\dfrac{100 \times 160}{800 \times 5}$ years.

∴ the required time $=$ 4 years. Ans

Hence from the asterisked line we have the following

Rule $Time = \dfrac{Interest \times 100}{Principal \times Rate\,per\,cent}$

Example 2. In what time will Rs 560 amount to Rs. 700 at 4 per cent ?

Sol. Interest = Rs 700 — Rs 560 = Rs. 140.

$\because$ Time $= \dfrac{Interest \times 100}{Principal \times Rate\,per\,cent}$.

$\therefore$ reqd time $= \dfrac{140 \times 100}{560 \times 4} = \frac{25}{4}$

$= 6\frac{1}{4}$ years. **Ans.**

EXERCISE 139

1   In what time will Rs 750 at simple interest produce Rs. 150 at 5 per cent ?

2   In what time will the interest on Rs 825 amount to Rs 99 at 4 per cent ?

3.   In what time will Rs. 1025 amount to Rs. 1312 at 7 per cent.?

4.   In what time will Rs 720 at simple interest produce Rs 120 at $2\frac{1}{2}$ per cent ?

5.   In what time will the interest on Rs. 575 amount to Rs. 362 $4a$. at 9 per cent ?

6   In what time will the interest on Rs 721. $4a$. amount to Rs 144 $4a$ at 5 per cent. ?

7   In what time will Rs 729. $4a$ amount to Rs 957. $2a$ $3p$ at $6\frac{1}{4}$ per cent ?

8.   In what time will a sum of money double itself at 10 per cent ?

9   In what time will Rs. 227 $10a$. $8p$. trible itself at 8 per cent.?

10.   In what time will Rs. 481. $1a$. produce interest equal to the principal at 15 per cent.?

11.   In what time will the interest on any sum of money at 5 per cent. be ˙125 of the principal ?

12   In what time will Rs. 845 4a amount to Rs 1056 9a at $1\frac{1}{2}p$ per rupee per month ?

13   In what time will a sum of money trible itself at $12\frac{1}{2}$ per cent ?

14   A person borrowed Rs 1525 at $2\frac{1}{2}$ per cent. on July 5 and after a certain period paid Rs. 7 10a. as interest, find the date of payment

15.   On February 1, 1926 a person borrowed £500 at 5 p c promising to return it as soon as the interest amounted to £10 , on what date did the loan expire ?

16.   In how many years will 450 fr amount to 576 fr. at $3\frac{1}{3}$ p c. simple interest ?

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§15   Some Important Typical Examples

Example 1   If Rs 566 10a 8p amount to Rs 680 in 4 years, what will Rs 728 5a 4p amount to in $2\frac{1}{2}$ years at the same rate p c per annum ?

Sol   Int on Rs 566 10a 8p $=$ Rs. 680 $-$ Rs. 566. 10a 8p.
$$=\text{Rs } 113.\ 5a\ 4p=\text{Rs. } \tfrac{340}{8}$$

$$\text{rate p c}=\frac{\tfrac{340}{4}\times100}{566\tfrac{2}{3}\times4}=\frac{340\times100\times3}{1700\times4\times3}=5$$

Now int. on Rs 728 5a 4p $=\dfrac{(\text{Rs. }728\ 5a\ 4p\,)\times5\times5}{100\times2}$

$$=\frac{\text{Rs }728\ 5a\ 4p}{8}$$

$$=\text{Rs } 91\ 0a\ 8p$$

∴ the reqd. amount $=$ Rs 728 5a 4p + Rs. 91 8p.
$$=\text{Rs } 819\ 6a\quad\text{Ans}$$

Example 2.   A sum of money in $3\frac{1}{2}$ years at $6\frac{1}{4}\%$ amounts to £781 19s. What will it amount to in 4 years at $7\frac{1}{2}$ % ?

Sol.   Amount $=$£781 19s. $=$£$781\tfrac{19}{20}=$£$\tfrac{15639}{20}$.
    Time $=\frac{7}{2}$ years
    Rate $=\frac{25}{4}$ p c

∴ Principal $=$£$\dfrac{\tfrac{15639}{20}\times100}{(\tfrac{7}{2}\times\tfrac{25}{4})+100}=$£$\dfrac{15639\times5}{\tfrac{975}{2}}$

$$=\pounds\frac{15639\times5\times8}{975}=\pounds\frac{3208}{5}=\pounds641\ \ 12s.$$

Now interest on £641. 12s. for 4 years at $7\frac{1}{2}\%$

$$=\frac{\pounds641\ \ 12s\times4\times15}{100\times2}$$

$$=\frac{\pounds641.\ \ 12s\times3}{10}$$

$$=\frac{\pounds1924\ \ 16s.}{10}=\pounds192.\ 9s.\ 7\cdot2d.$$

$\therefore$ Amount$=\pounds641\ \ 12s+\pounds192\ 9s.\ 7\cdot2d.$
$$=\pounds834\ \ 1s.\ 7\cdot2d.\ \ \mathbf{Ans.}$$

Example 3   A certain sum of money amounts to Rs 768 in 4 years and to Rs. 832 in 6 years.   Find the sum and the rate %.

Sol. Rs 832—Rs. 768=Rs 64,
$i,\ e.,$ Rs. 64 is the interest for 2 years.
$\therefore$ interest for 4 years=Rs. 128
$\therefore$ Principal=Rs 768—Rs. 128=Rs. 640.   **Ans**
$\because$ Rate$=\dfrac{\text{Int}\times100}{\text{Principal}\times\text{Time}}$

$\therefore$ reqd. rate$=\dfrac{128\times100}{640\times4}=5$ p. c. **Ans**

Example 4.   A certain sum amounts to £583 at $2\frac{1}{2}\%$ but at $\frac{1}{2}\%$ less rate to £572. 8s , find the time and sum

Sol. Int. at $\frac{1}{2}$ p. c. on the Principal$=\pounds583-\pounds572\ 8s$
$$=\pounds10.\ 12s.=\pounds\tfrac{53}{5}$$
$\therefore$ Int at $2\frac{1}{2}$ p. c.   ,,   ,, $=\pounds\tfrac{53}{5}\times2\times\tfrac{5}{2}=\pounds53.$
$\therefore$ Principal$=\pounds583-\pounds53=\pounds530$ $\Big\}$ **Ans.**
and Time $=\dfrac{53\times100}{530\times\frac{5}{2}}=4$ years

Example 5.   £320 amount to £384 at a certain rate in a certain time, also £625 amount to £775 in the same time at 1% higher rates , find the rates and time.

Sol   (1) Int. on £320$=\pounds384-\pounds320=\pounds64$
$$\therefore\ R\times T=\frac{64\times100}{320}=20$$

(2) Int on £625=£775−£625=£150.

$$\therefore (R+1)\times T=\frac{150\times100}{625}=24$$

$$\therefore \frac{R\times T}{(R+1)\times T}=\tfrac{20}{24}=\tfrac{5}{6}, \text{ or } \frac{R}{R+1}=\tfrac{5}{6}$$

$$\therefore 5(R+1)=6R \text{ or } 5R+5=6R$$

$$\therefore R=5\%, \text{ higher } R=6\% \text{ and } T=4 \text{ yrs Ans.}$$

Example 6   A person invested Rs 1200 for 4 years and Rs 900 for 3 years and the total interest from these investments was Rs 375, find the rate per cent

Sol.   In the first case he invested Rs 1200×4

$$= \text{Rs 4800 for one year}$$

In the second case he invested Rs. 900×3

$$= \text{Rs 2700 for one year.}$$

$\therefore$ total investment for one year=Rs. 4800+Rs 2700

$$= \text{Rs. 7500.}$$

$$\therefore \text{Rate} = \frac{\text{Int.}\times100}{\text{Principal}\times\text{Time}}$$

$$\therefore \text{reqd rate} = \frac{375\times100}{7500\times1}=5 \text{ p c. Ans.}$$

Example 7   A person invested £3200 at a certain rate per cent. and £2800 at 1 per cent higher rate. The total interest from these investments in 4 years was £952, find the rates of interest.

Sol   Int on £3200+£2800 for 4 years at the certain rate+int. on 2800 for 4 years at 1 per cent =£952

or int. on £6000 for 4 years+$\dfrac{2800\times4\times1}{100}$=£952

or int on £6000 for 4 years+£112 =£952

$\therefore$ int on £6000 for 4 yrs. at the certain rate =£840

$$\left.\begin{array}{l}\therefore \text{certain rate}=\dfrac{840\times100}{6000\times4}=3\tfrac{1}{2} \text{ p. c.}\\[2mm]\therefore \text{ second rate}= 3\tfrac{1}{2}+1 =4\tfrac{1}{2} \text{ p. c}\end{array}\right\} \text{Ans.}$$

Example 8   A man had Rs. 1200, part of which he lent at 5 per cent and a part at 4 per cent   He got Rs. 106 as interest after 2 years.   How much did he lend at 5 p c.?

Sol.   If he had lent the whole sum at 5 per cent , the interest would have been Rs $\dfrac{1200 \times 5 \times 2}{100} = $ Rs   120 , but the actual interest is Rs 106, i e , the interest would have been increased by Rs 14

This increase of Rs. 14 will be decreased in the interest of the second sum

Interest on Rs 100 at 5 per cent for 2 years $=$ Rs. 10
Interest on Rs 100 at 4 per cent. for 2 years $=$ Rs   8
i e , there is an increase of Rs 2 per hundred

increase of Rs 14 will be on Rs $\dfrac{100}{2} \times 14 = $ Rs 700.

∴ he lent Rs 1200 − 700 $=$ Rs 500 at 5 p c   **Ans**

Aliter   Suppose he lent Rs 1200 at 5 p c for 2 yrs.

∴   interest $= \dfrac{1200 \times 5 \times 2}{100} = $ Rs. 120

i. e , the int will increase by Rs 120 − Rs. 106 $=$ Rs 14

Now suppose he lent Rs. 1200 at 4 per cent. for 2 yrs.

∴   int. $= \dfrac{1200 \times 4 \times 2}{100} = $ Rs 96

i e , the int will decrease by Rs 106 − Rs 96 $=$ Rs 10

∴ the ratio between the two investments will be inverse of 14 : 10, i e , 10 : 14 or 5 : 7

investment at 5 p c $=$ Rs. $\frac{5}{12} \times 1200 = $ Rs.500 **Ans**

Aliter   The mean int | int at 5 p. c   int at 4 p c
for 2 yrs per Rs 100 | Rs 10   Rs 8
$= $ Rs $\dfrac{106 \times 100}{1200}$ | Rs $\frac{53}{5}$
$= $ Rs $\frac{53}{5}$ | Rs $\frac{53}{8}$ − Rs 10 − $\frac{53}{8}$
                            | $\frac{5}{8}$ $\frac{7}{8}$
                            | 5   7

By the Alligation Rule, ratio between the investments at 5 p c and 4 p. c $= 5 : 7$

investment at 5 p c $=$ Rs $\frac{5}{12} \times 1200 = $ Rs. 500. **Ans**

Example 9   A person had Rs 1343 $5a$ $4p$, part of which he lent at 5 p c for 4 years and the rest at $6\frac{1}{4}$ p c for 3 years , the interests he received, in both the cases, were equal , how much did he lend at $6\frac{1}{4}$ p c ?

Sol Suppose he lent Rs 1343 $5a$ $4p$. at 5 p c for 4 yrs.

$\therefore$ int $= \dfrac{\text{Rs. } 1343\ 5a\ 4p \times 5 \times 4}{100}$

$\qquad = \text{Rs } 268\ 10a\ 8p = \text{Rs } \frac{808}{3}$

Now suppose he lent Rs 1343 $5a$ $4p$ at $6\frac{1}{4}$ p c. for 3 yrs

$\therefore$ int $= \dfrac{\text{Rs } 1343\ 5a\ 4p. \times 25 \times 3}{100 \times 4}$

$\qquad = \text{Rs } \frac{4030}{16} = \text{Rs } \frac{2015}{8}$

Since both the interests are equal,

$\qquad$ ratio between the two investments $= \frac{2015}{8} : \frac{806}{3}$.

$\qquad\qquad\qquad\qquad\qquad = 6045 : 6448.$

$\qquad\qquad\qquad\qquad\qquad = \quad 15 : 16$

$\therefore$ investment at $6\frac{1}{4}$ p c $= \frac{16}{31}$ of Rs. 1343 $5a$ $4p$

$\qquad\qquad\qquad\quad = \text{Rs } 693\ 5a\ 4p$ **Ans.**

Aliter   Suppose first investment $=$ Rs 100.

$\qquad$ int $= \dfrac{100 \times 5 \times 4}{100} = \text{Rs } 20.$

$\qquad$ int on the second investment at $6\frac{1}{4}$ p c for 3 yrs. is also Rs 20

$\therefore$ second sum $= \dfrac{\text{Int} \times 100}{\text{Time} \times \text{Rate}} = \dfrac{20 \times 100}{6\frac{1}{4} \times 3}$

$\qquad\qquad\qquad = \dfrac{20 \times 100 \times 4}{25 \times 3} = \text{Rs } \frac{320}{3}$

$\therefore$ if first investment $=$ Rs 100, the second $=$ Rs $\frac{320}{3}$

$\therefore$ the ratio between the two investments $= 100 : \frac{320}{3}$

$\qquad\qquad\qquad\qquad\qquad\qquad = 300 : 320$

$\qquad\qquad\qquad\qquad\qquad\qquad = \ 15 : 16.$

We can now proceed further and get the answer

Example 10   A person had Rs 3500. He lent a part of it at 4 p. c. for 3 years and the remaining at $4\frac{1}{2}$ p c. for $2\frac{1}{2}$ years   The interest in the first case is Rs 48 more than the interest received in the second case. Find both the sums.

Sol. Principal when int. is Rs. 48, time 3 yrs, and rate 4 p. c $=\dfrac{48 \times 100}{3 \times 4}=$Rs 400

∴ remaining sum $=$ Rs 3500$-$Rs. 400$=$Rs 3100.

Now divide Rs 3100 into two parts such that the interest on the first part at 4 p c in 3 years may be equal to the interest on the second part at $4\frac{1}{2}$ p. c in $2\frac{1}{2}$ yrs [*See* Ex 9] and then add Rs. 400 to the first part.

Example 11. What annual payment will discharge a debt of Rs 848 due in 4 years, the rate of interest being 4% ?

Sol. Let x rupees be the annual payment.

Amount of Rs. x in 3 yrs. at 4 p c $=$ Rs $x + \dfrac{x \times 3 \times 4}{100}=\dfrac{112x}{100}$

„   „   2 yrs   „ $=$ Rs $x + \dfrac{x \times 2 \times 4}{100}=\dfrac{108x}{100}$

„   „   1 yr.   „ $=$ Rs. $x + \dfrac{x \times 1 \times 4}{100}=\dfrac{104x}{100}$

These amounts and the last payment of Rs x will discharge the debt.

∴ $\dfrac{112x}{100}+\dfrac{108x}{100}+\dfrac{104x}{100}+x=$Rs 848

$\tfrac{424}{100}x=$Rs. 848

$x=$Rs $848 \times \tfrac{100}{424}=$Rs 200   Ans

Example 12 Divide Rs 945 12a 4p into three parts such that the amounts of these parts after $2\frac{1}{2}$, 4 and 5 years respectively may all be equal, the rate of interest being 5% per annum

Sol   Amount of first part$=\dfrac{\text{first part} \times 5 \times 5}{100 \times 2}+$first part

$=\dfrac{\text{first part}}{8}+$first part

$=$first part $(\tfrac{1}{8}+1)$

$=\tfrac{7}{8} \times$first part

Similarly amount of 2nd part $=\frac{6}{5}\times$ second part

and   ,,   ,, 3rd part $=\frac{4}{5}\times$ third part

$\frac{9}{8}\times$ first part $=\frac{6}{5}\times$ 2nd part $=\frac{4}{5}\times$ third part

$\therefore$ first part   2nd part   3rd part $=\frac{8}{9}$ $\frac{5}{6}$. $\frac{4}{5}=80$ 75   72

Now divide the given sum in the ratio of 80   75 . 72.

MISCELLANEOUS EXERCISE 140

1. Find the simple interest on Rs. 365 8a from August 14, 1920 to March 21, 1921 at $3\frac{1}{8}$%

2 Find the simple interest on Rs 756 4a from January 5, 1924 to May 30, 1924 at $3\frac{3}{4}$%.

3 Find the amount of Rs 2560 at $2\frac{1}{4}$% from June 15, 1920 to January 20, 1921

4. Indar borrowed Rs 785 for 3 years at 3p per rupee per month and Sundar borrowed Rs 665 for 4 years 2 months at the same rate , who will pay more interest and by how much ?

5. What sum of money will amount to Rs. 540. 10a. in $2\frac{1}{3}$ years at $3\frac{1}{4}$% ?

6. At what rate p c will Rs. 325 amount to Rs 349. 6a in $2\frac{1}{3}$ years ?

7 In what time will Rs. 521. 15a. 11p double itself at 8% ?

8 The interest of a certain sum amounts to Rs 70 5a in 5 years at $2\frac{1}{4}$% , find it.

9 At what rate p c will the interest on Rs 457. 8a. amount to Rs 114. 6a. in 8 years ?

10. In what time will Rs. 965 amount to Rs. 1206. 4a at 5% ?

11 Amount of a certain sum in 3 years is Rs 545 and in 4 years, Rs 560 , find the sum and the rate p c

12 Amount of a certain sum in 3 years is Rs. 575 and in 5 years, Rs. 625 , find the sum and the rate p c.

13 A certain sum amounts to Rs. 600 in 4 years , if the rate of interest be increased by one p c it amounts to Rs 620. Find the sum and the rate per cent.

14   A certain sum amounts to Rs 456 in $3\frac{1}{2}$ years and Rs. 464 in 4 years , find it and also find the rate p. c

15   What sum of money will amount to Rs 1062 in $5\frac{1}{4}$ years at $3\frac{3}{7}$ p. c.

16   What sum of money will produce Rs. 21 $8a$ in 146 days at 5 p c

17   Rs 1000 were borrowed on July 1, 1923 at $3\frac{1}{2}$ p. c., and after a time the account was cleared by paying Rs 1014 , find the date of payment.

18   At what rate per cent. will the interest of a sum in $2\frac{1}{2}$ years amount to $\frac{1}{12}$ of the principal ?

19   At what rate per cent will the interest of a sum in $2\frac{2}{3}$ years amount to $\frac{3}{16}$ of the principal ?

20   What sum of money will amount to Rs 829 $8a$. $7\frac{1}{4}$ from July 15 to September 26 at $2\frac{3}{4}$ per cent ?

21   What sum of money will produce Rs. 121 $4a$. $6p$ from March 13 to July 16 at $9\frac{1}{3}$ p c ?

22   If Rs 621. $4a$. amount to Rs 745. $8a$ in 4 years, what will Rs. 662 $10a$ $8p$ amount to in $2\frac{1}{2}$ years at the same rate per cent per annum simple interest ?

23   If Rs 525 amount to Rs. 616 $14a$. in $4\frac{2}{3}$ years, what will Rs 780 amount to in 4 years at the same rate per cent per annum simple interest ?

24.   A person lent Rs 1200 for 4 years and Rs 900 for 3 years and thus got Rs 375 as interest, find the rate p c.

25.   A person lent Rs 800 for 5 years and Rs 900 for 4 years and thus got Rs 304 as interest , find the rate p.c.

26   A person lent Rs 625 for 4 years, Rs 300 for 5 years and Rs. 820 for 2 years and thus got Rs. 282 as interest , find the rate per cent

27   A sum of money in 4 years at 5 p. c amount to £864. $12s$, what will it amount to in 3 years at $6\frac{1}{4}$ p c ?

28   A person had Rs 2000, part of which he lent at 5 p c and the remainder at $4\frac{1}{2}$ p c He got Rs 282 as interest after 3 years. How much did he lend at $4\frac{1}{4}$ p c ?

29. A person had Rs 2100 He lent a part of it at 5 per cent. and the remainder at 4 per cent , after 3 years he found that the dieffrence between the interests was Rs. 9 only Find both the sums

30 Interest of a certain sum in 5 years at 5 per cent is more than the interest of the same sum in 6 years at $3\frac{1}{4}$ per cent by Rs 260 , find the sum

31 Interest of a certain sum in 8 years at 5 per cent. is greater than the interest of the same sum in 9 years at 3 per cent by Rs 169 , find the sum.

32 Divide Rs 2000 into two parts such that if the first be put out at S. I for 6 years at $3\frac{1}{2}$ p c and the second for 3 years at $4\frac{1}{2}$ p. c the interest on the first sum shall be double that on the second

33. A person invested £2500 at a certain rate per cent and £1500 at 1 p c higher rate and the total interest from these investments in 3 years was £525 , find the rates

34 A person invested £1200 at a certain rate per cent. and £1600 at $2\frac{1}{2}$ p c less rate and the total interest from these investments in 4 years was £680 , find the rates

35 A person had Rs 1566 10a 8p , part of which he lent at 5 p c for 3 years and the rest at $4\frac{1}{2}$ p. c. for $4\frac{1}{2}$ years The interest realized in both the cases was the same. How much did he lend at 5 p c ?

36 What annual payment will discharge a debt of Rs 1620 due in 5 years at 4 p c ?

37 (a) Divide £182 3s 4d into three parts in such a way that their amounts after 1, 2 and 3 years respectively may be equal, the rate of interest being 4 per cent

(b) A, B and C borrowed Rs 9746 in all from a money-lender at 5 p interest and paid back equal amounts in full settlement of their respective debts after 2, 5 and 6 years respectively Find the sum borrowed by each

38 £450 amount to £504 at a certain rate per cent. and in a certain time, also £720 amount to £828 in the same time at 1 p. c. higher rate Find the rate and time

39. £360 amount to £432 in a certain time at a certain rate per cent , also £540 amount to £621 in a year less at the same rate , find the rate and time

40 A certain sum amounts at 5 p c to £780 but at 4 p. c less rate to £760, 10s, Find the time and sum,

F 29

CHAPTER XXVI.

COMPOUND INTEREST.

§1   We have mentioned in Chapter XXV that when the consideration for the use of money borrowed is paid yearly, half-yearly or monthly as agreed upon and the principal remains the same, the consideration thus paid is called the simple interest   But sometimes the interest is not paid as stated above and is added yearly or half-yearly to the principal and this amount becomes the principal for the next period.   This system *of adding the* interest to the principal continues till the   end of the specified time   Money, in this case, is said to be at compound interest and the difference between the final amount and the original principal is called the **Compound Interest**

Thus simple interest on Rs   200 at   5 per cent   in 2 years $=$ Rs   20

But the compound interest as stated above will be reckoned as follows —

Interest on Rs. 200 at 5 per cent in one year $=$ Rs.   10

Now the principal for the next year would   be Rs. 200 $+$ Rs. 10 $=$ Rs   210

Interest on Rs. 210 at 5 per cent in one year $=$ Rs. 10   8*a*

$$\therefore \quad \text{final amount} = \text{Rs } 210 + \text{Rs } 10.\,8a. = \text{Rs } 220 \; 8a.$$
$$\text{compound interest} = \text{Rs } 220 \; 8a. - \text{Rs } 200$$
$$= \text{Rs } 20 \; 8a. \text{ Ans.}$$

Note 1   Compound interest is evidently always more than the simple interest

2   The compound interest might also be obtained by adding together the interests for the 1st year and 2nd year

§2   It is convenient to work sums in compound interest in decimals   The following examples will illustrate the method clearly —

Example 1 Find the compound interest on Rs. 500 for 3 years at 5 p c

Sol. Rs.   500   =Principal for 1st year.
                  5 (multiply by 5 and divide by 100)

Add {   25 $\emptyset\emptyset$ =interest for 1st year
           500

           525   =Principal for 2nd year
             5   (multiply by 5 and divide by 100)

Add {   26 25   =Interest for 2nd year
           525

           551'25   =Principal for 3rd year
               5   (multiply by 5 and divide by 100)

Add {   27 5625   =Interest for 3rd year
           551'25

Subtract {   578 8125   =Total amount
                500   =Principal

                78 8125   =Compound int for 3 years.
                         =Rs. 78. 13a Ans

Note Division by 100 is done mentally as explained in Art 5, Chapter VIII

Aliter. Amount of Rs 100 at the end of 1 year = Rs 105

∴ „ „ Re 1 „ „ = Rs $\frac{105}{100}$

∴ „ „ Rs. 500 „ „ = Rs. $\frac{105}{100}$ of Rs 500

   „ „ Rs 500 „ 2 years
                = $\frac{105}{100}$ of $\left(\frac{105}{100}$ of Rs 500$\right)$
                = $\left(\frac{105}{100}\right)^2$ of Rs 500.

   „ „ Rs. 500 at the end of 3 years
                = $\frac{105}{100}$ of $\left\{\left(\frac{105}{100}\right)^2$ of Rs. 500$\right\}$.
                = $\left(\frac{105}{100}\right)^3$ of Rs 500

Hence the following Formulae—

Amount = Principal $\left(1+\frac{rate}{100}\right)^{No\ of\ yrs.}$

Compound interest = Principal $\left\{\left(1+\frac{rate}{100}\right)^{No.\ of\ yrs.} -1\right\}$

or briefly $A = P\left(1 + \dfrac{r}{100}\right)^n$ ... . (1)

$C.I. = P.\left\{\left(1 + \dfrac{r}{100}\right)^n - 1\right\}$ . .. (2)

We can apply decimals to this 2nd method also, thus :—

Amount $= \left(\frac{105}{100}\right)^3$ of Rs. 500

       $= $Rs $\frac{105}{100} \times \frac{105}{100} \times \frac{105}{100} \times 500$

       $= $Rs $1\cdot05 \times 1\cdot05 \times 1\cdot05 \times 500$

       $= $Rs $578\cdot8125$

∴ C. I.=Rs. 578·8125—Rs 500

       $= $Rs 78 8125—Rs. 78 13$a$ **Ans.**

Example 2. Find the compound interest on Rs. 512. 8a. for 3 years at $4\frac{1}{2}$ per cent.

Sol Rs. 512. 8a.=Rs 512·5 , $4\frac{1}{2}$=4 5

Rs.	512·5	=Principal for 1st year
	4 5	
	25625	
	20500	
Add {	23 0625	=Interest for 1st year
	512 5	
	535 5625	=Principal for 2nd year
	4 5	
	26778125	
	21422500	
Add {	24 1003125	=Interest for 2nd year
	535 5625	
	559 6628125	=Principal for 3rd year
	4·5	
	27983140625	
	22346512500	
Add {	25 1848265625	=Interest for 3rd year
	559·6628125	
Subtract {	584 8476390625	=Total amount
	512·5	
	72 3476390625	=Compound interest
		=Rs. 72 5a 6p.
		nearly. **Ans.**

. **Or by formula**

$$\text{Amount} = \left(1 + \frac{4\frac{1}{2}}{100}\right)^3 \times \text{Rs. 512 } 8a$$

$$= \tfrac{209}{200} \times \tfrac{209}{200} \times \tfrac{209}{200} \times \tfrac{1035}{2}$$

$$= \text{Rs.} \frac{9357562225}{16000000} = \text{Rs.} \frac{9357\,562225}{16}$$

$$= \text{Rs. } 584\,8476390625$$

But Principal $= \text{Rs } 512\,5$

$\therefore$ C I. $=$ Rs $72`3476390625 =$ Rs. 72. $5a$ $6p$. **Ans**

§3 Interest payable half-yearly or quarterly

When the interest is payable half-yearly, calculate the compound interest for *double* the number of years at *half* the rate per cent. and when it is payable quarterly, calculate it for 4 times the number of years at $\frac{1}{4}$ the rate per cent

Example 3. Find the compound interest on Rs, 666. $10a$ $8p$ for $1\frac{1}{2}$ years at 5 per cent. per annum payable half-yearly

Note   Since there are two half years in a year, there C I in $1\frac{1}{2}$ years at 5 p c payable half-yearly is the same as C I in 3 years at $2\frac{1}{2}$ per cent

Sol. $\because$ Rs 666, $10a$. $8p$. $=$ Rs, $\tfrac{2000}{3}$,

$\therefore$ find the C I on Rs 2000 at $2\frac{1}{2}$ per cent in 3 years and divide the result by 3.

$$\text{By formula C. I.} = \text{Rs.} \left\{ \left(1 + \frac{2\frac{1}{2}}{100}\right)^3 - 1 \right\} \times 2000$$

$$= \text{Rs.} \{ \tfrac{41}{40} \times \tfrac{41}{40} \times \tfrac{41}{40} - 1 \} \times 2000$$

$$= \text{Rs.} \{ \tfrac{68921}{64000} - 1 \} \times 2000$$

$$= \text{Rs. } \tfrac{4921}{64000} \times 2000 = \text{Rs } \tfrac{4921}{32}$$

$$= \text{Rs. } 153 \quad 12a \quad 6p.$$

Now divide this result by 3.

$\therefore$ required C I. $=$ Rs. 51 $4a$. $2p$. **Ans,**

Example 4. Find the compound interest on Rs. 333 $5a$. $4p$ for $1\frac{1}{2}$ years at 8 p. c per annum payable quarterly.

Note   Since there are 4 quarters in a year, therefore, C I in 1½ years at 8 p c payable quarterly is the same as C I in 6 years at 2 p c

Sol   Rs. 333. 5a 4p. = Rs $\frac{1000}{3}$,

∴ find the C. I on Rs. 1000 in 6 years at 2 per cent. and then divide the result by 3

C I = Rs $\{(1 + \frac{2}{100})^6 - 1\} \times 100$

    = Rs $\{\frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} - 1\} \times 1000$

    = Rs. $\{1\ 02 \times 1\ 02 \times 1\ 02 \times 1\ 02 \times 1\ 02 \times 1\ 02 - 1\} \times 1000$

    = Rs. $(1\ 126162419264 - 1) \times 1000$

    = Rs. 126162419264×1000

    = Rs $126\ 162419264$ = Rs. 126. 2a. 7p. nearly.

∴ required C. I.   = Rs 42 0a 10p nearly. **Ans.**

§4   To find C. I when the given time is not an exact number of years

Example 5   Find the compound interest on Rs. 1800 for 2 years 9 months at 4 per cent.

Sol.	Rs. 1800	= Principal for the 1st year.
	4	
Add {	72'00	= Interest for the 1st year.
	1800	
	1872	= Principal for the 2nd year.
Add {	74 88	= Interest for the 2nd year.
	872	
	1946'88	= Principal for the 3rd year.
	3	(See note below)
Add {	58'4064	= Interest for 9 months of the
	1946'88	3rd year.
Subtract {	2005 2864	= Total amount.
	1800	
	205'2864	= Required C. I.
		= Rs. 205. 4a 7p. nearly **Ans.**

Note   Interest for 9 months at 4 p c would be 3 p c, hence in calculating the interest for 9 months of the 3rd year we have taken rate 3 p c instead of 4

EXERCISE 141.

Find the compound interest on —

1	Rs. 100	for 2 years at 5 per cent.
2	Rs 200	for 2 years at 4 per cent.
3	Rs 250	for 3 years at 5 per cent.
4.	Rs 500	for 2 years at 6 per cent.
5	Rs 750	for 3 years at 5 per cent.
6	Rs. 1500	for 3 years at 4 per cent.
7	Rs 2100	for 3 years at 5 per cent
8	£1200	for 2 years at $2\frac{1}{2}$ per cent.
9	£1500	for 3 years at $3\frac{3}{4}$ per cent
10	£1800	for 4 years at $3\frac{1}{3}$ per cent

Find to the nearest pie or penny the compound interest on ·

11.	Rs 625 4a.	for 2 years at 5 per cent
12.	Rs 821 4a	for 3 years at 6 per cent.
13	£521 5s	for 2 years at $3\frac{1}{4}$ per cent.
14.	£625 10s	for 3 years at $4\frac{1}{2}$ per cent
15.	Rs. 833 5a 4p.	for 2 years at 5 per cent.
16	Rs 566 10a 8p	for 3 years at $2\frac{1}{2}$ per cent
17.	Rs 1266 10a. 8p	for 3 years at 8 per cent.
18.	£1233 6s. 8d.	for 2 years at 6 per cent.
19.	£1666 13s 4d	for 2 years at 4 per cent.
20.	£1353. 6s. 8d	for 3 years at 10 per cent.

Find the amount at compound interest of —

21	Rs 512 8a	for $2\frac{1}{2}$ years at 5 per cent.
22	Rs 621. 4a.	for $2\frac{3}{4}$ years at 4 per cent.
23	Rs. 1600	for $2\frac{1}{4}$ years at $4\frac{1}{2}$ per cent.
24	Rs 1533. 5a 4p	for $2\frac{1}{3}$ years at 5 per cent.
25	£1626 13s 4d	for $3\frac{1}{2}$ years at 5 per cent.
26.	£1582 10s	for $2\frac{1}{2}$ years at 8 per cent

27. Find the compound interest on Rs 1000 at 5 per cent. for $1\frac{1}{2}$ years payable half-yearly

28 Find the amount at compound interest on Rs. 1250 for 2 years at 6 per cent. payable quarterly.

29 Find the difference between the simple and compound interest on Rs. 1600 in $2\frac{1}{2}$ years at $3\frac{1}{4}$ per cent. per annum

30. Find the amount at compound interest on £1200 for 3 years if the interest for the first, second and third years be 8, 5 and 4 per cent respectively

INVERSE CASES OF COMPOUND INTEREST

§4   To find Principal

First case *When Interest, Time and Rate are given.*

Example 1. The compound interest on a certain sum of money in 3 years at 5 per cent amounted to Rs. 78. 13a., find the sum.

Sol   Let Rs 100 be the Principal,

$$\text{interest} = \text{Rs. } \left\{ (1 + \tfrac{5}{100})^3 - 1 \right\} \times 100$$
$$= \text{Rs } \left\{ \tfrac{21}{20} \times \tfrac{21}{20} \times \tfrac{21}{20} - 1 \right\} \times 100$$
$$= \text{Rs } \left\{ \left(\tfrac{9261}{8000} - 1 \right) \right\} \times 100$$
$$= \text{Rs. } \tfrac{1261}{8000} \times 100 = \text{Rs } \tfrac{1261}{80}.$$

If the int is Rs. $\tfrac{1261}{80}$, the sum = Rs. 100

∴   „   „   „   is Re 1,   „   = Rs $100 \times \tfrac{80}{1261}$

∴   „   „   „   is Rs. 78 13a   „   = Rs. $100 \times \tfrac{80}{1261} \times \tfrac{1261}{16}$
$$= \text{Rs } 500. \quad \textbf{Ans.}$$

From the above solution we deduce the following

Rule. $Principal = \dfrac{Given\ C\ I \times 100}{C\ I\ on\ Rs\ 100}$

EXERCISE 142.

1   What sum of money must I invest at 5 per cent. compound interest, so that I may gain Rs. 51. 4a. in 2 years ?

2   What sum will produce Rs 128 2a. in 2 years at 5 per cent per annum compound interest ?

3.   What sum put out at compound interest at 4 per cent would produce Rs 187 4a 8'832p. in 3 years ?

4   What sum of money must I invest at 4 per cent. compound interest, so that I may gain interest Rs. 390. 3a. $2\tfrac{2}{25}p.$ in 3 years ?

5 What sum of money must I invest at 5 per cent compound interest, so that I may gain Rs. 2586. 1a 2⅖p in 4 years ?

Second case. *When Amount, Time and Rate are given*

Example 2 Find what sum, lent at compound interest, will amount to Rs 1157 10a. in 3 years at 5 p. c.

Sol Let Rs 100 be the Principal,

Amount$=$Rs $(1+\frac{5}{100})^3\times100$

$=$Rs $\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\times100=\frac{9261}{80}$

If the amount is Rs. $\frac{9261}{80}$ the sum$=$Rs 100

"   "   Re. 1,   " $=$Rs $100\times\frac{80}{9261}$

"   "   Rs 1157 10a " $=$Rs $100\times\frac{80}{9261}\times\frac{9261}{8}$

$=$Rs 1000 **Ans**

From the above solution we have the following

Rule. $Principal=\dfrac{Given\ amount\times100}{Amount\ on\ Rs\ 100}$

EXERCISE 143

1 What sum will amount to Rs 882 in 2 years at 5 per cent per annum compound interest ?

2 Find what sum will amount to Rs 4134. 6a in 2 years at 5 per cent. per annum compound interest

3 The amount of a certain sum at compound interest in 1½ years at 5 per cent per annum amounts to Rs. 861, find the sum.

4. What sum put out at compound interest at 5 per cent. would amount in 3 years to £810 6s. 9d ?

5 Find what sum will amount to Rs. 7364 10a 9p. in 2 years at 3½ per cent. per annum compound interest.

6. What sum lent at compound interest will amount to Rs. 16143 12a in 2½ years at 5 per cent. per annum ?

7. What sum will amount to £1591. 13s. 2'16d. in 3 years at compound interest, the interest for the first, second and third years being 3, 2 and 1 per cent respectively?

8 What sum will amount to Rs 650 in one year and Rs. 676 in two years at compound interest?

§5   To find Time.

Consider the following example *when Principal, Amount and rate are given*

Example 3   In how many years will Rs. 625 amount to Rs 676 at 4 per cent compound interest?

Sol. $\because$ Principal $\left(1+\dfrac{\text{Rate}}{100}\right)^n =$ Amount

$$625\left(1+\tfrac{4}{100}\right)^n = 676$$
$$\left(1+\tfrac{4}{100}\right)^n = \tfrac{676}{625}$$
$$\left(\tfrac{26}{25}\right)^n = \left(\tfrac{26}{25}\right)^2$$
$$n = 2.$$

the required time is 2 years   **Ans.**

Or we may proceed from *thus

$$\left(1+\tfrac{4}{100}\right)^n = \tfrac{676}{625}$$
$$\left(\tfrac{26}{25}\right)^n = \tfrac{676}{625}$$

Now divide $\tfrac{676}{625}$ by $\tfrac{26}{25}$ and then the quotient again by $\tfrac{26}{25}$ and so on,

$$\tfrac{676}{625} \div \tfrac{26}{25} = \tfrac{26}{25} \qquad \text{1st year}$$
$$\tfrac{26}{25} \div \tfrac{26}{25} = 1 \qquad \text{.2nd year.}$$

Now the division terminates   No of years $= 2$. **Ans.**

§6. Sometimes the number of years contains fractional part also. The following examples will illustrate the method of solving such questions :—

Example 4   In what time will £6250 amount to £7592. 14s 6'48d at 6 p c. Compound Interest?

Sol. $£7592.\ 14s.\ 6'48d. = £\tfrac{7592727}{1000}$

$\because$ Principal $\left(1+\dfrac{\text{Rate}}{100}\right)^n =$ Amount

$\therefore$ $6250\left(1+\tfrac{6}{100}\right)^n$ $= \tfrac{7592727}{1000}$

$\because$ $\left(1+\tfrac{6}{100}\right)^n$ $= \tfrac{7592727}{1000} \times \tfrac{1}{6250}$

$\because$ $\left(\tfrac{53}{50}\right)^n$ $= \tfrac{7592727}{6250000}$

Now $\frac{7592727}{6250000}-\frac{53}{50}=\frac{7592727}{6250000}\times\frac{50}{53}=\frac{143259}{125000}$ [one year

$\frac{143259}{125000}-\frac{53}{50}=\frac{143259}{125000}\times\frac{50}{53}=\frac{2703}{2500}$ [two years

$\frac{2703}{2500}-\frac{53}{50}=\frac{2703}{2500}\times\frac{50}{53}=\frac{51}{50}$ [three years

Here the last quotient is less than the divisor, therefore the reqd No of years=3 years+a fraction of a year

Now find by simple interest the time in which £1 will amount to $£\frac{51}{50}$ at 6 p c, i e, the int of $£1=£\frac{51}{50}-£1$ $=£\frac{1}{50}$.

$$\text{Time}=\frac{\text{Interest}\times100}{\text{Principal}\times\text{Rate}}$$

$$=\frac{\frac{1}{50}\times100}{1\times6}=\frac{1}{3}\text{ year}$$

reqd time$=3\frac{1}{3}$ yrs An

Note   Such questions can easily be orked out in decimals

EXERCISE 144

1   In what time will Rs 500 amount to Rs. 551 4a at 5 per cent. compound interest ?

2.   In what time will Rs 1000 amount to Rs. 1157 10a at 5 per cent. compound interest ?

3.   In what time will Rs. 24000 amount to Rs. 27783 at 5% compound interest ?

4   Rs 500 produce Rs. 40 12a 9$\frac{3}{5}p$ at 4 p c. compound interest, find the time

5   In what time will £12500 amount to £15185 9s 0 96d. at 6 per cent. compound interest ?

6.   In what time will £15625 amount to £20470, 6s 4'8d. at 8 p. c compound interest ?

§7   To find Rate per cent

Observe the following examples *when Principal, Interest or Amount and Time are given.*

Example 5.   At what rate p c. compound interest will Rs. 500 amount to Rs 578 13a in 3 years ?

Sol   Principal $\left(1+\frac{r}{100}\right)^{n}=$Amount

∴ $500\left(1+\frac{r}{100}\right)^{3}=\frac{9261}{16}$

$$\left(1+\frac{r}{100}\right)^3=\frac{9261}{16}\times\frac{1}{500}$$

$$\therefore\quad\left(1+\frac{r}{100}\right)^3=\frac{21\times21\times21}{20\times20\times20}$$

$$\therefore\quad1+\frac{r}{100}=\frac{21}{20}\quad\left[\begin{array}{l}\text{By extracting}\\\text{cube root.}\end{array}\right]$$

$$\frac{r}{100}=\frac{1}{20},\quad r=5$$

$$\therefore\quad\text{Rate}=5\text{ p c. Ans}$$

EXERCISE 145

1   At what rate per cent compound interest, will Rs. 5000 amount to Rs. 6655 in 3 years ?

2   At what rate p c compound interest, will Rs 500 amount to Rs 551. 4a in 2 years ?

3   At what rate per cent compound interest will Rs 625 amount to Rs 676 in 2 years ?

4   At what rate p c. compound interest, will Rs 1250 produce Rs 128 2a in 2 years ?

5   Find the rate p c if Rs. 250 amount to Rs. 270. 6a 4⅘p at compound interest in 2 years

6   A sum of Rs 285 put out at compound interest for 3 years produces Rs. 29 7a 4⅞²p., find the rate per cent. of interest.

§8   Some Important Typical Examples.

Example 1   The difference between the simple and the compound interest on a certain sum of money for 3 years at 5 p. c is Rs. 133 7a , find the sum

Sol. S. I. on Re 1 for 3 yrs at 5 p c $=$ Rs $\dfrac{5\times3}{100}=$ Rs. ˙15.

Amount of Re 1 at the end of 3 years at comp int
$=$ Rs. $\left(1+\frac{5}{100}\right)^3=$ Rs $(1\;05)^3=$ Rs 1 157625

$\therefore$ comp int. on Re 1 $=$ Re ˙157625

$\therefore$ diff. between S I. and C I. of Re 1 $=$ Re. ˙007625.

But the given difference $=$ Rs 133 7a. $=$ Rs 133˙4375

$\therefore$ the required sum $=$ Rs. $\dfrac{133\;4375}{˙007625}$

$=$ Rs. 17500. **Ans.**

Example 2   If the C I on a certain sum for 2 years at 3 p. c. be Rs 57 1a 6p , what will be the S. I. ?

Sol.   C. I on Re 1=Re $\left(\frac{103}{100}\right)^2$−1=Re $\frac{609}{10000}$.

and S I. on Re. 1=Re $\frac{1}{100}$×2×3=Re $\frac{6}{100}$,

∴ S I.   C.I = $\frac{6}{100}$ $\frac{609}{10000}$=6×10000 609×100=200 203

.   S. I = $\frac{200}{203}$×C. I.

$\qquad$ = $\frac{200}{203}$×Rs 57. 1a 6p.

$\qquad$ =Rs $\frac{200}{203}$×$\frac{1827}{32}$=Rs $\frac{225}{4}$=Rs 56. 4a **Ans**

Example 3.   The interest on a given sum of money for one year is Rs 26 4a. and the compound interest for two years is Rs 53. 13a , find the rate per cent

Sol.   Simple int. for 2 yrs = Rs 26 4a.×2=Rs 52 8a

and compound interest for 2 years =Rs 53 13a

∴ difference between the C I. and S I =Re. 1. 5a.

in S.I. and C I. the int. for the first year is the same

∴ Re 1 5a. is the int on Rs. 26. 4a for one year

∴ rate = $\dfrac{\frac{21}{16}\times100}{26\frac{1}{4}\times1}$=5 p. c. **Ans.**

Example 4   A sum of money put out at compound interest for 2 years amounts to £540 16s and in 3 years to £562. 8s 7 68d Find the rate of interest.

Sol   Evidently £562 8s. 7 68d −£540 16s , i e., £21 12s. 7˙68d ($£\frac{1704}{136}$) is the interest on £540 16s ($£\frac{1704}{5}$) for 1 year

.   interest on Rs. 100 for 1 yr.=$\frac{5}{1704}$×100×$\frac{1704}{136}$=4

reqd rate is 4 p c **Ans.**

Example 5.   What annual payment will discharge a debt of Rs. 6305 in 3 years, the rate of interest being 5 per cent compound interest ?

Sol   Suppose the annual payment is Rs 100

. Rs 100 are to be paid after one year

∴ Principal=Rs. 100×$\frac{100}{105}$=Rs. $\frac{2000}{21}$

Again Rs 100 are to be paid after 2 years,

∴ Principal=Rs 100×$\frac{100}{105}$×$\frac{100}{105}$=Rs $\frac{40000}{441}$

Again Rs. 100 are to be paid after 3 years,

$\therefore$ Principal is Rs. $100 \times \frac{100}{104} \times \frac{100}{104} \times \frac{100}{104} =$ Rs $\frac{800000}{1261}$.

Total sum $=$ Rs $\frac{2000}{21} + \frac{40000}{441} + \frac{800000}{9261} =$ Rs. $\frac{2522000}{9261}$

If the Principal is Rs $\frac{2522000}{9261}$ the annual payment is Rs 100, if the Principal be Rs. 6305 then the annual payment will be $=$ Rs $\frac{9261}{2522000} \times 100 \times 6305$

$$= \text{Rs } \frac{9261}{4} = \text{Rs. } 2315. \, 4a \quad \text{Ans}$$

Example 6 A man borrows a certain sum and pays back in 2 years in two equal instalments If compound interest is reckoned at 4 per cent and if he pays back annually Rs 676, what sum did he borrow ?

Sol Since the annual instalment $=$ Rs. 676 and rate $= 4\%$.

$\therefore$ The reqd sum $=$ Rs. $676 \times \dfrac{100}{100+4} +$ Rs. 676

$$\times \left(\frac{100}{100+4} \right)$$

$$= \text{Rs } 650 + \text{Rs } 625$$

$$= \text{Rs } 1275 \quad \text{Ans.}$$

Example 7. Divide Rs. 5854 $8a$ between A and B so that A's share at the end of 7 years may be equal to B's share at the end of 9 years, C I being at 4 p c

Sol By the question

(A's share) $(1 + \frac{4}{100})^7 = (B$'s share) $(1 + \frac{4}{100})^9$

i.e. , $\dfrac{A\text{'s share}}{B\text{'s share}} = (1 + \frac{4}{100})^2 = \frac{676}{625}$.

Now divide Rs. 5854 $8a$ in the ratio of $676 : 625$

$\therefore 676 + 625 = 1301.$

$\therefore A$'s share $= \frac{676}{1301}$ of Rs. $5854\frac{1}{2} =$ Rs 3042 $\Big)$

and $\therefore B$'s share $=$ Rs. $5854 \, 8a -$ Rs 3042 $\Big\}$ Ans.

$= $ Rs. $2812 \, 8a$ $\Big)$

EXERCISE 146.

1. On what sum will the difference between the S I. and C I for 3 years at $2\frac{1}{2}$ per cent amount to £3. 15s $7\frac{1}{2}d$?

2 The difference between the simple and the compound interest on a certain sum for 3 years at $3\frac{3}{4}$ per cent is £3. 8s. $4\frac{1}{8}d$ Find the sum

3. A sum of money put out at C. I amounts to Rs 2420 in 2 years and to Rs 2662 in 3 years Find the sum and the rate

4. A certain sum put out at C I amounts in 2 years to £270 4 and in 3 years to £281 216. Find the sum and the rate.

5 On what sum of money will the C I for 2 years be the same as the S. I on £943 for 10 years, the rate of interest being 5% ?

6 I borrow money at 3% simple interest payable yearly and lend it immediately at 5 p c interest payable half-yearly receiving C I for the second half-year and gain thereby Rs. 660 at the end of the year What is the sum of money which I borrow ?

7 The C. I on a sum for 3 years at 5 per cent is £331. 0s 3d , what is the S. I. ?

8 The C. I. on a certain sum for 2 years at 5 p c. is £ 17. 1s 8d , find the C I on the same sum for the same time at 4 p c

9 The difference between simple and compound interest on a sum of money for 3 years at 5 per cent. is £7 12s 6d. Find the sum

10. I buy a house on the condition that I shall pay £500 now, £425 one year hence and £289 two years hence What would be the cash value of the house, compound interest being calculated at $6\frac{1}{4}$ p c ?

11 The interest on a given sum of money for one year is £26 and the compound interest for 2 years is £53 04 , find the rate per cent.

12   What annual payment will discharge a debt of Rs 8200 due in 2 years at the rate of 5 per cent. compound interest ?

13   A sum of £12193 15s. is borrowed to be paid in 3 equal annual instalments   Find the instalments, if the rate is 4 per cent compound interest

14.   Divide £820 into two parts so that their amounts after 2 and 3 years respectively at 5 per cent. compound interest may be equal.

15   What sum of money will amount to Rs 3528 in 2 years at 5 per cent and what will it amount to in 2 more years ?

16   A sum of money is borrowed and paid back in two equal instalments of Rs 8820 allowing 5 per cent. per annum compound interest.   What is the sum borrowed ?
[Burma, 1924]

17.   Divide £6375 between A and B so that A's share at the end of 3 years and B's share at the end of 2 years if put out at 4 per cent compound interest may be equal

18   The third year's interest of a sum put at 4 p. c compound interest was 16s 7˙68d. more than that of the preceding year.   Find the sum

19   Divide £6305 into three sums such that their amounts by compound interest at 5 p c per annum for 2, 3 and 4 years respectively shall be equal

20   A placed a sum of money for 3 years at 5 p. c. simple interest, and B placed out an equal sum at the same rate for the same time at compound interest and thereby gained £26. 13s 4d. more than A   What money was placed out by each ?

21   A man left Rs 45909 for his sons A and B who are 15 and 13 years old in such a way that of their shares be invested at 2 per cent. compound interest, they shall receive equal amounts on reaching 18 years of age   How did he divide the money ?

CHAPTER XXVII.

PRESENT WORTH AND DISCOUNT

§1 A sells a horse worth Rs. 500 to B but B instead of paying to A the price of the horse in cash money, promises to pay after 6 months.   Evidently A will charge something more than Rs 500 from B, namely the interest of this money for 6 months   Suppose that the rate of interest is 4 p c so that the interest on Rs 500 for 6 months is Rs 10 and hence B shall pay Rs. 510 for the horse after 6 months, but if he had paid cash then he would have paid Rs 500 only for the horse   In this case, Rs. 500 is said to be the Present Worth of Rs 510 due 6 months hence and Rs. 10 is called the Discount on Rs 510. Evidently, therefore, *the interest on the Present Worth is the discount on the amount due some time hence, i e ,*

Discount=Interest on Present Worth ,

and Amount= Present Worth+Discount

§2   Def   The Present Value or Present Worth of a sum due at the end of a certain period, is the sum which with its interest for the given period, will be together equal to the sum due

The sum due is also called the Amount

The True discount (briefly, discount) is the difference between the sum due and its present worth.   Hence discount is the deduction made for the payment of a sum of money before it is due   It is of two kinds

(i)   True discount

(ii)   Banker's or commeraial discount

§3   To find the discount.

Observe the following examples —

Example 1.   Find the discount on Rs 360 due at the end of 4 years at 5 p, c

465

F 30.

Sol. ∵ the int. on Rs. 100 for 4 yrs. at 5 p. c.=Rs. 20

∴ the amount of Rs 100=Rs. 120,

i.e., the discount on Rs 120=Rs. 20,

∴ „   „   „ Re. 1 =Re. $\frac{20}{120}$.

· „   „   „ Rs 360=Re. $\frac{20}{120}$×360

                                =Rs. 60. **Ans.**

Note   Interest is calculated on the Principal but the discount is calculated on the amount

§4   To find the Present Worth.

The following solved examples will illustrate the method —

Example 2. Find the present worth of £721 due 9 months hence at 4 p c.

Sol. ∵ the int. on £100 for 9 months at 4 p. c.=£3,

amount of £100 after 9 months=£103 ,

i e , the present worth of £103   =£100 ,

„   „ of £1   =£$\frac{100}{103}$

··   „   „ of £721   =£$\frac{100}{103}$×721

                                    =£700. **Ans.**

§5   When Present Worth and Discount are reckoned at Compound Interest, we shall find the amount of Rs 100 or £100 at C. I. for the given time at the given rate, and proceed as before

Example 3. Find the Present Worth and Discount of Rs 2112. 8a. due 2 yrs. hence, at 4 p. c Comp. Interest.

Sol. At 4 p. c C I for 2 yrs. Re 1 amounts to
Rs $(1\cdot04)^2$, *i e* , Rs 1 0816.

Present Worth of Rs. 1·0816=Re. 1

·   „   „   „ Re. 1   =Re. $\frac{1}{1\cdot0816}$

∴ „   „   „ Rs. 2112 5=Rs. $\frac{2112\ 5}{1\ 0816}$

∴   P W =Rs 1953. 2a   **Ans.**

Discount=Rs 2112. 8a −Rs 1953. 2a

       =Rs 159 6a   Ans

EXERCISE 147.

Find the discount on (1—5) —

1   Rs. 1270 due 7 months hence   at 10 p. c. S I.
2   £1051 5s. due a year hence   at 5⅜ p c. S.I.
3   £520. 17s 6d. due 3½ yrs hence   at 4½ p c. S.I.
4   Rs 135 14a. 8p due 3¾ yrs hence at 4½ p. c. S I.
5. £2450. 18s. 9d due 3⅓ yrs hence   at 3¾ p c C I.
6   What discount at 5 p. c. simple interest must be allowed for present payment of £2200 due 2 yrs hence ?

Find the present worth of (7—11) —

7. £870   due 2½ years hence at 3½ p c. S I.
8. Rs. 223 7a   due 10 months hence at 3¾ p c. S.I
9. £769 5s   due 8⅓ years hence at 4 p c. S.I
10 £587 18s 9d. due 6 months hence at 4½ p. c S I
11. £9724 1s   due 4 years hence at 5 p. c C I
12 Find the present worth of Rs 545 11a. 8p due 2 yrs hence at 4% compound interest.
13 If the discount on a sum of money due 6 months hence at 8 per cent be £7. 10s. 11¾d., find the Present Worth of the sum
14. What must be paid in cash in order to clear off a bill Rs 297. 7a. 6p. due in 3 years 6 months, simple interest being calculated at 4 p c per annum ?
15 If money is worth 5¼ p c per annum, what sum of money will discharge a debt of Rs. 8175 which falls due after 5 months ?
16 Find the present worth of £241. 12s. 4d. 146 days hence at 4⅓ p c per annum ?
17 Find the difference between the interest on Rs 2466 10a 8p. for 2¾ years at 5½ p. c. and the discount on Rs 2839. 12a. due 2¾ years hence at the same rate. Explain the result
18 What sum of money paid down will discharge a debt of £1000 due in two equal half-yearly instalments, simple interest being at 5 per cent ?
19 If Rs. 6760 be due three years hence, allowing compound interest at 4 per cent per annum, what sum will be due at the end of the first year ?

INVERSE CASES OF DISCOUNT

§6   All inverse questions on discount can be easily solved just like those on interest, if it is remembered that the interest on present worth is equal to the discount on amount due

§7   To find the sum or Amount.

Example 1   What is the sum due $8\frac{1}{3}$ years hence, whose discount at 4 per cent is £192 6s. 3d ?

Sol   Suppose the sum is £100.

. int for $8\frac{1}{3}$ yrs at 4 p. c $= £\frac{25}{3} \times 4$ $= £\frac{100}{3}$

the amount of £100 $= £)100 + \frac{100}{3})$ $= £\frac{400}{3}$

and ∴ £192 6s. 3d $= £192\frac{5}{16}$ $= £\frac{3077}{16}$,

. (a) By "Rule of Three," we have

$£\frac{100}{3}$ $£\frac{3077}{16}$ $£\frac{400}{3}$ required amount.

∴ the amount $\frac{3}{100} \times \frac{3077}{16} \times \frac{400}{3} = £\frac{3077}{4} = £769$ 5s   Ans.

(b) By Unitary Method,

$£\frac{100}{3}$ is the discount on $£\frac{400}{3}$

$£1$ is   „   „   on $£\frac{400}{3} \times \frac{3}{100}$

∴ $£\frac{3077}{16}$ „   „'   „   on $£\frac{400}{3} \times \frac{3}{100} \times \frac{3077}{16}$

$= £\frac{3077}{4} = £769. 5s$ Ans.

Aliter.   Present worth $= \dfrac{£192\ 6s.\ 3d.}{4 \times 8\frac{1}{3}} \times 100$

$= £576. 18s\ 9d.$

.   sum due $=$ P W. $+$ Discount

∴   reqd sum $= £576. 18s\ 9d + £192$ 6s 3d.

$= £769. 5s$ Ans

Example 2.   If the present value of a bill due 8 months hence at $4\frac{1}{2}\%$ is Rs. 66 10a 8p., find the bill.

Sol,   . interest on present worth $=$ discount on the sum

∴ discount $= \dfrac{(\text{Rs } 66\ 10a\ 8p) \times 8 \times 9}{100 \times 12 \times 2}$

$= \dfrac{200 \times 8 \times 9}{3 \times 100 \times 12 \times 2} =$ Rs. 2.

∴   bill $=$ Rs 66 10a. 8p $+$ Rs 2

$=$ Rs 68. 10a. 8p   Ans

Example 3   On what sum of money due at the end of 2 years does the discount at 5 per cent compound interest amount to £54 13s 4d ?

Sol.   Amount of £100 for 2 yrs. at C I $=£100(\frac{105}{100})^2$
$$=£100\times\tfrac{105}{100}\times\tfrac{105}{100}=£\tfrac{441}{4}.$$

Discount of $\tfrac{441}{4}=£\tfrac{441}{4}-£100=£\tfrac{41}{4}.$

   $£\tfrac{41}{4}$ is the discount on $£\tfrac{441}{4}$

. £1   is ,,   ,, on $£\tfrac{441}{4}\times\tfrac{4}{41}$

∴ £54 13s. 4d or $£\tfrac{164}{3}$ is ,,   on $£\tfrac{441}{4}\times\tfrac{4}{41}\times\tfrac{164}{3}$
$$=£588.\quad\text{Ans}$$

EXERCISE 148

What is the sum due (1—3) —

1 9 months hence, whose discount at 4 per cent. is Rs 1200 2a ?

2. 3½ years hence, whose discount at 4½ per cent. is £70. 17s. 6d. ?

3. 245 days hence whose discount at 3⅞ per cent is £12 16s 8d. ?

4. If the discount of a bill due 1 year 8 months hence at 4½ per cent. be £35 5s , find the sum

5 What sum due 9 months hence at 3½ per cent is worth Rs 850 now ?

6 If the present value of a bill due 15 months hence at 2⅔ per cent. be £631 10s , find the sum.

7. On what sum of money due at the end of 2 years at 4 per cent compound interest does the discount amount to £34 ?

8. The true discount on a bill due 8 months hence at 2½ per cent. is £176 14s 8d , find the amount of the bill

§8　To find Time.

Example 4　When is the sum due, if the discount on Rs 23. 5*a*. 4*p*. at $1\frac{2}{3}$ per cent amount to Re 1. 7*a* 4*p* ?

Sol　　　Amount — Discount = Present worth,

here the P W is Rs 23 5*a* 4*p*. — Re 1. 7*a*. 4*p*
$$= \text{Rs. } 21. \ 14a = \text{Rs. } \tfrac{175}{8},$$

and ∴ int on Rs $\tfrac{175}{8}$ at $1\frac{2}{3}$ p c is Re 1 7*a* 4*p*. or Rs $\tfrac{35}{24}$.

$$\therefore \ \text{Time} = \frac{\text{Int.} \times 100}{\text{Principal} \times \text{Rate}} = \frac{\tfrac{35}{24} \times 100}{\tfrac{175}{8} \times \tfrac{5}{3}} = \frac{35 \times 100}{24} \times \frac{3 \times 8}{175 \times 5}$$
$$= 4 \text{ years}\quad \textbf{Ans.}$$

Rule　　$Time = \dfrac{Discount \times 100}{P \ W \ \times Rate}$

EXERCISE 149

When is the sum due, if the (1—5) —

1.　P W. of Rs 9265　　at $4\frac{1}{2}$ p c. be Rs 8500 ?
2.　,,　,, £658　　　at $3\frac{3}{4}$ p c. be £640 ?
3.　,,　,, £241 12*s*. 4*d*　at $4\frac{1}{3}$ p c. be £437 10*s*.?
4.　Dis on Rs 2516 4*a*　at $6\frac{1}{4}$ p c be Rs 482. 14*a*. 8*p* ?
5　Discount on £355 5*s* at $4\frac{1}{2}$ p c be £5 5*s*
6.　If the present worth of £668 5*s* $7\frac{1}{2}d$. at 5 per cent. be £568. 15*s* , find when the sum is due.
7　If Rs 165 14*a* $1\frac{7}{17}p$ be the discount of a debt of Rs 2820 at simple interest at the rate of $3\frac{3}{4}$ p. c., how many months before due was the debt paid ?

§9.　To find Rate

Example 3.　At what rate per cent does the discount on £1378 in one year four months amount to £53 ?

Sol　∴　Present worth = Amount — Discount

∴ in this case P W. = £1378 — £53 = £1325 and the interest on it in 1 year 4 months is £53

∴　Rate $= \dfrac{\text{Int} \times 100}{\text{Principal} \times \text{Time}} = \dfrac{53 \times 100}{1325 \times 1\frac{1}{3}} = 3$ years　**Ans**

Hence the **Rule**　$Rate = \dfrac{Discount \times 100}{P \ W \ \times Time}$

EXERCISE 150.

1. If the discount on Rs 2261 5a 4p. due 1½ years hence be Rs 128, what is the rate?

2 What is the rate of interest if the discount on £387 7s 7½d payable at the end of 3 years be £41 10s. 1½d ?

3 What is the rate, if the present worth of Rs 8175 due in 5 months is Rs. 8000 ?

4 What is the rate of interest, when discount on Rs 41204 +a 8p due 9 months hence is Rs 1200 2a. ?

5 The P W. of Rs 1321. 8a due 2¼ years hence is Rs 1200, find the rate of interest.

6 What is money worth when the discount on £481. 8s 2¼d due 4 months hence is £5 3s 2½d.?

---

§10. Some Important Typical Examples.

(a) To find the sum when difference between interest and discount on the same sum, rate and time are given.

Example 1 If the difference between the interest and the discount on a certain sum of money for 2 years at 5 p.c. is Rs. 2 13a 4p , find the sum

Sol Suppose the sum   =Rs 100

∴ Interest for 2 years at 5 p. c =Rs   10

∴ Amount „   „   „   „   =Rs. 110

. Discount on Rs 110   =Rs   10

.. Discount on Re. 1   =Rs $\frac{10}{110}$

Discount on Rs 100   =Rs. $\frac{10}{110} \times 100 = \frac{100}{11}$

Diff between discount and interest on Rs 100

=Rs 10− Rs $\frac{100}{11}$=Re. $\frac{10}{11}$, Rs. 2. 13a 4p.=Rs. $\frac{17}{6}$

Re $\frac{10}{11}$ is the difference on Rs 100

Re 1   „   „   on Rs. 100×$\frac{11}{10}$

Rs. $\frac{17}{6}$ „   „   on Rs 100×$\frac{11}{10}$×$\frac{17}{6}$=Rs. $\frac{935}{3}$

=Rs 311 10a. 8p   Ans

An important Rule

The diffeience between the interest and the discount on a sum of money is equal to the interest on the discount

$$\text{Sum} = \text{Present Worth} + \text{Discount},$$

$$\text{Int on Sum} = \text{Int on P W} + \text{Int on Discount},$$
$$\text{but Int on P. W} = \text{Discount on the Sum.}$$

$$\therefore \text{Int on Sum} = \text{Discount on Sum} + \text{Int. on Discount,}$$
i e, Interest on Sum − Discount on Sum = Interest on Discount

It follows therefore that interest is always greater than the discount.

(*b*) *To find sum and time when interest and discount on the same sum and rate are given*

Example 2   The interest on a sum of money at 4 p c. is £67. 4s and the discount on the same sum for the same time at the same rate is £60   What is the sum and time?

Sol. ∵   Int on Sum − Discount = Int on Discount

∴   £67. 4s − £60, *i e*, $£7\frac{1}{5}$ = Int. on £60.

$$\therefore \qquad\qquad £1 = \text{Int on } £\frac{60}{7\frac{1}{5}}$$

$$\therefore \quad £67\frac{1}{5} = \text{Int. on } £\frac{60 \times 67\frac{1}{5}}{7\frac{1}{5}} = £560,$$

i e, the required sum = £560   **Ans**

Hence the **Rule.** $Sum = \dfrac{Interest \times Discoun}{Interest - Discount}$

Now the question is " In what time the interest on £560 at 4 p c. will be £67 4s ?"

$$\therefore \quad \text{Time} = \frac{\text{Interest} \times 100}{\text{Principal} \times \text{Rate}} = \frac{(£67. \ 4s.) \times 100}{£560 \times 4}$$
$$= \frac{336 \times 100}{5 \times 560 \times 4} = 3 \text{ years.} \quad \textbf{Ans.}$$

(*c*)   *To find sum and rate when interest and discount on the same sum and time are given*

The method is the same as in Example 2.

(d) *To find the time when interest and discount on different sums are given.*

Example 3 If the interest on Rs. 8500 at $4\frac{1}{2}$ per cent. be equal to the discount on Rs 9265 for the same time at the same rate, when is the latter sum due ?

Sol. Int on Rs 8500 = Discount on Rs. 9265,

.   Rs 8500 is the P. W of Rs 9265

.   Int on Rs 8500 = Rs. 9265 − Rs 8500 = Rs 765,

.∴ Time $=\dfrac{765\times100}{8500\times4\frac{1}{2}}=2$ yrs   Ans.

(e) *To find the rate when interest and discount on different sums are given*

The method is the same as in Example 3 above

(f) *To find discount for a greater period when the discount for the less period is given*

Example 4 If Rs 8 be allowed as discount off a bill of Rs 80 due 6 months hence, how much should be allowed off a bill of the same amount due 15 months hence ?

Sol Rs 8 is 6 months' discount   on Rs 80

.   Rs. 8 is 6 months' interest   on Rs 72

.   Rs 20 is 15 months' interest on Rs 72

i. e , „   „   „   ,   discount on Rs 92.

Discount on Rs. 92 = Rs. 20

.∴   „   on Re. 1 = Re $\frac{20}{92}$

     „   on Rs 80 = Rs $\frac{20}{92}\times80=$ Rs $\frac{400}{23}$

∴ required discount = Rs 17 6*a* $3\frac{3}{18}p$ Ans.

EXERCISE 151.

1. If the interest on Rs 50 at $4\frac{1}{2}$ p c be equal to the discount on Rs. 59 for the same time at the same rate, when is the latter sum due ?

2   If the discount on £2830. 15*s* $7\frac{1}{2}d$. be equal to the interest on £2784 7*s*. 6*d*. for the same time, find the time, the rate of interest being 5 per cent

3   The difference between the interest and discount on a certain sum for 9 months at 4 per cent. is Rs. 2 13*a*. Find the sum.

4. The interest on Rs 250 at 3 p. c is equal to the discount on Rs. 265 at the same rate and for the same time. Find the time

5. The interest on Rs 1462 8a. at a certain rate for 4 years is equal to the discount on Rs 1725 12a. for the same time at the same rate   What is the rate per cent ?

6 The discount on a certain sum at $2\frac{1}{3}$ p c. is £19. 12s and the interest on the same sum for the same time and rate is £26. 19s. Find the time.

7. If the present value of a bill of £3820 6s 3d. due 5 months hence is £3750, what is the present value of a bill of £3876 11s 3d , due 9 months hence at the same rate   Find also the rate of interest

8   At what rate per cent will the interest on £3729. 7s. 6d. in 4 years, be equal to the discount on £4661 14s. $4\frac{1}{2}d$ for the same time ?

9. The interest on a certain sum of money for a certain time is Rs 36 and the discount for the same time is Rs. 30 , find the sum

10   If Rs. 12 be allowed at 6 months' discount off a bill of Rs 132 and at the same rate of interest Rs. +0 be allowed off a bill of Rs 240, for how long a period had the latter bill to run ?

11. The discount on Rs. 275 for a certain length of time is Rs 25 , what is the discount on the same sum (i) for double the length of time, (ii) for half the length of time ?

12. A tradesman marks his goods with two prices, one for cash money and the other for credit of 6 months , what ratio should the two prices bear to each other, the rate of interest being $7\frac{1}{3}$ per cent ?   If the credit price of an article be Rs. 332, what is its cash price ?

13. A man bought a house for Rs 1100 and sold it at once for Rs. 1217. 8a. allowing the buyer 5 months' credit. Money being worth $3\frac{1}{2}\%$ per annum, what was the gain per cent ?

14   If Rs. 10 be allowed as discount off a bill of Rs 50 and at the same rate Rs 25 be allowed off a bill of Rs 150, due at the end of 8 months, for how long a period had the first bill to run ?

15   If £6 be allowed as discount off a bill of £56 for 8 months, find the amount of another bill, of which £30 is allowed as discount for 20 months

16.   If Rs. 5 be allowed as discount off a bill of Rs 125 due a certain time hence, what should be the discount allowed off, if the bill had twice as long to run ?

17   If £2652 5s be due 3 years hence, what sum will be due at the end of 1 year, if compound interest is allowed at 3 per cent ?

18   A grocer buys 480 mds. of sugar for Rs 6135 payable at the end of 3 months and on the same day sells them at Rs. 12 11a per maund ready money. What per cent does he gain or lose by the transaction reckoning interest at 9 per cent. per annum ?

19   I buy a horse for £40 and sell it for £45 at a credit of 8 months   What do I gain per cent , reckoning money worth 6 per cent. per annum ?

20.   Eight copies of a book can be bought for a certain sum payable at the end of a year, but for a cash payment 10 copies can be had at the same price   What is money worth ?

21   The interest on £720 for a certain time is £18 : find the discount on the same sum for the same time

22   The interest on a certain sum for two years is £60, while the discount on the same sum due three years hence is £80   Find the sum and the rate per cent.

23   £259 7s. is due 4 years hence and £173 18s 5 years hence   What sum at the present time is equivalent to both these sums, calculating interest at $3\frac{1}{2}$ per cent. ?

24   A book sent from England costs me (including 1s 6d postage) 16s. 1d , my book-seller allowing me two pence in the shilling as discount on the published price What is the published price ?

25   The interest on a certain sum of money for 3 months at 5 per cent. exceeds the discount on the same sum due 3 months hence at the same rate by 11a. 3p Find the sum

§11   Banker's Discount.

In all commerical transactions when a man buys goods, he frequently does not pay "*cash*" for them but gives what is known as a *Bill of Exchange*, which is an agreement for paying the price after a certain time   The following is a form of a bill of exchange, which is always drawn on a stamped paper

---

Bill of Exchange.   Rs 10100 Due on June 7, 1916.

STAMP

Accepted, payable at The Punjab
National Bank Ltd , Calcutta
Brij Lal

Lahore
January 4, 1916.

Five months after or order the sum Hundred Rupees for

date   pay   to   myself of Ten thousand One value received.

To
     L Brij Lal,
                Calcutta

Harprasad.

Here Brij Lal has bought from Harprasad goods worth Rs 10100 (*face value* of the bill), but does not pay in cash. He promises to pay later on, say after five months Harprasad, therefore, draws up the bill of exchange as above and forwards it to Brij Lal for his "acceptance."   If the latter agrees to it, he signs his name across the bill with the note that the money can be obtained from the bank named there, on the date agreed upon   Brij Lal is said to "accept" the bill, which he returns to Harprasad. It is said to be *drawn* "at five months "

The date when the bill is "drawn" is January 4. Five months (always calendar months) from that date will

be June 4   But Harprasad cannot legally claim the money till three days after this, called "Three Days of Grace", pass

The date when the bill (matures," *i e*, when the money becomes legally due, is June 7 Harprasad, therefore, cannot claim the money of the bill till June 7, when he will present it to the Punjab National Bank and receive Rs 10100 of Brij Lal's money   The whole transaction is then closed

But there is another way, which is very generally adopted, in which Harprasad might proceed   Suppose in the interval between January 4 and June 7, Harprasad wants money very badly—he may then take the bill to a banker or a broker and ask him to discount it.   If the banker is willing and can pay the money, then he keeps the bill with him and, in exchange, pays over to Harprasad some money in cash   On the 7th June, when the bill "matures," the banker presents the bill to the Punjab National Bank and receives Rs 10100 of Brij Lal's money and here the whole transaction is closed

Now the question arises,—how much does the banker (or broker) who "discounts" the bill pays to Harprasad ? If he pays the *true* present value of the Rs. 10100, he makes no profit for himself   It is, therefore, the custom for the banker to pay in cash the face value of the bill *minus the simple interest* on it for the interval the bill has yet "to run," *i e.,* the number of days from the date on which the banker discounts it up to June 7

Banker's discount is, therefore, the same as the simple interest on the bill for the unexpired time.

It is also known *Practical* or *Commercial Discount*

§12   The person who draws the bill is called the drawer, and the person on whom it is drawn is called the drawee or acceptor In the above example Mr. Harprasad is the drawer and Mr. Brij Lal the drawee   The person to whom the money is to be paid is called the payee

A Promissory Note is also a contract (or promise) for a certain sum of money on a certain date to a person named, or to his order, or to bearer   It differs from a bill

of exchange only in this respect that it is written by the debtor instead of by creditor. The form of a promissory note is given below.

To Mr Harprasad   January 4, 1916

STAMP   Five months after date, I promise to pay to Mr Harprasad or his order, the sum of Ten Thousand and One Hundred Rupees, for value received.

Rs. 10100   Brij Lal

The bill shown above leads to the following question —

Example 1   Harprasad draws a bill, whose face value is Rs 10100 on January 4 at 5 months and offers it to a banker on March 26 to be discounted   If the banker's rate of discount be 5 p c. per annum, what will he pay for the bill ?

Sol   ·· the bill is drawn on January 4, ' it is legally due on June 7   Since it is discounted on March 26 . it has yet "to run," from March 26 to June 7, *i e*., 73 days.

Now the banker's discount=simple interest on Rs. 10100 for 73 days at 5 per cent =Rs. 101 ,

. the banker discounts (or buys) the bill for Rs. 10100 *minus* Rs 101=Rs 9999.   **Ans**

Note 1   The banker keeps the bill with him up to June 7 when on presenting it to the bank, he gets Rs 10100

2   The *true* present value of Rs 10100 for 73 days at 5 p c can be found to be Rs 10000 and hence *true* discount is Rs 100 Since the banker pays for the bill only Rs 9999 instead of Rs 10000 he makes a gain of Re 1

The fact is expressed by saying that banker's discount is greater than true discount by Re 1

Note 3   One month from January 31 (the last day of the month) will lead to February 28 or 29 (the last day of the month) *i e* , the word month always means 'calendar month '

Example 2   A banker discounts a 2 months' bill of Rs. 1000 at 3% discount   What rate of interest per annum is he charging ?

Sol.   Banker's discount $=$ Rs. $\dfrac{1000 \times 3 \times 2}{100 \times 12} =$ Rs 5

he advances Rs 995 in payment of the bill and in 2 months' time he gets Rs 1000, $i.\,e$, the interest on Rs 995 for 2 months is Rs. 5

rate of interest $= \dfrac{5 \times 100}{995 \times \frac{1}{6}} = 3\frac{3}{199}$ per cent   **Ans.**

§13   The following facts should be clearly understood

1   The discount on a bill$=$int on the true P W of the bill.

2   Banker's discount on a bill$=$interest on the bill

and . banker's discount$-$true discount$=$interest on (bill$-$true P W)$=$int on true discount , and

banker's discount$=$true discount$+$int on true discount $=$amount of true discount

Example 3. If Rs 120 is the true discount on Rs 4120 for 9 months, for how long is Rs 120 the true discount on Rs 6120 at the same rate ?

Sol.   Rs 120 is the true discount on Rs 4120 ,

. Rs 4000 is the true P W. of Rs 4120

Rs 120 is the interest   on Rs. 4000 for 9 months at a certain rate

$$\text{Rate} = \dfrac{120 \times 100}{4000 \times \frac{9}{12}} = 4\%$$

Again, Rs. 120 is the true discount on Rs. 6120,

Rs 120 is the int on Rs 6000 for a certain time at 4 per cent ,

. required time $= \dfrac{120 \times 100}{6000 \times 4}$ year $= 6$ months   **Ans.**

Example 4   The banker's discount on £604 at 5% is the same as the true discount on £611 11s. for the same time at the same rate   Find the time

Sol.   int on £604$=$true discount on £611 11s $=$Int on (true P W of £611. 11s.)

£604   = true P W of £611 11s.

£7 11s   = true discount on £611 11s

i. e £7. 11s = int. on £604 at 5% for a certain time,

$$\text{the required time} = \frac{£\frac{151}{20} \times 100}{604 \times 5} \text{ year}$$

= 3 months.   Ans

EXERCISE 152.

1   What rate of interest do I get for my money when, in discounting a bill due in 10 months, I deduct as discount 4% of the total amount of the bill ?

2   At what rate p c will the banker's discount on £6452. 10s. 8d. due 4 years hence amount to £1613 2s 8d ?

3.   The difference between banker's discount and true discount on a sum of money for 3 months at 5 per cent. is Rs 16 10a 8b   Find the sum

4.   Find the banker's discount on a bill of £142 17s 10d drawn on May 1 at 4 months and discounted on June 15, rate of interest being $1\frac{1}{2}$ per cent

5   A banker in discounting a bill due in 6 months at 6 per cent charges 9s more than the true discount. Find the amount of the bill

6.   The present worth of a certain sum due in 3 years at 5 p c compound interest is £416 13s 4d , what is the discount ?

7.   If the true discount on a sum due in 1 year, be $\frac{400}{419}$ of the banker's discount, what is the rate p c of simple interest ?

8.   The true discount on a certain sum due 7 months hence is £8 15s. and the banker's discount on the same sum for 7 months £8 18s $\frac{3}{4}d$, find the sum and the rate per cent per annum.

9.   How much less than the true P W will a banker give for a bill of Rs 9504, due in $7\frac{1}{2}$ months, interest at 5 p c ?

10.   What will a banker retain on discounting a

bill of Rs 12750, drawn on the 4th March at 10 months and discounted on 14th August at 5 per cent ?

11   A bill was drawn on May 14 at 2 months and was discounted on July 2 at $8\frac{1}{9}$ per cent   If the Banker's gain was 1s 4d , for what sum was the bill drawn ?

12.   A person discounting a bill 8 months before it is due, at 6 per cent receives £2 7s 6d less than the amount of the bill , what was the bill drawn for ?

13   A bill for £126 5s was drawn on March 9 at 5 months, interest being calculated at 5% , the discount on the bill was £1 5s 3d   On what date was the bill discounted ?

14   A banker discounted a bill 9 months before due and found that he would have $\frac{12}{13}$ of what he deducted as discount if he had reckoned true discount   Find the rate at which interest was calculated

15   A bill broker pays £242 19s 8d for a bill of £243. 6s. 8d. discounted 12 days before it is nominally due At what rate did he reckon the discount ?

16   A bill for Rs. 2625 is discounted and the discount charged amounts to Rs 18 6a   If the rate of interest be $3\frac{1}{2}$ per cent , find how many days the bill has yet to run

17   The interest on a certain sum for 10 years is greater than the true discount on the same sum, due in 10 years, by one-quarter of the discount   Find the rate which is the same in both cases.

18   The true discount on a sum of money for 3 months is Rs 150, and the commercial discount for the same time and rate is Rs 151 2a   Find the sum of money and the rate.

19   The true discount on a certain sum due in 3 months at 4% is £17 5s   Find the banker's discount and the sum.

20.   The true discount on a sum of money for 6 months is £12 and the banker's discount for the same time and at the same rate is £12 10s , what is the sum and the rate of interest ?   [*Nagpur 1929*]

F. 31.

CHAPTER XXVIII

BANKRUPTCY, TAXES, RATES, COMMISSION, BROKERAGE AND INSURANCE.

§1 Bankruptcy or Insolvency.

A tradesman becomes bankrupt or insolvent when his debts exceed his property. His debts are called liabilities and his property is called his assets By property we mean cash, book debts, i e , the money other persons owe to the bankrupt and immovable property such as buildings, machinery, land, etc.

It is clear that the bankrupt cannot pay to his creditors the full amount of their money   If his assets are $\frac{3}{4}$ of his liabilities he shall pay $\frac{3}{4}$ of their money, i e , $12a$ per rupee or $15s$ per $£$. This $12a$ per rupee or $15s$ per $£$ is called dividend

Suppose a tradesman owes his creditors Rs. 6000 and his assets are Rs 2000 , what dividend shall he pay in the rupee ?   We shall solve the example thus

for Rs. 6000 he can pay Rs 2000

for Re 1   he shall pay Rs $\frac{2000}{6000}$ = Re $\frac{1}{3}$,

i e , $5a$. $4p$ Ans

Note   Dividend in the $£$ or Re = Assets − Liabilities

whence   Assets = Liabilities $\times$ Dividend

and   Liabilities = Assets − Dividend

§2. We shall now solve some examples for illustration.

To find dividend in the rupee or $£$.

Example 1   A bankrupt's assets amount to Rs 5497, $8a$. and his liabilities are Rs. 7330   What can he pay in the rupee, and what shall a creditor receive for his claim of Rs 1275 ?

Sol. (*ι*) Dividend in the Re.=Assets—Liabilities

$$=\text{Rs. } 5497 \; 8a - \text{Rs } 7330$$
$$=\text{Re } \tfrac{3}{4} \text{ or } 12a. \quad \textbf{Ans}$$

(*ιι*) For Re 1 he can pay Re $\tfrac{3}{4}$

∴ for Rs. 1275 he shall pay Rs $1275 \times \tfrac{3}{4} = \tfrac{3825}{4}$

$$\iota. e, \text{ Rs. } 956 \; 4a \quad \textbf{Ans}$$

To find Liabilities.

Example 2   A bankrupt's assets were Rs 5333 5a +b and he paid a dividend of 10a 8p. in the rupee, what were his liabilities ?

Sol. Liabilities=Assets—dividend

$$=\text{Rs } 5333 \; 5a. \; 4p. - 10a \; 8p$$
$$=\text{Rs } \tfrac{16000}{3} \times \tfrac{8}{5}$$
$$=\text{Rs. } 8000. \quad \textbf{Ans.}$$

To find Assets

Example 3.   A bankrupt's liabilities were Rs. 1866 10a. 8p. and he paid 13a 4p in the rupee, find his assets.

Sol   Assets=Liabilities × Dividend

$$=\text{Rs } 1866 \; 10a \; 8p. \times 13a. \; 4p.$$
$$=\text{Rs } \tfrac{5600}{3} \times \tfrac{5}{6}$$
$$=\text{Rs } \tfrac{14000}{9} = \text{Rs. } 1555. \; 8a \; 10\tfrac{2}{3}p. \quad \textbf{Ans.}$$

Example 4   A bankrupt can pay 10a in the rupee, had he Rs 2000 more, he could have paid 12a in the rupee Find the amount of his debts and assets

Sol. `    `$12a - 10a = 2a = \text{Re. } \tfrac{1}{8}$

he could have paid Re $\tfrac{1}{8}$ more on Re 1 of his debts.

∴ he could have paid Re. 1 more on Rs 8 of his debts

„   „   „   „ Rs 2000 „ Rs 2000×8 „

Hence his debts are Rs $2000 \times 8 = $ Rs 16000 }
and his assets are $10a \times 16000 = $ Rs 10000 } **Ans.**

Example 5   A bankrupt has book debts, equal in amount to his liabilities, but on Rs 6000 of such debts he can recover only 12a. in the rupee and on Rs 2400

only $4a$ in the rupee and the expenses of the liquidation are Rs 366 $10a$ $8p$, if he pays $13a$ $4p$ in the rupee, what is the amount of his liabilities ?

Sol (i) On Re 1 the bankrupt loses $(16a - 12a)$ or $4a$
on Rs. 6000 he will lose Rs $6000 \times \frac{1}{4} =$ Rs 1500.

(ii) On Re. 1 he loses $(16a - 4a)$ or $12a$
on Rs 2400 he will lose Rs. $2400 \times \frac{3}{4} =$ Rs 1800

$\therefore$ the total loss including the expenses of bankruptcy
$=$ Rs. $1500 +$ Rs. $1800 +$ Rs 366. $10a$ $8p$
$=$ Rs 3666 $10a. 8p =$ Rs $\frac{11000}{3}$

His creditors lose $(16a - 13\frac{1}{3}a)$ or $\frac{8}{3}a$, in the rupee
$\frac{8}{3}a =$ Re $\frac{1}{6}$.

Now Re. $\frac{1}{6}$ is the loss on Re 1 of liabilities

$\therefore$ Re 1 $,,$ on Rs 6

$\therefore$ Rs $\frac{11000}{3}$ $,,$ on Rs $6 \times \frac{11000}{3}$ $i.e$ Rs. 22000.

$\therefore$ his liabilities $=$ Rs 22000. Ans.

§3   Preferential Claims.

Rent, servants' wages, etc are called preferential claims and they are always paid in full before the dividend is declared

Expenses of winding up a concern $i. e$

Expenses incurred in getting a bankruptcy sanctioned in collecting the book debt, and in arranging payment to the creditors are also, like the preferential claims, deducted before the dividend is declared.

Example 6. The assets of a bankrupt amount to Rs. 5621 and the liabilities to Rs 7682   If Rs 525 $8a$ be the preferential claims and Rs 324 $8a$. the expenses of the winding up, find the dividend declared in the rupee

Sol. Assets after paying the preferential claims and expenses of winding up $=$ Rs $5621 - ($Rs 324 $8a +$ Rs. 525. $8a.) =$ Rs. 4771,

and liabilities after paying the preferential claims
$=$ Rs $7682 -$ Rs 525 $8a. =$ Rs 7156 $8a$

$\therefore$ Dividend $= \dfrac{\text{Rs } 4771}{\text{Rs. } 7156 \ 8a.}$ $=$ Rs $\dfrac{4771 \times 2}{14313}$

$=$ Re $\frac{2}{3}$ or $10a$ $8p$, Ans

Example 7.   A creditor receives on a debt of £1280, a dividend of 11s. 3d in the £ and he receives a further dividend of 3s 6d. in the £ on deficiency   What amount does he receive in all ?

Sol   The first payment $=11s$ $3d$ or $£\frac{9}{16}$ on £1 of debts,

$\therefore$ the deficiency $=1-\frac{9}{16}$ or $£\frac{7}{16}$ on £1 of debts.

The 2nd payment $=3s$ $6d.$ or $£\frac{7}{40}$ on £1 of debts,

the 2nd payment $=\frac{7}{40}\times\frac{7}{16}$ or $£\frac{49}{640}$ on £1 of debts,

$\therefore$ both payments $=\frac{9}{16}+\frac{49}{640}$ or $£\frac{409}{640}$ on £1 of debts.

Now in £1 of debt the creditor receives $£\frac{409}{640}$,

..   ,, £1280   ,,   ,,   ,, $£\frac{409}{640}\times 1280$

i. e , £818. **Ans.**

EXERCISE 153.

1   A bankrupt's assets amount to Rs 3216 and his liabilities are Rs 6432   What can he pay in the rupee ?

2   A bankrupt's estates amount to Rs 1200 and his debts to Rs 1600 , how much can he pay in the rupee ?

3.   A bankrupt's debts amount to Rs 1624 $10a$ and his assets to Rs 541 $8a$ $8p.$ only , what dividend can he pay in the rupee ?

4   A bankrupt owes Rs. 1575. $8a$ but his assets are Rs. 1050. $5a$. $4p$ How much in the rupee can he pay ?

5   A bankrupt's effects amount to Rs 7521 and debts to Rs 10028   What dividend can he pay in the rupee and what amount of money will he pay to a creditor for a claim of Rs 7128 ?

6   A bankrupt owes A Rs. 26520, B Rs 46338 and C Rs 15114 $8a$, his estate is worth Rs 29324 $2a$. $8p.$, how much can he pay in the rupee and what will each creditor receive ?

7.   A bankrupt's liabilities amount to Rs 38700 and his creditors lose Rs 12900 , what dividend in the rupee does he pay and what will he pay to a creditor for a claim of Rs 8972 ?

8   A bankrupt's assets are Rs. 21000 and he can pay a dividend of $13a$ $4p$ in the rupee , find the liabilities.

9 A bankrupt's liabilities amount to Rs 18538. 5a. 4p. and he pays a dividend of 12a. in the rupee , find his assets.

10. A creditor received 16s 8d. in the £ and thereby lost £150 , how much was due to him ?

11. A bankrupt's effects amount to Rs. 15126 3a and he declares a dividend of 9a. in the rupee   What amount does he owe ?

12 A bankrupt's effects amount to Rs 20136 and liabilities to Rs 26521 4a , after paying the expenses of winding up the concern he declares a dividend of 12a. in the rupee , find the amount of expenses

13. A bankrupt can pay a dividend of 13a 4p. in the rupee , had he Rs 1200 more he could have paid 14a 8p. in the rupee   Find the amount of his debts and assets

14 A bankrupt can pay a dividend of 13a 4p in the rupee , had he Rs 500 less the dividend would have decreased by 2a 8p. in the rupee   Find the amount of his debts and assets

15. A bankrupt has book debts equal in amount to his liabilities, but on £1400 of such debts he can only recover 15s in the £ and the expenses of the winding up are £110 , if he pays his creditors 16s 8d in the £, what is the amount of his liabilities ?

16 A bankrupt has book debts equal in amount to his liabilities, but on £500 of such debts he can only recover 12s. in the £ and on £800 only 10s in the £ and the expenses of the winding up are £150 , if he pays his creditors 18s in the £, what is the amount of his liabilities ?

17. The assets of a bankrupt amount to Rs. 1680. 8a., the liabilities to Rs. 2184 4a   If the preferential claims be Rs. 324 4a. and the expenses for winding up the concern Rs. 116 4a., what dividend can he declare in the rupee ?

18. The effects of a bankrupt amount to £2137. 12s. 4d and the debts to £2625. 10s   If the preferential claims be £225. 10s. and the expenses for winding up the con-

cern £112 2s 4d , how much can be paid in the £ to the creditors ?

19   A creditor receives on a debt of £480 a dividend of 10s. in the £ and he receives a further dividend of 6s 8d in the £ on the deficiency , what amount does he receive in all ?

20   A creditor receives on a debt of £384 a dividend of 12s 6d in the £ and he receives a further dividend of 3s 9d in the £ on the deficiency , what amount does he receive in all ?

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§4. Incomes and Taxes.

The tax levied by the Government on the annual total gross income of a person or a company at the rate of so many pies or pence in every rupee or pound respectively is called the Income Tax   What remains after the payment of the income-tax is called the net income.   It follows therefore that

Net income   = gross income – income-tax,

Gross income = net income + income-tax

§5. Rates

The sums of money paid at the rate of so many pies or pence in a rupee or a pound respectively to the local authorities such as Municipalities, District Boards, etc by the house-holder or land-owner are called Rates

The *rates* are charged on the annual value (called also the rateable value) of the property.

Example 1   A man's gross income is Rs. 1600 , find the income-tax at the rate of 4p. in the rupee.

Sol. $4p. = \frac{1}{3}a = $ Re. $\frac{1}{45}$,

If the gross income is Re 1, the income-tax = Re $\frac{1}{45}$

. if the gross income is Rs. 1600, income-tax = Rs. $\frac{1}{45} \times 1600$

i e , Rs. 33 5a. 4p.   Ans.

From the above solution we learn that the

Income-tax = gross income × the rate of tax

Example 2   A man's gross income is £2100   Find his *net income* when he pays the income-tax at the rate of 5d. in the £

Sol. $5d = \frac{5}{12}s. = £\frac{1}{48}$

Income-tax = gross income × the rate of tax

$$= £2100 \times \frac{1}{48} = £\frac{175}{4} = £43 \quad 15s$$

∴   net income   = gross income − income-tax,

∴   reqd net income = £2100 − £43 15s

$$= £2056 \quad 5s. \quad \textbf{Ans}$$

Aliter. $£1 - 5d = £1 - £\frac{1}{48} = £\frac{47}{48}$

If gross income is £1 , the net income = $£\frac{47}{48}$

       „   „   „ £2100 ,   , = $\frac{47}{48} \times 2100$

                 i e. = $£\frac{82425}{4} = £2056. 5s.$ **Ans**

Example 3   If the income-tax be at the rate of 5p. in a rupee and a man has to pay a tax of Rs. 15 8a 4p , find the amount of his *gross income*

Sol $5p = \frac{5}{12}a = $ Re $\frac{5}{192}$, Re. 15 8a 4p $=$ Rs. $\frac{745}{48}$.

If the income-tax is Re. $\frac{5}{192}$, gross income = Re. 1

„   „   „   Re 1   „   „   = Rs. $\frac{192}{5}$

„   „   „   Rs. $\frac{745}{48}$ „   „   = Rs $\frac{192}{5} \times \frac{745}{48}$

                               = Rs. 596. **Ans**

From the solution we conclude that

Gross income = income tax − rate of tax

Example 4.   A man's net income after the payment of income-tax at the rate of 7p. in the rupee is Rs. 1665. Find his *gross income*

Sol   Re. $1 - 7p = 185p. =$ Re $\frac{185}{192}$

If the net income is Re. $\frac{185}{192}$, gross income = Re 1

„   „   „   Re 1   „   „   = Rs $\frac{192}{185}$

„   „   „   Rs 1665 „   „   = Rs $\frac{192}{185} \times 1665$

                             = Rs 1728. **Ans.**

Here gross income = given net income − net income of Re 1

Example 5   A man's net income after payment of income tax at the rate of $6p$. in the rupee is Rs 2421. 14a What will it be, if the income-tax be reduced by $1\frac{1}{2}p$ in the rupee ?

Sol.   (i) Re $1-6p =$ Re $\frac{31}{31}$ Rs 2421. 14$a =$ Rs. $\frac{19375}{8}$

(ii) $6p - 1\frac{1}{2}p =$ Re. $\frac{3}{128}$, Re $1 -$ Re. $\frac{3}{136} =$ Re $\frac{125}{128}$.

If net income is Re $\frac{31}{31}$, the gross income $=$ Re 1,

if net income is Re 1, the gross income $=$ Rs $\frac{32}{31}$,

$\therefore$ if net income is Rs $\frac{19375}{8}$,, $=$ Rs $\frac{32}{31} \times \frac{19375}{8}$

$=$ Rs 2500

Also, if gross income is Re 1, the net income $=$ Re $\frac{125}{128}$,

$\therefore$ if gross income is Rs. 2500, the net income

$=$ Rs $\frac{125}{128} \times 2500 =$ Rs 2441 6a. 6p.   Ans

EXERCISE 154.

1   A man's gross income is Rs 1500 and he pays Rs 26 8a as income-tax.   Find his net income.

2.   A man's net income is Rs 1123. 8a 4p , if he pays Rs. 21 5a 4p. as income-tax, what is his gross income ?

3   A man's gross income is Rs 1875. Find the income-tax at the rate of $4p$ in the rupee.

4   A man's gross income is Rs. 2800.   Find his net income after paying an income-tax at the rate of $7p$ in the rupee

5.   A man's salary is Rs 400 per mensem. Find his annual net income after paying an income-tax at $4p$. in the rupee.

6   A man pays income-tax of Rs 56 7a 5p at the rate of $6p$. in the rupee , find his net income

7.   A man pays an income-tax of Rs 39 1a. at the rate of $5p$ in the rupee , find his net income

8.   A man after paying an income-tax at $6p$. in the rupee has Rs 1550 8a 6p. What was his gross income ?

9. A person after paying an income-tax at 5*d* in the £ has £2447 18s. 4*d*. Find his gross income

10. A man's net income after paying an income-tax at the rate of 4*d* in the £ is £2261 14s. What will it be, if the income-tax be 5*d* in the pound ?

11. A person's net income after paying an income-tax at 7*p* in the rupee is Rs 1252. 9*a* 8*p* What will it be, if the income-tax be reduced by 1*p*. in the rupee ?

12 When the income tax is 5*d* in the £ a man pays £15 less than when the income-tax is 7*d*. in the £. Find his income

13 When the income-tax is 9*p* in the rupee, a man has to pay Rs 19 3*a* 4*p*. more than when the income-tax is reduced by 2*p*. in the rupee Find his income.

14 A person's gross income is £1125 and his net income is £1106 5*s* Find the rate of income-tax

15 Income upto Rs 1500 being subject to an income-tax of 4*p* in the rupee and incomes above Rs 1500 to 6*p*. in the rupee, find what income above Rs. 1500 a man must have to be just as rich as a man with Rs 1488 a year

16 Incomes upto £ 1600 being subject to an income-tax of 5*d* in the £ and incomes above £1600 to 8*d* in the £, find what income above £1600 a man must have to be just £ 15 a year richer than the man with £ 1584 a year

17. Incomes upto £ 1200 are subject to an income-tax of 7*d* in the £ and incomes above £ 1200 to an income-tax of 10*d* in the £ Find what income above £1200 a man must have to be just 10s. a year poorer than the man with £1190 a year

18 The rent of a man's bungalow is Rs. 720 per annum It is assessed to the rates at half of this, the school rate is 2*a*. 9*p*. in the rupee, the roads rate is 3*a*. 3*p* What does he pay altogether for his residence ?

§6 Commission or Brokerage.

A person who buys or sells goods, property etc., for another person or company is called an agent or more commonly a broker, and the remuneration (certain percentage) he receives in return of his labours is called commission or brokerage

§Insurance

Insurance is a contract made between a company and a person, by which the former undertakes to pay to the latter or his heirs, the losses caused to him by some accident such as fire, shipwreck or death of a person and the latter promises to pay yearly, half-yearly, quarterly or monthly, as agreed upon, a certain percentage of the sum insured

The Company which makes such contracts is called the Insurance Company and the person or property protected against accidents by such contracts is called the insured   The sum of money (a certain percentage paid yearly, half-yearly, etc) is called the premium

§8. When a man insures so as to recover not only his property but also the premium and other expenses of insurance it is said to be covered

All questions under Commission, Brokerage and Insurance are solved by the Unitary Method We shall now solve some examples by way of illustrations

Example 1   A broker sells 28 shares of a bank at Rs 25 per share , find his brokerage at $1\frac{1}{2}$ per cent

Sol   Value of 28 shares $=$ Rs $28 \times 25 =$ Rs 700

$1\frac{1}{2}$ per cent $= \dfrac{1\frac{1}{2}}{100} = \dfrac{3}{100}$ of the whole,

$\therefore$ brokerage $=$ Rs $\dfrac{3}{100} \times 700$

$\qquad\qquad = $ Rs. $\dfrac{21}{2} =$ Rs 10. 8a.   **Ans**

Example 2   An agent sells a house and pays the owner Rs 3840 after deducting his commission at 4% , find the value of the house.

Sol   If he pays Rs 96, the value of the house $=$ Rs 10

$\therefore$ if he pays Re. 1, the value of the house $=$ Rs $\frac{10}{96}$

$\therefore$ if he pays Rs 3840, the value of the house

$$= \text{Rs } \tfrac{100}{96} \times 3840$$
$$= \text{Rs } 4000. \text{ Ans}$$

Example 3   For what sum should goods worth Rs. 3750 be insured at $6\frac{1}{4}\%$ so that in case of loss the owner may recover the premium as well as the value of the goods.

Sol.   If the goods worth Rs $100 - $ Rs. $6\frac{1}{4}$, i.e , Rs $93\frac{3}{4}$ be insured for Rs 100 he will recover the loss as well as premium.

Now Rs $93\frac{3}{4}$ must be insured for Rs 100,

.   Re. 1   „   „   „   „   Rs $100 \times \frac{4}{375}$,

      Rs. 3750 „   „   „   „   Rs. $100 \times \frac{4}{375} \times 375$

      which is equal to Rs. 4000   Ans

covered insurance $= \dfrac{100}{100 - \text{rate}} \times$ value of the property

EXERCISE 155.

1. Find the commission on goods worth Rs. 324 at $1\frac{1}{4}$ per cent.

2   I purchase goods worth Rs. 521 and pay a broker age at $1\frac{1}{2}\%$.   What do the goods cost me altogether ?

3. I purchased goods worth Rs. 5325 and paid Rs. 106 8a. as commission , find the rate per cent of commission.

4. If the brokerage on goods at 5a. 4p. per cent. amounts to Rs 12 5a. 4p , find the value of the goods.

5   Commission at $1\frac{1}{8}$ per cent amounts to Rs. 6. 12a. On what sum is the commission paid ?

6. A broker after deducting his brokerage at 12a per cent pays Rs 14887 8a to the owner of the house At what price was the house sold ?

7   A man whose life is insured for Rs 2000 has to pay an annual premium of Rs. 70   What is the rate per cent. ?

8   A man insures his life for Rs 3000 , what is the annual premium at 3 per cent ?

9   A man insures his life for Rs 2000 , what is the annual premium at Rs 3 10*a* 9*p*. per cent ?

10   For what sum must a house-holder insure his house worth Rs 6998 4*a* at 7 per cent so that he may in case of loss, be able to recover both the value of the house and the premium ?

11.   For what sum must a cargo worth Rs 23500 be insured at 6 per cent so that in case of loss, both cargo and premium may be recovered ?

12   A house is insured for Rs 15000 so as to cover both the value of the house and the amount of the premium at 3%   Find the actual value of the house.

13   A cargo is insured for £2000 so as to cover both the value of the cargo and the amount of the premium at 5 per cent , find the actual value of the cargo

14   A man's annual income is Rs 5700 , after paying insurance premium which are exempted from income-tax, he pays Rs 166 10*a* 8*p* as income-tax at 3⅓ per cent What amount of money does he pay for premium ?

15   A man's annual income is Rs. 4100   He pays no income tax on the money paid for premiums   On the remainder he pays Rs 133 5*a*. 4*p* as income-tax at 4 per cent   What amount of money does he pay for premiums ?

CHAPTER XXIX

EXCHANGE

§1 **Exchange** *means the process of paying or receiving the money of one country for an equivalent sum in another.*

This process can be effected by remitting

(i) specie or coined money ,

(ii) bullion or gold, silver in bars ,

(iii) a bill of exchange

The payment of a sum of money, by means of specie or bullion, is a costly process , remittances, therefore, are usually made by sending bills of exchange

§2 **Def.** A written order, to some person at a distance, to pay a certain sum, at a particular time, to another person, or to his order, is called a *bill of exchange*

Bills of exchange are of two kinds—**foreign bills** and **inland bills**

§3 The equality between the standard coins of two countries obtained by comparing their weight and fineness, is called the **par of exchange** between those countries Thus, if it is found that the quantity of pure silver in one rupee is exactly equal to that of pure silver in 1 25s. then Re 1 $=$ 1 25s

§4. The **rate or course of exchange** is such a variable sum of money of one country as is proposed to be given for a fixed sum of that of another , thus, for instance, £1, a fixed sum of English money, is given for a variable sum of French money, *viz* , 25 16 francs, 25'18 francs, 25'23 francs, etc more or less than the par of exchange

Note that when the rate of exchange is *above* the par of exchange, it is said to be at a **premium**, and when it is *below* the par of exchange, it is said to be at a **discount**

§5　Arbitration of exchange is the method of determining a course of exchange, called the direct or arbitrated rate, between any two places, when the rate of exchange between each of them and some other place or places is known

Example 1.　What is the exchange value of £78. 11s $7\frac{1}{2}d$ at 2s $\frac{3}{4}d$ per rupee ?

Sol　£78 11s $7\frac{1}{2}d = £78\frac{93}{160}$, 2s $\frac{3}{4}d = £\frac{33}{320}$
Reqd value $= £78\frac{93}{160} ÷ £\frac{33}{320} = 1\frac{2573}{160} × \frac{320}{33}$
$= $ Rs 762　Ans

Example 2　If Rs 160 $4a$ $8p$ are equivalent to £15. 15s $6\frac{57}{84}d$, find the exchange value per rupee

Sol　Rs. 160 $4a$ $8p =$ Rs $160\frac{7}{3\tau}$
£15 15s $6\frac{57}{84}d. = 315\frac{147}{356}s$.
．Reqd exchange value $= 315\frac{147}{356}s. ÷$ Rs $160\frac{7}{3\tau}$
$= \frac{80787}{256} × \frac{24}{3847} = \frac{60}{33}s = 1s$ $11\frac{5}{8}d$ per rupee　Ans

Example 3　I pay Rs 51000 to a bank for a bill of exchange payable in London　The rate of exchange is 1s $10\frac{1}{2}d$ for the rupee and the bank charges me 2 per cent on the amount payable in England　How much will my -agent in London receive ?

Sol　1s $10\frac{1}{2}d = £\frac{3}{32}$

The exchange value $= £51000 × \frac{3}{32}$

Now the agent receives £10 out of £102 sent by me

∴ the sum received by the London agent
$= £51000 × \frac{3}{32} × \frac{100}{102}$
$= £\frac{9375}{2} = £4687$ 10s　Ans.

Example 4.　If the par of exchange be two English shillings for the Indian rupee, but if an Indian bill of exchange for Rs. 540 $12a$. be negotiated in London for £51 10s, how much per cent below par is the rate of exchange ?

Sol.　Rs 540 $12a. =$ Rs. $\frac{2163}{4}$, £51. 10s $= £\frac{103}{2}$.
rate of exchange $= £\frac{103}{2} ÷$ Rs $\frac{2163}{4} = \frac{103}{2} × \frac{4}{2163}$
$= £\frac{2}{21}$ or $\frac{40}{21}s$.
$2s - \frac{40}{21}s$ or $\frac{2}{21}s$. is below par on every 2s
Reqd percentage $= \frac{2}{21} × 100 ÷ 2$
$= \frac{100}{21}$ or $4\frac{16}{21}\%$　Ans

Example 5. When the rate of exchange is 1s $7\frac{3}{4}d$ for the rupee, what is the nearest sum of Indian money equivalent to £79 3s. $7\frac{3}{4}d$. ?

Sol Since Re $1 = 1s\ 7\frac{3}{4}d = \frac{79}{48}s$ or $\frac{79}{4}d$.

Rs $2 = 3s\ 3\frac{1}{2}d$

Now £79 3s. $7\frac{3}{4}d = £79 + 3s\ 3\frac{1}{2}d + 4\frac{1}{4}d$

$= $ Rs $(79 \times 20) - \frac{79}{48} + $ Rs.2 + Re $\frac{17}{4} - \frac{79}{4}$

$= $ Rs $960 + $ Rs $2 + 3a\ 5p$ nearly

$= $ Rs. $952\ 3a\ 5p$ Ans.

Example 6 An American merchant sends goods worth 58870 dollars to an English merchant when the course of exchange is 3 36 dollars to a pound sterling but receives his money after 2 months when the rate of exchange is 3 48 dollars to a sovereign How much does the English merchant gain or lose by the 2 months' delay in payment ?

Sol. The exchange value in the 1st case

$= £58870 \times \frac{100}{336}$

„   „   „   „   2nd case

$= £58870 \times \frac{100}{348}$

gain $= £58870 \times (\frac{100}{336} - \frac{100}{348})$

$= £58870 \times 100 \times \frac{12}{336 \times 348} = £\frac{3625}{6}$

$= £604\ 3s.\ 4d$ Ans

Example 7 The exchange between London and Calcutta is 1s. $7\frac{1}{3}d$ per rupee, that between Paris and London is 25 2 francs for a pound sterling and that between Rome and Paris is 11 lira for 10 francs (1) Find the arbitrated rate between Italy and India (2) If a Calcutta merchant owes Rs 4000 to a merchant of Rome what sum in lira will liquidate the debt ?

Sol. 1s $7\frac{1}{3}d = £\frac{13}{160}$

Re. $1 = £\frac{13}{160}$

£ $1 = 25\ 2$ francs

francs $10 = 11$ lira

How many lira $=$ rupee 1

∴ Reqd No. of lira $= \frac{13 \times 252 \times 11}{160 \times 10 \times 10} = 2\ 25225.$ **Ans.**

Rs. $4000 = 2\ 25225 \times 4000$

$= 9009$ lira   Ans.

Example 8   If the short rate of exchange in London on Paris is 25 l, and the rate of discount for two months' bills in London is at $4\frac{1}{2}\%$ per annum, what debt in Paris can be discharged by a person in London who has a two months' bill on London for £2400 ?

Sol   Discount on £1 $=$ £1 $\times\frac{9}{4}\times\frac{1}{100}\times\frac{2}{12}$

$$= £\frac{3}{400}$$

∴ Present value of £1 $= £(1-\frac{3}{400})$ or £ $\frac{397}{400}$

Hence £ $\frac{397}{400}=25\cdot2$ francs.

∴ The amount of reqd debt

$$= £2400\times\frac{397}{400}\times\frac{25\cdot2}{1\cdot0}\text{ francs.}$$

$$=60046\cdot4\text{ francs.   Ans,}$$

Example 9   How much sterling should be given for 1920 marks each equivalent to $11\frac{1}{4}d$ at par, when English money bears a premium of 4% in Germany ?

Sol   Since 1 mark $=11\frac{1}{4}d$ or $£\frac{45}{960}$

1920 marks $= £\frac{45}{960}\times1920$ or $£90$.

The reqd sum $= £90\times\frac{104}{100}$

$$= £93\ 12s.\text{ Ans}$$

Example 10   What is the short rate of exchange, when the price in Hamburg of six months' bills in Calcutta is 1·18 marks per rupee and the rate of discount in Calcutta is $3\frac{1}{3}$ p c ?

Sol   Present value of Re 1 $=$ Re $(1-1\times\frac{10}{3}\times\frac{1}{100}\times\frac{6}{12})$

$$=\text{Re. }\frac{59}{60}.$$

Re $\frac{59}{60}=1\ 18$ marks

Hence the reqd. short rate $=1\ 18-\frac{59}{60}$

$$=1\ 2\text{ marks.   Ans}$$

EXERCISE 155 A.

1.   How much Indian money can be paid for £1560, when the rate of exchange is 1s 4d per rupee ?

2.   Find the exchange value in pound sterling of Rs 19920 at 1s $10\frac{1}{2}d$ for the rupee

3.   A merchant of Bombay indents from London goods worth £4840   What must he pay, when the course of exchange is 1s. 10d a rupee ?

F. 32.

4 If London exchanges with Lahore at a gain of $8\frac{1}{2}$% when the course of exchange is Rs. 24. 6*a* per £, what is the par of exchange ?

5 Bombay exchanges with England at 1*s*. 10*d*. a rupee and with New York at Rs 4   2*a* a dollar, find the estimated rate of exchange between England and New York.

6 If the exchange between England and Spain be 10$\frac{3}{4}d$ per franc and that between England and India be 1*s*. 4*d* per rupee, what is the arbitrated rate between Spain and India ?

7 A New York dollar at par of exchange is worth 4*s* 8*d*. What is the value of 391 dollars when exchange on London is at a premium of $6\frac{1}{4}$ per cent ?

8. An English officer, who gets his annual pension in rupee suffers a loss of £115. 17*s* owing to a fall in the exchange from 1*s* 4*d* to 9*d*. per rupee. Calculate his annual pension in rupees

9 Express $\frac{2}{3}$ of Rs. 17 8*a*. and $\frac{5}{6}$ of £1. 14*s* 6*d*. as the fraction of Rs 170 , a rupee being worth 2 shillings

10. When a rupee is worth 1*s* $4\frac{3}{16}d$., how many rupees can be bought for £13 9*s* $9\frac{1}{2}d$. ?

11 The course of exchange between Bombay and London varies at different times from 1*s* 2*d* to 1*s* 3*d* per rupee   A merchant wants a table giving the values in Indian money of £1 and £10 in the form given below. Copy this table and fill up the blank spaces correct to the nearest anna

When Re 1=	1*s* 2*d*.	1*s* $2\frac{1}{4}d$	1*s* $2\frac{1}{2}d$.	1*s*. $2\frac{3}{4}d$.	1*s*. 3*d*.
£ 1=	Rs. *a*	Rs *o*.	Rs. *a*.	Rs *a*	Rs *a*
£ 10=					

12 At a certain rate of exchange a sum of Rs. 2400 is required for remitting a certain sum in pounds from

Bombay to London , but if the rate of exchange were to
rise by 2d per rupee Rs 300 less would suffice. Find the
original rate of exchange per rupee.

13   A person in England has a certain sum invested
in India to derive an annual income at the rate of 4½ per
cent   If, after deducting 2 per cent as agent's charges
for drawing and remitting the money, it brings him an
income of £429 19s 6d per annum, what is the amount
of the investment in rupees, the rate of exchange being
1s 7½d per rupee ?

14   Find the cost in rupees of one mile of railway,
which consists of two rails each weighing 40 lbs per yard
on wooden sleepers weighing 70 lbs each placed 2 ft 8 in.
apart   The rails cost in England £6. 13s per ton, and the
sleepers 2s 4½d each   The rate of freight is £1 5s per,
ton, and landing charges amount to Rs 2 8a per ton
Rate of exchange 1s 8d per rupee

15   If the rupee is worth 1s. 6¾d , express Rs. 6 5a
4p as a fraction of £1 , and find the least number of
rupees equal in value to an integral number of pounds.

16   Find the value of £1 in Indian money when the
exchange is at 1s 5½d per rupee, the price of a bill for
£70 at the same rate of exchange and the alteration in the
price of the bill owing to a fall of ½d. in the value of the rupee

17   A shilling equals 12 pence and a guinea equals
21s , how many guineas are Rs. 200 equal to when 1s
7½d are equal to Re 1 ?

18   An American dollar is equal to Rs 2 2⅖a and is
also equal to 5 375 francs , how many francs are there in
one rupee ?

19   If Rs. 1000 a month is equivalent to £1112. 10s
a year, what is the value of a rupee in English money ?

20   13 lbs of tea at 3s 6d a lb are mixed with 19 lbs.
at 3s 11d. a lb , what is the price of the mixture in rupees,
annas, pies, when a rupee is equivalent to 1s. 11d ?

CHAPTER XXX

STOCKS AND SHARES

§1   Stock.

When the Government of a country requires large sums of money for any purpose, *e g*, to construct a railway, or to carry on a war etc., where does it get money from ?   Evidently it must *borrow* or *contract a loan* from the people.   The people lend money to the Government which give to the *Lenders* in exchange for their money, *Bonds* or *Acknowledgments* for the amount lent but it reserves to itself the option of the *Time* of paying off the *Principal* on the clear understanding to pay the interest of the money regularly at fixed periods. *Stock* is the term applied to this money lent to the Government of a county or to a Trading firm at some fixed rate of interest.

Thus, if the Government of India borrows 50 crores of rupees at 5 per cent. and if a person *A* lends Rs. 1000 of this amount, *A* is said to have Rs. 1000, 5 per cent. Stock.   He receives a document to this effect and is entitled to receive the interest *viz.*, Rs 50, upon this document from year to year until the Government repays the loan to *A*   This interest is paid *half-yearly* and the document may be sold and transferred from one person to another like any other kind of property.

§2   Suppose the holder of a stock wants cash money for some urgent business or for a change in his circumstances, what will he do ?   He cannot demand the payment of the loan from the Government before the due date but he can *sell* his stock to another person whereby his claim to half-yearly interest is transferred to that person.

The cash value of stock is not affected by any change in the rate of interest, since it is fixed once for all by the Government at the time when the money is borrowed.

500

But it varies from time to time, sometimes even twice or thrice daily, owing to political or commercial causes ; for instance, if the current rate of interest is smaller, the investment is free from all possible risks and the number of investors is large, then a holder of Rs 100 stock could sell it for more than Rs 100   Similarly, if the current rate of interest is greater or if the investment is risky, then the same stock is to be sold at perhaps less than Rs 100   Hence remember the following terms :—

(1) At par   If by selling a Rs 100 stock, a person receives Rs. 100 cash, the stock is said to be at par.

(2) At a premium or above par. If the selling price of Rs 100 stock is more than Rs 100 cash, the stock is said to be at a premium or above par

(3) At a discount or below par. If the selling price is less than Rs. 100 cash, it is said to be at a discount or below par.

§3. Brokerage.

Stock is generally bought and sold through a broker, who usually *charges* $\frac{1}{8}$ *per cent on the stock* bought or sold   Thus if the market value of Rs 100 stock is Rs 102, the seller of a stock receives Rs $(102-\frac{1}{8})$ and the purchaser has to pay Rs $(102+\frac{1}{8})$ This charge of a broker is called *brokerage*   Hence

The brokerage must be added to the price of stock which is bought and subtracted from the price of that which is sold through a broker. The broker's charge is often quoted "*brokerage* $\frac{1}{8}$", the words 'per cent' being omitted   Unless the brokerage is expressly stated, it need not be considered when doing examples in stocks.

§4   Money borrowed by a Government is called National Debt   Money lent to the Government of India is said to be invested in Government of India Securities. Money lent to the Government of England is said to be invested in Funds   A large portion of the National Debt of England is called Consols

§5   Students should very carefully distinguish between the *paper or nominal value* of stock and the *cash or actual price*, as well as between the *amount of stock*

purchased and the *sum invested* for its purchase   Thus if the 3 per cent. stocks are at 95, a man who invests Rs 950 will be able to purchase Rs 1000 stock *i e*, the amount of stock held by the man is Rs 1000, but the *actual* or *cash* value of that stock is only Rs. 950.

The student should also clearly understand the meaning of an expression of the type "Rs 4000 in the 5 per cen s" It does *not* mean that the man invested Rs 40)0. in the 5 per cent stock but that he holds stock of the *nominal value* of Rs. 4000 in the 5 per cent   He might have actually invested more or less than Rs 4000 according as the stock was at a premium or at a discount.

§6. Shares.

Suppose a big railway line or some such work is to be done, which requires a large amount of money. Evidently one or two individuals cannot provide all the money   Several persons meet together and decide to start a company   These men, called **promoters**, first decide the amount of money or **capital** required for the purpose and then elect a few responsible men called **directors** who are considered fit to manage the affairs of the company which is called a **joint stock company.** The directors then divide the capital into a large number of equal parts or **shares** and invite the public to **subscribe** to the fund.   A man can take as many shares as he likes and thus he becomes a **share-holder**   The whole amount of the shares is not generally paid in one instalment , for instance, suppose that the capital of a Railway Company Rs 1,000,000 is divided into 100,000 shares of Rs 10 each , since the construction cannot be completed in a few days or months, the whole amount is not required at once   The company, therefore, might ask its share holders to pay at first only Rs 3 per share and the remaining Rs 7 when **called upon** The Rs. 300,000 thus raised is called the **paid-up capital** of the Company

Now when the railway has been completed, it begins to work and thus earn money, a portion of which is spent in paying working expenses and the remainder is usually divided amongst the share-holders   Profit thus divided among share-holders is called **dividend.**

§7. Different Varieties of Stock

When a company finds that the capital subscribed by its shareholders is not sufficient, it does not issue more shares but it usually borrows money at a fixed rate and agrees to pay interest on this money before paying dividends to its share-holders, who subscribed to its original capital   Money thus raised is called preference stock, whereas the original capital is called ordinary or deferred stock.

Again certain companies issue what are called debentures   The holder of a debenture stock receives a fixed rate o interest and not a share of its profits, but if the interest is not regularly paid, he can have claim on the property of the company

§8. Warning against common mistakes.

In solving questions on stock, the student must clearly distinguish between cash and stock, as explained in Art 5

A stock is often denoted by the rate of interest it yields. Hence *"5 per cent at 98"* means —

  (*i*) There is a certain stock which pays a dividend of Rs 5 on every Rs 100 stock ,

  (*ii*) the market value of Rs 100 stock is Rs 98 *i. e* , a person can purchase a Rs 100 stock for Rs. 98 only , and therefore

  (*iii*) there is a dividend of Rs. 5 on an investment of Rs 98, *i e* , by spending Rs. 98 only, a person can have an income of Rs 5

§9   All examples in stock can be solved by the principle of *Rule of Three* or *Unitary Method*   We shall now solve a few questions of different types on stock.

  (*a*) Amount to be invested.

Example 1.   What sum of money will be required to buy Rs. 16,000 stock in 3½ per cents at 91 ?

Sol   Rs. 91 are required for buying a Rs 100 stock

$\cdot$ Rs 100 stock $\cdot$ Rs. 16000 stock. Rs. 91 reqd. sum

$\cdot$ the reqd sum $=$ Rs. $\dfrac{16000 \times 91}{100} =$ Rs 14560. **Ans.**

or by *Unitary Method* —

$\cdot$ cost of Rs. 100 stock $=$ Rs. 91
. cost of Re 1   ,, $=$ Re $\frac{91}{100}$
. cost of Rs. 16000,, $=$ Rs. $\frac{91}{100} \times 16000$
                          $=$ Rs 14560   Ans

Example 2.   What sum must I invest to buy £1700 stock in $4\frac{1}{2}$ per cents. at $101\frac{3}{8}$, brokerage being $\frac{1}{8}$ p. c. ?

Sol.   Here £$101\frac{3}{8} + \frac{1}{8}$ *i e*, £$101\frac{1}{2}$ is required for buy-ing a £100 stock ,

£100 stock £1700 stock $\cdot$ £$101\frac{1}{2}$ required sum

$\therefore$ required sum $=$ £$\dfrac{1700 \times 101\frac{1}{2}}{100} =$ £1725 10s Ans.

EXERCISE 156.

How much must be invested to purchase (1—9):—

1   Rs 2000 stock at 92 ?

2   £2400 stock at 103 ?

3.   Rs. 4500 stock in the $3\frac{1}{2}$ per cents. at 88 ?

4.   5550 stock in the 4 per cents. at $97\frac{5}{8}$?

5.   £650 stock in the 3 per cents   at $90\frac{3}{4}$ through a broker ?

6   Rs. 3075 in the $4\frac{1}{2}$ per cents. bonds at par (broker-age as usual) ?

7.   £8833. 6s 8d. in the 5 per cents. at $3\frac{3}{4}$ premium, brokerage $\frac{1}{8}$ p c ?

8   Rs 1229 2a 8p. in the 4 per cents. at $\frac{3}{8}$ dis-count, brokerage as usual ?

9   Rs. 1250 stock at $50\frac{1}{2}$ premium (dividend $5\frac{1}{2}$ p c) ?

10.   When a certain stock is selling at 19 premium, what money will a person require to buy £2025 stock at $5\frac{1}{2}$ p c., brokerage being $\frac{1}{8}$ p. c. ?

(b) Amount obtained by the sale of stock.

Example 3. A person has Rs. 10000 stock in a certain company whose shares are at 135 per cent premium What amount does he get by selling his stock through a broker, brokerage being $\frac{1}{8}$ per cent ?

Sol. Here by selling Rs. 100 stock, the man gets Rs 235 $-$ Re $\frac{1}{8}=$ Rs $234\frac{7}{8}$ i e, Rs $\frac{1879}{8}$

$\therefore$ by selling Re 1 stock he will get Rs $\frac{1879}{8}\times\frac{1}{100}$

$\qquad$,, $\quad$ Rs 10000 ,, $\quad$,, $\quad$ Rs $\frac{1879}{8}\times\frac{1}{100}\times 10000$

$\qquad\qquad\qquad\qquad\qquad\qquad =$Rs 23487 8$a$ Ans

EXERCISE 157

How much money do I derive from the sale of (1—3) —

1. Rs 16000 in the $3\frac{1}{2}$ per cents. at $88\frac{1}{8}$?

2 £15850. 16s 8d in the $3\frac{1}{4}$ per cents. at $98\frac{3}{8}$, brokerage being as usual ?

3 £9533 6s. 8d in the 3 per cents. at $95\frac{5}{8}$, brokerage being as usual ?

4 A person has Rs 5000 stock in a certain company whose shares are at 116% premium What amount does he get by its sale through a broker, brokerage being $\frac{1}{8}$ p c ?

5. A person has Rs. 7500 stock in a certain company whose shares are at $8\frac{4}{5}$ p c. discount What amount does he get by its sale through a broker ?

(c) Amount of stock purchased.

Example 4 How much stock can be purchased by investing £506 9s 2d in the $3\frac{3}{4}$% at $89\frac{1}{4}$, (brokerage $\frac{1}{8}$% ?)

Sol. Here in order to purchase £100 stock, a person should invest £$(89\frac{1}{4}+\frac{1}{8})$ i e, £$\frac{715}{8}$, £506. 9s 2$d_r=$£$\frac{12155}{24}$

$\qquad$ amount of stock bought for £$\frac{715}{8}$ $=$£100

$\therefore$,, ,, ,, for £ 1 $\quad=$£$100\times\frac{8}{715}$

,, ,, ,, for £$\frac{12155}{24}=$£$100\times\frac{8}{715}\times\frac{12155}{24}$

$\qquad\qquad\qquad\qquad\qquad\qquad =$£566. 13$s$. 4$d$. Ans·

Example 5   I have Rs 3600 of 3 % stock . I sell out at 110 and buy $4\frac{1}{2}$ % stock at 120 , how much $4\frac{1}{2}$ % stock do I hold ?

Sol.   By selling Rs. 100 stock I get Rs 110 ,

∴ by selling Rs 3600 stock I get Rs $\frac{110}{100} \times 3600 =$ Rs 3960.

Again, by investing Rs 120, I buy Rs 100, $4\frac{1}{2}$ p c. stock

∴ by investing Rs 3960 I buy Rs $\frac{100}{120} \times 3960$,,   ,,

<div align="center">i e.   Rs. 3300 stock. Ans</div>

EXERCISE 158

Find the amount of stock purchased by investing (1—4):—

1   Rs 8280 in the $3\frac{1}{2}$ per cents. at 92

2.   £821 5s in the $4\frac{1}{2}$ per cents at $82\frac{1}{8}$.

3.   Rs. 5062 8a when the price is at a premium of Rs $12\frac{3}{8}$ and brokerage as usual,

4   £6909 18s in the 3 per cents. at £$92\frac{3}{4}$, brokerage being 2s 6l per £100

5   Find the price of 4% paper when Rs 5230 stock can be bought for Rs 5753 cash

6   A person invests 3000 fr in 5 per cents. at 87'73 , what amount of stock will he hold ?

7.   A man with £ 5330 purchased stock at £91 and when the price of the stock increased by £ $1\frac{3}{4}$, he sold it, and with the money thus obtained, he again purchased Debentures at £$102\frac{1}{2}$. How much debenture stock did he purchase ?

(d) Gain or loss by sale of stock

Example 6   A person invests £14340 in consols when they are at $89\frac{5}{8}$ and sells when they are at $93\frac{3}{8}$ What is his gain ? (Brokerage as usual)

Sol.   Here, he bought the consols at $(£89\frac{1}{2}+\frac{1}{8})$ i e , £$89\frac{5}{8}$ and when he sold he got £$(93\frac{3}{8}-\frac{1}{8})$ i e., £$93\frac{1}{4}$, so that for every £$89\frac{5}{8}$ or £ $\frac{717}{8}$ that he invested he gained £ $(93\frac{1}{4}-89\frac{5}{8})$ i e , £$3\frac{5}{8}$ or £$\frac{29}{8}$

∴ for investing £14340 his gain$=\frac{8}{717} \times \frac{29}{8} \times 14340$

<div align="right">$=$£580   Ans.</div>

EXERCISE 159

1. A person invests £13597. 10s in the purchase of the 3 per cents at $9\frac{7}{8}$ and afterwards sells it at $94\frac{1}{4}$ Brokerage being as usual, what profit does he make ?

2   I lay out Rs 2653 8a in buying $3\frac{1}{2}$ per cent. consols at $97\frac{3}{4}$ and then sell it at $96\frac{1}{8}$ Find my loss, usual brokerage being charged on each transaction

3   I invest Rs 126540 in the 3 per cents. at $99\frac{3}{4}$, at what price must I sell to gain Rs 15817. 8a. ?

4   A man invests Rs 3750 in buying stock at $93\frac{5}{8}$ and sold it at $95\frac{3}{4}$ If the whole transaction was done through brokers, how much did he gain ?

5   How much stock in the 4 per cents should be bought at 96, in order that by selling out at par Rs 250 may be gained ?

6   How much must I invest in the 3 per cent. stock when they are at 10 per cent below par, that by sell-ing when they are at 5 per cent. premium, I may gain Rs 1125 ?

7   A person invests £1365 in the three per cents at 91 , he sells out £1000 stock when they have risen to $93\frac{1}{2}$ and the remainder when they have fallen to 85. How much does he gain or lose by the transaction ?

8. A person invests Rs 7560 in the 3 per cent stock at $94\frac{1}{2}$ and when it falls to 90, he sells out $\frac{1}{4}$ of his stock , afterwards when it is at $94\frac{3}{4}$, he sells the remainder. Find the alteration in his capital

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§10   Income

In all examples given above, it will be seen that we did not take into account the rate of interest which any stock yielded, for the question of *income* did not arise in any case   But in fact the question of income is the most important factor in all transactions on stocks or shares   Two persons might possess two different kinds of stocks by investing unequal amounts of money but yet

they might be of the same value to the holders in point of income. It will be easily noticed from the examples solved below that all questions of income and transfer of stock from one kind to another belong to the theorem of "Rule of Three" or "Unitary Method".

(b) Income derived from investment.

Example 1. What income is derived from investing Rs. 7560 in the 3 per cents at $94\frac{3}{8}$, brokerage being $\frac{1}{8}$ p c ?

Sol. Here, by investing Rs $(94\frac{3}{8} + \frac{1}{8})$ i e, Rs $94\frac{1}{2}$, a man could get a Rs 100 stock, which will bring in an annual income of Rs 3 ,

Income on Rs $94\frac{1}{2}$ cash $=$ Rs. 3

      ,,   ,, Re 1   ,, $=$ Re $3 \times \frac{2}{189}$

      ,,   ,, Rs. 7560 ,, $=$ Rs. $3 \times \frac{2}{189} \times 7560$

                         $=$ Rs 240   **Ans.**

EXERCISE 160.

1. What annual income will be derived by investing Rs 35190 in the $3\frac{1}{2}$ per cents at 90 ?

2. A invests £1695 16s. 8d in the $3\frac{1}{2}$ per cents at $101\frac{3}{4}$. B invests £1767. 3s. 9d in the 3 per cents at $94\frac{1}{4}$, what is the difference of their yearly incomes ?

3 What is the total income derived by investing Rs 686 in the 3 per cents at 98 and Rs 765 in the $2\frac{1}{5}$ per cents. at 85 ?

4 What half yearly dividend is obtained by an investment of £5000 in the 3 per cent stock at $87\frac{3}{8}$, after deducting 7d. in the £ for income-tax ?

5 What amount of $2\frac{3}{4}$ per cent. consols will yield a net income of £152 17s 1d., when the income-tax is 8d in the £ ?

(b) Sum to be invested for a certain income.

Example 2 How much must I invest in the $3\frac{1}{2}$ per cents at 85 to have an annual income of £73. 10s ?

Sol   To get £ $3\frac{1}{2}$, I shall have to invest   £85 ,

..   „   „ £ 1   „   „   £85 × $\frac{2}{7}$

.   „   „ £ $73\frac{1}{2}$ „   „   £85 × $\frac{2}{7}$ × $14\frac{7}{1}$

=£1785   Ans

EXERCISE 161

1   How much must I invest in the 3 per cents. at $90\frac{1}{3}$ (brokerage as usual) to have an annual income of Rs 1465 ?

2   How much must be invested in $3\frac{3}{4}$ per cent. stock at $112\frac{1}{2}$ to get an income of Rs. 300 ?

3.   How much $3\frac{1}{2}$ per cent Government Securities at $95\frac{1}{4}$ must be sold out in order to purchase enough 5 per cent Municipal Debentures at $119\frac{3}{4}$ to produce an annual income of Rs 665 ?   (A brokerage of $\frac{1}{4}$ per cent. is charged on each transaction)

4   What sum of money invested in the 4 per cents at par would realise the same income on Rs 10200 invested in the $4\frac{1}{2}$ per cents at 102 ?

5   How much money must be invested in a 3 per cent stock at $92\frac{1}{2}$ to produce the same income at £1710 invested in a $3\frac{1}{2}$ per cent. stock at 95 ?

6.   A man invests one-third of his capital in the $3\frac{1}{2}$ per cent Government Securities at $96\frac{1}{2}$ and the remaining two-thirds in the $4\frac{1}{3}$ per cent. Debentures at $105\frac{1}{2}$. If the difference of the two annual incomes be Rs 1997, find his capital.

(c) Price of a stock to produce a certain income

Example 3.   Find the price of the 4 per cent. stock, so that an investment of Rs 4680 may produce an annual income of Rs. 180, brokerage being as usual

Sol   Income of Rs 180, is on Rs 4680,

„   Re 1 „ „ Rs $\frac{4680}{180}$

„   Re. 4 „ „ Rs. $\frac{4680}{180}$ × 4 = Rs 104

i. e , the holder of the stock altogether spent Rs. 104 for Rs 100 stock and out of this, he paid Re $\frac{1}{5}$ to the broker, hence the price of the stock is Rs $(104 - \frac{1}{5})$, *i e* Rs. $103\frac{4}{5}$. Ans.

EXERCISE 162

1   What is the price of 4 per cent. stock, when an investment of £17430 produces an income of £830 ?

2   A person invests Rs. 4095 in the 4 per cent. stock and obtains an income of Rs 173 5a 4p   What is the price of the stock ?

3   Find the price of $4\frac{1}{2}$ per cent. Government Promissory Notes when an investment of Rs 59422. 8a. produces a monthly income of Rs 213 12a

4.   A person sells £5000 of 3 per cent stock and buys $3\frac{1}{4}$ per cent. stock at $87\frac{1}{2}$ If the increase in his income be £5, what is the price of the 3 per cent stock ?

5   By selling £4500 of India 5 per cent. stock at $112\frac{1}{2}$ and investing the proceeds in China 7 per cent stock, the income is increased by £168 15s , what is the price of the latter stock ?

6.   Rs 210,000 invested in the 4 per cent stock produces annually Rs. 2166. 10a 8p more than if it had been invested in the $2\frac{3}{4}$ per cent stock at 99. Find the price of the 4 per cent. stock

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(d) Change in income

Example 4   A person invests Rs 44100 in the $3\frac{1}{2}$ per cent Government Securities at 98 and when they rise to $98\frac{1}{2}$ he sells out and invests the proceeds in the 5 per cent Municipal Debentures at $110\frac{13}{16}$ Find the change in his income.

Sol   Let us find what income he had at first

On Rs. 98 cash, gain = Rs $3\frac{1}{2}$

$\therefore$,, Re 1   ,, ,,   = Re $\frac{7}{2} \times \frac{1}{98}$

$\therefore$,, Rs 44100 ,, ,,   = Rs. $\frac{7}{2} \times \frac{1}{98} \times 44100$,
                                    = Rs. 1575

Then let us see how much money he got by the sale of his Securities

A bond bought at Rs. 98 was sold for Rs. $98\frac{1}{2}$,

-   $\therefore$,, ,,   ,, Re 1   ,,   Rs. $\frac{197}{2} \times \frac{1}{98}$

.   A bond bought at Rs. 44100 was sold for

$$\text{Rs. } \tfrac{197}{3} \times \tfrac{1}{98} \times 44100 = \text{Rs } 44325$$

Now this money was invested in the debentures

For every Rs. $110\tfrac{13}{18}$ invested he got an income $=$ Rs   5

.   „   „   Re 1   „   „ $=$ Rs $5 \times \tfrac{18}{1773}$

.   „   „   Rs 44325   „   „ $=$ Rs. $5 \times \tfrac{18}{1773} \times 44325$

$$= \text{Rs. } 2000$$

Hence his income increased by Rs 2000 $-$ Rs 1575,

i.e , Rs 425.   Ans

EXERCISE 163

1   A person transfers Rs 11000 from the 4 per cents at 92 to the 5 per cents at 110 , find the change in his income

2   A person invested £9075 in the 3 per cents at $90\tfrac{3}{4}$ and on the stock rising to 91 transferred it to the $3\tfrac{1}{2}$ per cents at $97\tfrac{1}{2}$ What increase does he produce in his yearly income ?

3   Find the change in income produced by shifting Rs. 11000 from the 3 per cents at $101\tfrac{3}{8}$ to $4\tfrac{1}{2}$ per cent stock at $137\tfrac{3}{8}$, the usual brokerage being charged on each transaction

4   Find the alteration in income caused by transferring £3200 stock from the 3 per cents at $86\tfrac{3}{8}$ to the 4 per cents at $11\tfrac{7}{8}$, the brokerage $\tfrac{1}{8}$ per cent being paid in each transaction

5   A person transfers Rs 1000 stock from 4 per cent stock at 90 to 3 per cent stock at $7\tfrac{1}{2}$ How much of the latter stock will he hold and what will be the difference in his income ?

6   If I hold £10000 of a 3 per cent. stock and sell out at $94\tfrac{1}{2}$, investing the money obtained in the 4 per cent. railway debentures at 105, what change would I make in my income ?

7   I invest Rs 12805 in the 4 per cents. at $98\tfrac{1}{4}$ and when they have risen to $102\tfrac{3}{4}$, I sell out and invest in the

$4\frac{1}{2}$ per cents. at $105\frac{3}{8}$, what is the change in my income ?
(Brokerage $\frac{1}{4}$ per cent on all transactions)

8. A person invests Rs. 19700 in the $3\frac{1}{2}$ per cent. stock at $98\frac{1}{3}$ and when they rise to $101\frac{1}{3}$, he sells out and invests the proceeds in the $4\frac{1}{2}$ per cent. consols at $114\frac{3}{16}$. Find the change in his income

9 The price of Govt. stock which pays $6\frac{1}{2}$ per cent is $185\frac{3}{4}$, Rs 6250 of this stock is transferred to a 3 per cent stock at $92\frac{7}{8}$, find the loss in annual interest

10 A person bought Rs. 14500, $3\frac{1}{2}$ per cent stock at $72\frac{1}{4}$ and when the price fell to 68 he sold it , with the proceeds of the sale he bought 4 per cent stock at $75\frac{5}{9}$ Find the change in his income.

11 By selling out £3000 in the 4 per cent at 90 and investing the proceeds in the India 5 per cent. stock at $106\frac{1}{10}$, what change in income has been effected ?

12. I invested Rs 49700 in shares paying Rs. 7 per share, when the Rs 100 share is at $122\frac{1}{2}$. If I had invested it in the $5\frac{1}{4}$ per cent bonds at 105, what increase in my annual income would have been obtained ?

13. A man invests Rs 42780 in the 5 per cents. at $114\frac{7}{8}$ but afterwards sells at $135\frac{1}{8}$ and invests the proceeds in the 4 per cents. at $92\frac{7}{8}$ What is the change in his income ? (Brokerage $\frac{1}{8}$ on each transaction)

14. A man buys £3430 stock in the $3\frac{1}{2}$ per cents. When they are at $85\frac{1}{3}$, he sells out and invests his capital in the 4 per cents at 98 Find the alteration in his income

15 A man has Rs 31000 in the 3 per cents at 92 but sells out and with the proceeds purchases 5 per cent. Railway stock at 115. Find the change in his income

16. A person invests £18150 in the 3 per cents at $90\frac{1}{3}$ and on the stock rising to $91\frac{1}{4}$, transfers it to $3\frac{1}{2}$ per cents. at $97\frac{1}{4}$ Find the alteration in his income (Brokerage $\frac{1}{4}$ per cent.)

17. A man transfers Rs 3600 from the 4 per cent. stock at $95\frac{3}{8}$ to the $3\frac{1}{2}$ per cents at $89\frac{7}{8}$ How much of the latter stock does he hold and what is the alteration in his income, allowing $\frac{1}{8}$ per cent brokerage in each case ?

18   A person invested £1000 in the 3 per cents at 90⅝ , but the price rising to 91¼, he sold out and invested the proceeds in the 3⅓ per cents at 97⅕.   Find the increase in his income

19.   A person invests Rs. 21390 in the 5 per cents at 115 , he afterwards sells at 135 and invests the proceeds in the 4 per cents at 93.   Find the change in his income

20.   A man has £6680 stock in a 4 per cent at 98 When the price has fallen 2 per cent he transfers his capital to 3½ per cent stock at 83½.   Find the change in his income.

21   A person sells out of the 3⅓ per cents at 92¾ and realises Rs 18550   If he invests ⅔ of the produce in the 4 per cents at 96 and the remainder in the 3 per cents at 90, find the alteration in his income

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§11.   Some Important Typical Examples

Example 1   What rate of interest will a person receive if he invests in 6⅓ per cents. at 129⅞ ?   (Brokerage ⅛ per cent)

Sol   Since 129⅞ + ⅛, ı e , 130 yields an interest of 6½

    130   100   6⅓   required rate

$$\text{required rate} = \frac{100 \times 13}{2 \times 130} = 5$$

ı e., the rate of interest is 5 per cent   Ans.

Example 2. Which stock is the better to invest, in the 4 per cents. at 102 or the 3½ per cents at 96 ?

Sol   (1)   Income on Rs 102 = Rs 4
    "   on Re.   1 = Re $\frac{4}{102}$
(2)   Income on Rs   96 = Rs $\frac{7}{2}$
    "   on Re.   1 = Re $\frac{7}{192}$

ın $\frac{4}{102}$, $\frac{7}{192}$ first fraction ı e , $\frac{4}{102}$ is greater

∴ the 4 per cents at 102 is the better investment Ans.

Example 3   What is the price of a 3 per cent stock, if after paying an income-tax of 5d in the £ a man derives an income of 3½ per cent on his money ?

Sol. The tax being $5d.$ in the $£$, a man's net income in a $£$ is $235d.$ *i.e.*, $£\frac{47}{48}$.

∴ $£\frac{47}{48} : \frac{7}{3}$ :: $£1$: gross income,

∴ gross income $= £\frac{7}{3} \times \frac{48}{47} = £\frac{168}{47}$.

∴ $£\frac{168}{47} : £3$ :: $£100$: price of stock ,

∴ required price $= £\dfrac{3 \times 100 \times 47}{168} = £\frac{1175}{14} = £83\frac{13}{14}$,

i.e., the 3 p. c stock is at $83\frac{13}{14}$ **Ans**

Example 4. A person invests £14970 in the purchase of the 3 per cents at 90 and the $3\frac{1}{4}$ per cents at 97. His total income being £500, how much of each stock did he buy ?

Sol. Suppose he invested $£x$ in the 3 per cents and

∴ $£(14970 - x)$ in the $3\frac{1}{4}$ per cents.

His income from the first $= £\ \dfrac{x}{30}$

and that from the second $= £\ \dfrac{13\,(14970 - x)}{388}$,

∴ by the question,

$$\frac{x}{30} + \frac{13(14970 - x)}{388} = 500 ,$$

which being solved gives $x = 9150$, *i e.*, he invested £9150 in the 3 per cents and   £5820 in the $3\frac{1}{2}$ per cents.

Now let us find the amounts of stock he purchased,

£90 : £9150 :: £100 : 3 p. c stock bought,

3 p. c stock bought $= £\frac{91500}{9} = \frac{30500}{3}$

$= £10166\frac{2}{3}$ **Ans.**

and £97 : 5820 :: £100 : $3\frac{1}{4}$ p. c stock bought,

∴ $3\frac{1}{4}$ p c. stock bought $= £\frac{582000}{97} = £6000$ **Ans.**

Note Questions of this type are easily solved, as shown above, by means of algebraical equations

Aliter. Suppose he invested £14970 in the purchase of 3 per cents. at 90

∴ income = £$\frac{3}{90}$ × 14970 = £499

i. e , a decrease of £500 − £499 = £1

Now suppose he invested £14970 in the purchase of 3¼ per cents. at 97

∴ income = £$\frac{3\frac{1}{4}}{97}$ × 14970 = £$\frac{97305}{194}$

i. e , an increase of £$\frac{97305}{194}$ − £500 = £$\frac{305}{194}$

he invested at 3 per cents and 3¼ per cents. in the ratio of $\frac{305}{194}$, 1 respectively i. e , 305　194

∴ investment at 3 per cents. at 90 = Rs. $\frac{305}{449}$ × 14970
　　　　　　　　　　　　　　　= Rs. 9150

and 　　„ 　　„ 3¼ per cents. at 97 = Rs. $\frac{194}{499}$ × 14970
　　　　　　　　　　　　　　　= Rs. 5820

Now proceed further.

Example 5. I invest Rs 163000 partly in a 4 per cent stock at 108 and partly in a 5 per cent. stock at 109½ so that my income is the same from each stock. How much do I invest in each stock ?

Sol. To obtain an income of Rs 5 from the first stock I must invest

Rs. 4 · Rs 5 . 108　required investment,

∴ investment = Rs $\frac{5 \times 108}{4}$ = Rs 135 , but I get Rs. 5

from the second stock by investing only Rs. 109¼.

Hence Rs 163000 must be divided in the ratio of 135 ꞉ 109½ i.e , 270 . 219 or 90　73,

∴ investment in the 1st stock = Rs. 163000 × $\frac{90}{163}$
　　　　　　　　　　　　　　= Rs 90000 ⎫
and ∴ 　　„ 　the 2nd stock = Rs　73000 ⎭　**Ans.**

EXERCISE 164.

1　A man invests £4031 10s in the 3 per cents at 94¾ , what will be his net income after an income-tax of 10d in the pound has been deducted, ⅞ p c. brokerage being allowed ?

2. A person investing in the 4 per cents. receives 5 per cent. on his money. What is the price of the stock?

3 Which is the better investment, the 3 per cents. at $83\frac{1}{2}$ or the $3\frac{1}{2}$ per cents. at 3 per cent discount?

4 Which is the better investment, bank stock paying 10 per cent at 319 or 3 per cent consols at 96? What will be the cost of £1500, 3 per cent consols at $89\frac{3}{8}$, brokerage being $\frac{1}{8}$ per cent? What rate of interest will such investment obtain?

5. A man holds $15\frac{1}{4}$ shares of a bank and receives £19 1s. 3d. per quarter If the interest he receives be 5 per cent per annum, find the value of a share.

6 Which is the better investment, $4\frac{3}{4}$ p c stock at 5 below par, or $5\frac{1}{4}$ p. c. stock at 5 premium?

7 What is the price of a 4 per cent. stock if it yields 5 p c. after paying an income-tax at 6d in the £?

8. A man invests a certain sum in $4\frac{1}{2}$ per cent. Government Paper at 104. The price falling to 101, he sells out and loses Rs 600 by the transaction, inclusive of brokerage Find the sum invested

9 A person invests Rs 44100 in the $3\frac{1}{2}$ per cent. Government Securities at 98 and when they rise to $98\frac{1}{4}$, he sells out and invests the proceeds in the 5 per cent Municipal Debentures His income thereby increases by Rs 425. What is the price of the Debentures?

10 A person invests Rs 19700 in the $3\frac{1}{2}$ per cent. Government Securities at $98\frac{1}{2}$ and when they rise to $101\frac{1}{2}$, he sells out and invests the proceeds in Municipal Deben tures at $114\frac{3}{16}$ His income thereby increases by Rs. 100. What is the rate of dividend of the Debentures?

11. Which is the better investment—the $3\frac{1}{4}$ per cent. stock at $95\frac{2}{8}$, or the 4 per cent. Debentures at $101\frac{1}{2}$? What will be the difference in the annual income by investing Rs. 22127 in each of them?

12. If by investing Rs 59422 8a in Government stock at $104\frac{1}{4}$, I derive a monthly income of Rs 213 12a, what is the rate of interest of the stock?

13　The difference between the incomes derived from investing a certain sum in 6 per cent stock at 126, and in 9 per cent stock at 210, is £22 10s　Find the amount invested

14.　One-third of a certain capital is invested in the 3½ per cent. Government Securities at 105, one-fourth in the 3 per cent Government Securities at 97½ and the remainder in the 4⅓ per cent Calcutta Municipal Debentures at 112¼　If the total income is Rs 830, what is his capital ?

15.　The difference between the income derived from investing a certain sum in 5 p c stock at 127 and in 5¾ per cent stock at 135 is £4. 14s　Find the amount invested and the income derived from each stock.

16　At what price must a person invest in the 4 per cent Government Promissory Note, so that after paying income-tax at 5 pies in the rupee, he may receive 4½ per cent on his money ?

17　A man invested Rs 5800 in the purchase of 5 per cent Debenture stock at par.　After he got the half yearly dividend he sold at 2½ per cent. premium and with the whole money purchased 4 per cent stock at Rs. 95 2a 6p.　What change was produced in his annual income ?

18.　A person had an annual income of Rs 480 from a 4 per cent stock　He sold at Rs 95. 14a and with the proceeds of the sale bought 5 per cent. Railway Debentures at Rs 119 9a.　Assuming that usual brokerage was charged on each transaction, find the change in his income.

19.　£ 3000, which I held in the 4 per cents was sold for me when they were at 82¾ by a broker, whose commission is ¼% and the proceeds were re-invested by him in the 4⅓ per cents. at 98¼.　What amount of the latter stock did he purchase ?

20　A man invests a sum of money in a stock at 91 and an equal sum in another stock at 117.　After some time, the price of the stocks have exchanged values, when he sells and makes a profit of Rs. 52.　What sum did he invest ?

21. A man invested the same sum in two different stocks $3\frac{1}{2}$ per cent. Govt. stock at $103\frac{1}{8}$ and 4 per cent. consols at 105 , his income from one was Rs. 93 more than from the other , what sum was invested in each stock ?

22. By selling out £3000 in the 4 per cents at 90 and investing the proceeds in the India 5 per cent. stock, a person finds his income increased by £6. 13s. 4d What is the price of the India stock ?

23 If I invest my money in shares paying Rs. 7 per share when the Rs 100 share is at $122\frac{1}{2}$, I find that I get Rs 355 a year more than if I invest it in the $5\frac{1}{4}$ per cent bonds at 105 Find my capital.

24 A man invests £1980 in the $3\frac{1}{8}$ per cents. at 99 and £3220 in the $4\frac{1}{2}$ per cents. at 105. What is the average rate of interest on his whole investment ?

25. What amount of stock must be purchased in English 5 per cents at $111\frac{1}{8}$ to produce the same yearly return as 3 lacs of rupees (a rupee $=1s\ 10\frac{1}{2}d$) invested in the Government $4\frac{1}{3}$ per cents. at $101\frac{1}{4}$?

26 A person finds that if he invests a certain sum in Railway shares paying £6 per share when the £100 share is at 132, he will obtain £10 16s. a year more for his money than if he invests in 3 per cent consols at 93 What sum has he to invest ?

27. By investing a certain sum of money in the $3\frac{1}{2}$ per cents at 72 a man gets Rs 35 less income than he would get by investing the same sum in the $4\frac{1}{2}$ per cents at 90 , find the sum invested.

28. A person invests a certain sum in the $3\frac{1}{2}$ per cent Government Securities when they are at $97\frac{3}{8}$, had he waited till they had fallen to $97\frac{1}{8}$, he would have had Rs. 400 more of Government Securities. How much money did he invest, $\frac{1}{8}$ per cent being charged as brokerage in both cases ?

29. What sum must a person invest in the 3 per cents. at 90, in order that by selling out Rs. 20000 stock, when they have risen to $93\frac{1}{2}$ and the remainder when they have fallen to $84\frac{1}{4}$, he may gain Rs. 125 by the transaction ?

STOCKS 519

If he invests the proceeds in the 4 per cents. at par, what will be the difference in his income ?

30   A person invests £6200 in the 3 per cents. at 89¼ and pays income-tax of 10d in the £, on the stock rising to 92, he sells out and invests the proceeds in £50 Railway shares at par which yield an annual dividend of 3½% clear of income-tax   Find the alteration in the income

31   I invest Rs 40000, partly in the 3 per cents. at 80 and partly in the 4 per cents at 96, and then I find that on the whole I received 4% interest on the sum invested.   What sums have I invested in the two stocks ?

32   A person invests £34539 in the 3 per cents at 87   After receiving one year's dividend he sells out at 89   He then invests the whole in Railway Stock paying 5 p c at 115.   What will be the difference in his income ?

33.   If 4% paper be at 110, what sum must I invest in order to secure a yearly income of Rs. 470, after paying an income-tax of 4p. in the rupee ?

34   A invests £3500 in buying equal amounts of 3 per cent stock at 78¼ and 6% stock at 109¾.   B invests the same sum, half in one stock and half in the other   Find (i) the difference in their incomes (ii) the ratio of their rates of interest

35.   Find what amount must be invested in the 2½ per cents at 95¾ to produce a clear income of Rs. 1000 a year, brokerage being ⅛% and income tax 4⅓ pies in the rupee.

36   A person bought consols at 94 and sold the same at 95⅝ thereby gaining Rs 550.   What cash did he receive for the stock, reckoning ⅜% brokerage on each transaction ?

37   A man invests £1668. 15s in the 3 per cents. at 89⅝   His first year's income he invests in the same stock but at a different price   His next year's income is £57. 10s. At what price did he invest in the second case ? (Brokerage ⅛ in both cases)

38   I buy £500 stock at 66 and afterwards £500 of the same stock at 69 , I sell out the whole when the price has risen to 89   What is the increase in my capital ?

39   A man possesses £22400 of 2¾ per cents con-sols   He sells them at 99⅜. He then invests one-half of the proceeds in 4 % Railway debenture stock at 132½, lends £7266 13s. 4d on mortgage at 4½ % and loses the rest   What alteration is produced in his annual income ?

40   A person bought Railway stock at 88 and after receiving the half year's dividend at 4½ % per annum, sold out at 92⅜, making a total profit of Rs 1987. 8a.   How much stock did he buy ?

41.   A invests a certain sum in the 3 per cents. at 80 and B invests half the sum in the 4 per cents   If A's income is to B's income as 8 is to 5, find the price of the 4 per cents.

42.   If money invested in the 3 per cent consols give exactly 3 per cent after the payment of income-tax at the rate of 1s per £ , find the price of the consols allowing ⅛ per cent to a broker for purchase

43   I invest ⅓rd of my property in Bank Stock, ⅛th in consols and the remainder in Railway Shares   I sell out when I might make a profit of 5 per cent , 3 per cent and 2 p c respectively on the investments and I altogether get £6190   What is the original amount of my property ?

44   A person after paying an income-tax of 7d. per £ has a clear income of £262. 2s. 6d. derived from stock in the 4½ per cents. , he sells out ⅔rd of this stock at 93¾ and invests the money in N W R. Stock at 112½, which pays 5¼ per cent per annum . what is his clear income now after paying income-tax as before ?

45.   A person invests Rs 10000 partly in the 4 per cents at 96 and partly in the 4½ per cents at 117   What amount does he invest in each stock, if he has the same income from each ?

46.   What is the price of the 4 per cent stock, if after paying an income-tax of 4 pies in the rupee, a man's in-come may be 1/31st of his capital.

47 A man had some 3 per cent stock. He sold the same at 90 and with the proceeds purchased 4 per cent. stock at 95 His income increased by Rs 243 How much 3 per cent stock did he possess ?

48 144 Shares in a company are worth £856 16s. when the dividend is at the rate of 7 per cent , how many shares ought to be worth £1193 8s when the dividend is at 9 per cent

49 A person has 3 per cent. stock which yields Rs 240 a year. He sells out $\frac{3}{4}$ of the stock at $87\frac{1}{4}$ and invests the proceeds in Railway Stock at $17\frac{1}{2}$. What dividend ought the latter to pay that he may thereby increase his income by Rs 40 ?

50 A person invests Rs. 36135 in the purchase of 4 per cent. stock at 63 and $3\frac{1}{2}$ per cents at 90 His total income is Rs 1428 8a How much money did he invest in each case ?

51 I invest £28787, partly in the purchase of 5 per cent stock at 115 and partly in 3 p c stock at 90 My total income is £938 10s , how much do I invest in each stock ?

52 A person has Rs. 24180 to invest in the $5\frac{1}{2}$ per cent Government loan at 108 and in the 6 per cent. Calcutta Municipal loan at 102 How must he divide his capital between them so as to obtain the same income from each ?

53. One year I receive a dividend of 6 per cent on my stock and pay an income-tax of Rs 1. 10a. 8p. per Rs 100 The next year I receive a dividend of $6\frac{1}{2}$ per cent and pay an income-tax at Re 1 4a per Rs 100 and find that my net income has increased by Rs 2490 How much stock do I hold ?

54 A man has £30043. 16s. $4\frac{1}{2}d$. Part of his money is invested by him in the purchase of 3 per cent. stock at 90 and the remainder in the 4 per cents. at $104\frac{3}{4}$ His total income is £1021 8s How much does he invest in the 3 per cent stock

55. If the 3 per cent consols are at $81\frac{7}{8}$, what must be the price of the 5 per cents that there may be no loss of income in selling out the former and investing the proceeds in the latter, allowing the usual brokerage on each transaction ?

CHAPTER XXXI

APPLICATION OF ALGEBRA AND GEOMETRY TO ARITHMETIC

1. Application of Algebra.

§1 Formulae

Some examples in Arithmetic can be easily solved by the application of Algebraical formulæ. The following examples may be noted carefully :—

Example 1   Simplify ˙121×˙997+˙121×˙003.

Sol.   Apply formula $ax \div bx = x(a+b)$.

The expression $= ˙121 (˙997 + ˙003)$
$$= 121 \times 1 = 121. \text{ Ans.}$$

Example 2   Simplify

˙526×˙526+2×˙526×˙474+˙474×˙474.

Sol.   Apply formula $a^2 + b^2 + 2ab = (a+b)^2$

The expression $= (˙526)^2 + (˙474)^2 + 2(˙526 \times ˙474)$
$$= (˙526 + ˙474)^2 = (1)^2 = 1 \quad \text{Ans.}$$

Example 3.   Simplify

˙529×˙529+˙379×˙379−˙758×˙529

Sol.   Apply formula $a^2 + b^2 - 2ab = (a-b)^2$

The expression $= (˙529)^2 + (˙379)^2 - 2(˙379 \times ˙529)$
$$= (˙529 - ˙379)^2 = (˙15)^2$$
$$= ˙0225. \text{ Ans.}$$

Example 4.   Simplify $\dfrac{(˙821 \times ˙821) - (˙179 \times ˙179)}{˙821 - ˙179}$

Sol.   Apply formula $a^2 - b^2 = (a+b)(a-b)$.

The expression $= \dfrac{(˙821 + ˙179)(˙821 - ˙179)}{˙821 - ˙179}$
$$= ˙821 + ˙179 = 1. \quad \text{Ans.}$$

Example 5 Simplify $\dfrac{85\frac{1}{4}\times 85\frac{1}{4}-34\frac{5}{6}\times 34\frac{5}{6}}{120\frac{1}{12}}$

Sol   The expression $=\dfrac{(85\frac{1}{4})^2-(34\frac{5}{6})^2}{120\frac{1}{12}}$

$\qquad\qquad =\dfrac{(85\frac{1}{4}+34\frac{5}{6})(85\frac{1}{4}-34\frac{5}{6})}{120\frac{1}{12}}$

$\qquad\qquad =\dfrac{120\frac{1}{12}\times 50\frac{5}{12}}{120\frac{1}{12}}=50\frac{5}{12}.$ **Ans.**

Example 6   Simplify $\dfrac{\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}+\frac{3}{80}+\frac{3}{100}}{\frac{1}{16}+\frac{1}{10}+\frac{1}{25}}$

Sol.   Apply formula $(a+b)^3=a^3+b^3+3a^2b+3b^2a.$

The expression $=\dfrac{(\frac{1}{4}+\frac{1}{5})^3}{(\frac{1}{4}+\frac{1}{5})^2}$

$\qquad\qquad =\frac{1}{4}+\frac{1}{5}=\frac{9}{20}$ **Ans.**

Example 7   Simplify

$\dfrac{121\times 121\times 121-\cdot 021\times\cdot 021\times\cdot 021-3\times\cdot 121\times\cdot 121\times\cdot 021+3\times\cdot 021\times\cdot 021\times\cdot 121}{\cdot 121\times\cdot 121-2\times\cdot 121\times\cdot 021+\cdot 021\times\cdot 021}.$

Sol   Apply formula $(a-b)^3=a^3-b^3-3a^2b+3b^2a.$

The expression $=\dfrac{(\cdot 121-\cdot 021)^3}{(\cdot 121-\cdot 021)^2}$

$\qquad\qquad =\cdot 121-\cdot 021=\cdot 1$ **Ans.**

Example 8. Simplify $\dfrac{576\times 576\times 576+\cdot 424\times\cdot 424\times\cdot 424}{576\times 576-576\times\cdot 424+\cdot 424\times\cdot 424}$

Sol   Apply formula $a^3+b^3=(a+b)(a^2-ab+b^2)$

The expression $=\dfrac{(576)^3+(\cdot 424)^3}{(576)^2-576\times\cdot 424+(\cdot 424)^2}$

$\qquad\qquad =576+\cdot 424=1000.$ **Ans.**

Example 9   Simplify $\dfrac{\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}-\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}}{\frac{9}{16}+\frac{3}{16}+\frac{1}{16}}.$

Sol.   Apply formula $a^3-b^3=(a-b)(a^2+ab+b^2).$

The expression $=\dfrac{(\frac{3}{4})^3-(\frac{1}{4})^3}{(\frac{3}{4})^2+\frac{3}{4}\times\frac{1}{4}-(\frac{1}{4})^3}$

$\qquad\qquad =\frac{3}{4}-\frac{1}{4}=\frac{1}{2}.$ **Ans.**

EXERCISE 165.

Simplify .—

1 $5672 \times 937 + 5672 \times 1063.$

2. $1526 \times {}^{\cdot}729 + 1^{\cdot}526 \times 271$

3 $5625 \times 15\,221 + 5625 \times 779$

4. $521 \times 521 + 379 \times 379 + 379 \times 1042.$

5. $561 \times 561 + 439 \times 439 + 2 \times 561 \times 439.$

6 $625 \times 625 + 375 \times 375 - 2 \times 625 \times {}^{\cdot}375.$

7. $5\frac{1}{8} \times 5\frac{1}{8} + 4\frac{6}{8} \times 4\frac{6}{8} + 2 \times 4\frac{6}{8} \times 5\frac{1}{8}$

8. $15\frac{7}{12} \times 15\frac{7}{12} + 11\frac{5}{12} \times 11\frac{5}{12} - 2 \times 15\frac{7}{12} \times 11\frac{5}{12}.$

9. $\dfrac{({}^{\cdot}375)^2 + 2({}^{\cdot}375 \times {}^{\cdot}125) + ({}^{\cdot}125)^2}{{}^{\cdot}375 + {}^{\cdot}125}.$

10. $\dfrac{{}^{\cdot}675 \times 675 - 325 \times 325}{{}^{\cdot}675 - {}^{\cdot}325}$

11. $\dfrac{{}^{\cdot}729 \times 729 - {}^{\cdot}529 \times {}^{\cdot}529}{729 + {}^{\cdot}529}.$

12 $\dfrac{35\frac{2}{3} \times 35\frac{2}{3} - 21\frac{5}{6} \times 21\frac{5}{6}}{57\frac{1}{2}}.$ **13** $\dfrac{58\frac{3}{4} \times 58\frac{3}{4} - 41\frac{1}{4} \times 41\frac{1}{4}}{17\frac{1}{2}}.$

14. $\dfrac{({}^{\cdot}79)^3 + ({}^{\cdot}21)^3 + 3({}^{\cdot}79)^2({}^{\cdot}21) + 3({}^{\cdot}21)^2({}^{\cdot}79)}{79 \times {}^{\cdot}79 + 21 \times 21 + 2 \times {}^{\cdot}79 \times 21}.$

15 $\dfrac{({}^{\cdot}179)^3 - 3({}^{\cdot}179)^2({}^{\cdot}079) + 3({}^{\cdot}079)^2({}^{\cdot}179) - ({}^{\cdot}079)^3}{({}^{\cdot}179)^3 - 358 \times {}^{\cdot}079 + ({}^{\cdot}079)^2}.$

16. $\dfrac{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} - \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} - \frac{1}{4} + \frac{1}{6}}{\frac{1}{2} \times \frac{1}{2} - \frac{1}{4} + \frac{1}{3} \times \frac{1}{3}}.$

17. $\dfrac{\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} + \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} + 3 \times \frac{1}{25} \times \frac{1}{6} + 3 \times \frac{1}{36} \times \frac{1}{5}}{\frac{1}{5} \times \frac{1}{5} + \frac{1}{15} + \frac{1}{6} \times \frac{1}{6}}.$

18. $\dfrac{526 \times 526 \times {}^{\cdot}526 + {}^{\cdot}474 \times {}^{\cdot}474 \times 474}{526 \times 526 - 526 \times 474 + 474 \times 474}.$

19. $\dfrac{556 \times 556 \times {}^{\cdot}556 - {}^{\cdot}496 \times {}^{\cdot}496 \times 496}{556 \times 556 + {}^{\cdot}556 \times 496 + {}^{\cdot}496 \times {}^{\cdot}496}.$

20. $\dfrac{({}^{\cdot}0347)^3 + ({}^{\cdot}9653)^3}{({}^{\cdot}0347)^2 - ({}^{\cdot}347)({}^{\cdot}09653) + ({}^{\cdot}9653)^2}.$

§2   Equations

Example 1   What is the number from which if you take away 21 the remainder is $\frac{3}{4}$ of the original number ?

   Sol   Since the number $-21 = \frac{3}{4}$ of the number

   $\therefore$ the number $-\frac{3}{4}$ of the No $=21$ [by transposition]

   $\therefore$ $\frac{1}{4}$ of the No $=21$

   $\therefore$ the number $=84$ **Ans**

Example 2   A boy loses $\frac{1}{5}$ of his money and then gains $8p$, he then loses $\frac{1}{4}$ of what he has and then gains $6p$, he afterwards loses $\frac{1}{3}$ of what he has and then finds that he has $6a$ $8p$. left, how much had he at first ?

   Sol $\frac{1}{5}$ of my money is lost $\therefore$ $\frac{4}{5}$ of it remains, $8p$. is then gained.

   . money now remaining $=\frac{4}{5}$ of original money $+8p$, of this $\frac{1}{4}$ is lost.

   . $\frac{3}{4}$ of ($\frac{4}{5}$ of original money $+8p$) remains,

   $6p$ is then gained

   $\therefore$ money now remaining $=\frac{3}{4}$ of ($\frac{4}{5}$ of original money $+8p.)+6p$, of this amount $\frac{1}{3}$ is lost

   $\therefore \frac{2}{3}$ of $[\frac{3}{4}$ of ($\frac{4}{5}$ of original money $+8p.)+6p.]$ remains

   $= \frac{2}{3}$ of ($\frac{3}{4}$ of original money $+6p+6b$.)

   or $\frac{2}{5}$ of original money $+$ $8p =6a$ $8p$)

   or $\frac{2}{5}$ of original money $=6a$

      original money $=6a. \times \frac{5}{2}$ $=15a$ **Ans.**

Example 3   The price of 3 cows and 5 oxen is Rs. 295 and that of 4 cows and 6 oxen is Rs. 370, find the price of a cow and an ox.

   Sol   Price of 3 cows and 5 oxen $=$ Rs. 295   (1)

   and price of 4 cows and 6 oxen $=$ Rs 370   .. (2)

Multiplying (1) by 4 and (2) by 3 we have,

   price of 12 cows and 20 oxen $=$ Rs 1180   ... (3)

   price of 12 cows and 18 oxen $=$ Rs 1110   .   . (4)

Subtracting (4) from (3), we have

price of 2 oxen $=$ Rs. 70

$\therefore$ price of 1 ox $=$ Rs. 35

Subtracting the price of 5 oxen from Rs. 295, we have the price of 3 cows $=$ Rs 120,

$\therefore$ price of a cow $=$ Rs 40

$\therefore$ cow costs Rs 40 and ox Rs. 35.   **Ans.**

Example 4.   5 men and 6 boys can do $\frac{5}{8}$th of a work in 2 days , 2 men and 9 boy can do $\frac{7}{10}$th of the work in 2 days , in what time can a boy do the whole work ?

Sol. In 2 days, 5 men and 6 boys can do $\frac{5}{8}$th of a work

$\therefore$ „ 1 day,   5 men and 6 boys can do $\frac{5}{16}$th „   „

$\therefore$ „ 1 day, 10 men and 12 boys can do $\frac{5}{8}$th „ ,,   (1)

Again, in 2 days 2 men and 9 boys can do $\frac{7}{10}$th of a work

$\therefore$ „ 1 day,   2 men and 9 boys can do $\frac{7}{20}$th „   „

$\therefore$ „ 1 day,   10 men and 45 boys can do $\frac{7}{4}$ „   „   (2)

$\therefore$ „ 1 day,   33 boys can do $\left(\frac{7}{4}-\frac{5}{8}\right)$ of the work
$\qquad\qquad\qquad$ [Subtracting (1) from (2)

i e., 33 boys can do $\frac{11}{8}$th of the work in one day.

$\therefore$ 1 boy can do $\frac{11}{8} \times \frac{1}{33}$ or $\frac{1}{8}$th of the work in one day

$\therefore$ 1 boy can do the whole work in 36 days.   **Ans.**

EXERCISE 166

1.   What is the number from which if you take away 17, the remaining is $\frac{5}{8}$ of the original number ?

2   What is the number in which if you add 20 the sum is $\frac{6}{5}$ of the original number ?

3.   A boy loses $\frac{1}{3}$ of his money and then gains 4p, he then loses $\frac{1}{4}$ of what he has and then gains 5p., he afterwards loses $\frac{2}{3}$ of what he has and then finds that he has 3a. 2p left   How much had he at first ?

4.   If 5 sheep and 9 goats cost Rs 107 and 9 sheep and 5 goats cost Rs 103, how much will one sheep and one goat cost ?

5. If 3 horses and 5 cows cost Rs. 345 and 5 horses and 3 cows cost Rs. 335, find the price of a horse

6 The cost of 9 chairs and 10 tables is Rs 163, and that of 6 chairs and 15 tables Rs 192, find the price of a chair and a table.

7 2 men and 4 boys can do $\frac{7}{15}$th of a piece of work in 4 days, and 3 men and 5 boys can do $\frac{18}{24}$th of it in 5 days. In what time can a boy do the whole work?

8. Two boys and one man can do a piece of work in 4 days, and one boy and two men can do it in 3 days In what time can a boy do the whole work?

9 5 men and 2 boys can do a piece of work in 4 days, 2 men and 4 boys can do $\frac{9}{10}$th of it in 5 days In what time can a man do the work?

10 If 3 men with 4 boys earn Rs 51 in 6 days, and, 4 men with 5 boys earn Rs 55 in 5 days, in what time will 6 men and 9 boys earn Rs 144?

11 If 5 men and 6 women do a piece of work in 10 days, which 3 men and 10 children can do in 12 days, find how long 19 men, 30 children and 12 women working together will take to do it

12 12 men and 15 women can finish a certain piece of work in 20 days, if 10 more women are put on, the work is then finished 5 days earlier. In what time can a man and a woman together finish the work?

13 If 2 boys and 1 man can do a piece of work in 4 hours and 2 men and 1 boy can do the same in 3 hours, find in what time a man, a boy, and a man and a boy together, respectively can do the same

---

§3. Surds

If we are asked to find the value of expressions which involve surds in their denominators, e g,

$$\frac{1}{\sqrt{2}-1}, \quad \frac{2}{\sqrt{3}-\sqrt{2}}; \quad \frac{\sqrt{2}}{\sqrt{3}+1}, \quad \frac{2+\sqrt{3}}{\sqrt{3}-\sqrt{2}}, \text{ etc. etc.}$$

we should remember the following algebraical facts —

(i) $\sqrt{a} \times \sqrt{a} = a$

(ii) $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$.

(iii) $\sqrt{a^2 \times b^2} = ab$.

(iv) $\sqrt{a^2 \times b} = a\sqrt{b}$

(v) $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$.

(vi) $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$

(vii) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$.

(viii) $(a + b)(a - b) = a^2 - b^2$.

Example 1. Find the value of $\dfrac{1}{\sqrt{2} - 1}$

Sol. $\dfrac{1}{\sqrt{2} - 1} = \dfrac{\sqrt{2} + 1}{(\sqrt{2} - 1)(\sqrt{2} + 1)} = \dfrac{\sqrt{2} + 1}{2 - 1}$
$$= \sqrt{2} + 1$$

If we now substitute the approximate value of $\sqrt{2}$ we get the required result to be 2 414.   **Ans.**

Note   If we had substituted the value of $\sqrt{2}$ in the given expression, we would have to divide the number 1 by a decimal fraction, a process which is more tedious than what is shown above

Example 2. Find the value of $\dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$.

Sol.   The given expression $= \dfrac{(\sqrt{3} + \sqrt{2})(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})}$

$$= \dfrac{3 + 2 + 2\sqrt{6}}{3 - 2} = 5 + 2\sqrt{6}$$

in which the approximate value of $\sqrt{6}$ being substituted we will get the required result to be 9 898.   **Ans**

Note   In the following examples, the student will have to use the values of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, etc, etc , a record (1 414, 1 732, 2 236, 2 449 respectively) of which will, therefore, be very useful and convenient

EXERCISE 167

Find the value of the following to 3 places of decimals :—

1. $\dfrac{1}{\sqrt{5}+1}$.　　2 $\dfrac{1}{\sqrt{3}-\sqrt{2}}$.　　3. $\dfrac{2}{\sqrt{5}-\sqrt{3}}$

4 $\dfrac{4\sqrt{2}}{\sqrt{3}-\sqrt{2}}$　　5. $\dfrac{2+\sqrt{3}}{\sqrt{3}+1}$.　　6. $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$.

7. $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-1}$.　　8 $\dfrac{\sqrt{3}+\sqrt{7}}{\sqrt{11}-\sqrt{5}}$.

9 $(\sqrt{6}+\sqrt{3}+\sqrt{2}+2)\,(\sqrt{6}-\sqrt{3}+\sqrt{2}-2)$.

10 $\dfrac{\sqrt{(\cdot 05 - \cdot 005)}}{\sqrt{(\cdot 05)} - \cdot 005}$

11. $\dfrac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \dfrac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \dfrac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$

Which is the greater quantity :—

12. $\sqrt{2}$ or $\sqrt[3]{3}$?　　　　13. $\sqrt{3}$ or $\sqrt[3]{15}$?

14 Find the value of $\dfrac{15+\sqrt{(\cdot 009)}}{1-\sqrt{(\cdot 9)}}$ correct to 3 places of decimals

15. Simplify $\left(\dfrac{\sqrt{11}+\sqrt{7}}{\sqrt{11}-\sqrt{7}}\right)^{2} - \left(\dfrac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}\right)^{2}$

II   Application of Geometry.

§4.   We know that the square *on* the hypotenuse of a right-angled triangle is equal to the sum of the squares *on* the other two sides   Hence the square *of* the measure of the side opposite to the right angle is equal to the sum of the squares *of* the measures of the sides containing the right angle.   Therefore we have the following

Rules   (ı) $(Hypo.)^2 = (Perp.)^2 + (Base)^2$
　　　(ıı) $(Perp.)^2 = (Hypo.)^2 - (Base)^2$
and (ııı) $(Base)^2 = (Hypo.)^2 - (Perp.)^2$, ı e

if any two sides of a right-angled triangle be given, we can find the third side.

F 34.

Example 1 The sides of a right-angled triangle are 16 feet and 63 feet , find its hypotenuse.

Sol. ∵ $16^2 + 63^2 = 256 + 3969$
$$= 4225$$
and $\sqrt{(4225)} = 65$

∴ the hypotenuse is 65 feet **Ans.**

Example 2 The diagonal of a rectangular court-yard is 355 yds , its length is 284 yds , find its breadth

Sol ∵ $(Base)^2 = (Hypo)^2 - (Perp)^2$
∴ $(Breadth)^2 = 355^2 - 284^2$
$$= (355 + 284)(355 - 284)$$
$$= 639 \times 71 = 71 \times 9 \times 71$$

∴ the reqd breadth is 71×3 or 213 yards **Ans.**

EXERCISE 168.

1 The town A is 72 miles west of B and 135 miles south of C. What is the distance between B and C ?

2 A ladder 25 ft long has its foot placed in a street and its top resting against a wall on one side of the street at a height of 15 ft. from the ground If the ladder be turned over to the other side, its top reaches to a point 20 ft high on the opposite wall. Find the breadth of the street.

3. "The tip of a bud of lotus was seen a span above the surface of water in a lake Driven by wind, it gradually advanced and sank at a distance of 2 cubits. Tell me quickly, O mathematician, the depth of the water " *Lilavati*

4 The tip of a bud of lotus was seen 16 inches above the surface of water in a lake Driven by wind it gradually advanced and sank at a distance of 36 inches. Find the depth of the water.

CHAPTER XXXII
GRAPHS

§1   In all mathematical operations we find two different kinds of quantities—*constants* and *variables.* Quantities which have always the same value are called constants, *e g*, a rupee is a constant quantity since its value never changes, it is always equal to 64 pice. Quantities which have different values at different places or times or whose values depend on certain circumstances are known as variables, *e g*, the temperature of a room, the speed of a train, the price of a certain article are variables as they change from time to time.

Again, variables are of two kinds, **dependent** and **independent**, *e g*, the earnings of a labourer depends upon the number of days he works   Here the number of working days as well as the earning of the man are both variables, but since the latter depends upon the former, the latter is called the dependent and the former, the independent variable   Similarly the distance travelled by a man is dependent variable and the time required to cover the distance is the independent variable

§2   If a train runs uniformly at the rate of 30 miles an hour, then we can easily find the distance it travels in a certain time, *i e*, there exists a certain relation between the two variables, namely the time and the distance travelled. This relation can be shown clearly by means of a diagram called a **graph**

§3   Let the horizontal line XOX' be cut at right angles by the vertical line YOY' at the point O, thus dividing the place into four spaces called *quadrants viz*, $XOY, YOX', X'OY'$ and $Y'OX.$

Take any point P in the first quadrant XOY and draw PM and PN perpendiculars on OY and OX respectively.

531
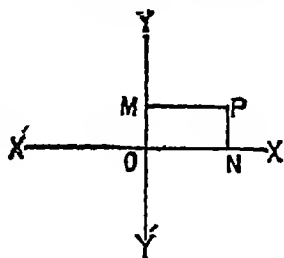
Let PM or ON$=x$,
and PN or OM$=y$,
x and y are called co-ordinates of P, the lines XX'and YY' are called the axes of co ordinates or more briefly the axes and are respectively called the axes of X and Y The point O is called the origin, x is called the abscissa and y the ordinate of P A point is denoted by the co-ordinates, the abscissa being named first. Thus the point P is called the point (x, y)

The values of x are measured from O along the axis of X, the values being positive when taken to the right of O along OX and negative when drawn to the left of O along OX'. The values of y are positive when drawn above XX' and negative when drawn below XX'. Thus the point $(4, 6)$ is plotted in the following way —

Measure ON$=4$ units along the axis of X to the right of O and then by drawing a perpendicular at N above OX and measuring PN$=6$ units of length, we get the point P, which is called the point $(4, 6)$
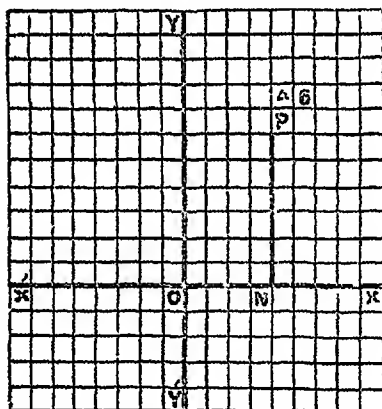
In the above figure the horizontal and vertical lines are drawn $\frac{1}{10}$th of an inch from one another. Such a paper is called a squared paper and questions on graphical method' are very conveniently done on this paper.

[For further description on the subject *see* our "Algebra Made Simple."]

§4. The graphical method of solving examples will be clearly understood from the following examples —

Example 1. A man walks westward 8 miles and then turns south and walks 6 miles, how far is he from the starting point ?

Sol Let O be the starting point Measure ON=8 units of length along OX′. Draw NP perpendicular and below the line XX′ and cut off NP=6 units of length Then evidently the man came to P. To measure OP, draw a circle from O as centre and OP as radius cutting OX′ at Q. Then OP=OQ and OQ=10 units, ı e, he is 10 miles away from the starting place.

Observation In this example, a mile has been denoted by $\frac{1}{10}$th of an inch which is the unit of measurement.
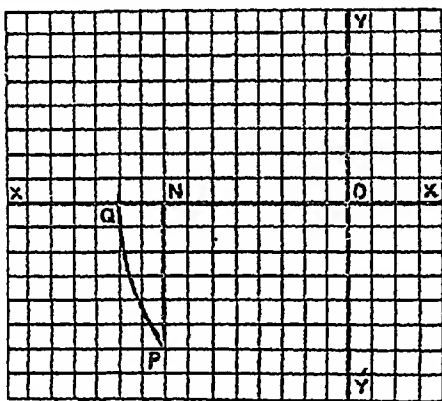
Example 2 Make a graph for converting inches to centimetres and *vice versa*

Sol. 1 cm $=\frac{2}{5}$ in approx , ∴5 cm $=2$ in. approx

Draw OX and OY at right angles and mark inches along OX and centimetres along OY.

To draw the graph, find the point A where the lines from 5 cm and 2 in. meet, and since 5 cm.=2 in.

.. the graph will pass through A But to keep the line straight, take another point before drawing the line, say

B where the 25 cm ãnd 10 in. lines meet   Then draw the graph OM

Explanation. How to find the equivalent of 12 cm. Dot the line from the number 12 on OY till it meets the graph at x, from x drop a perpendicular on OX and read as accurately as possible the number of inches along OX, $4\frac{1}{2}$ inches nearly,   12 cm $= 4\,8$ in approximately.

Again, to find the equivalent of $16\frac{1}{2}$ in in centimetres.

Dot the line from $16\frac{1}{2}$ on OX to meet the graph at y. From y dot the line to OY and see where it falls between 41 and 42 , it should be $41\frac{1}{4}$ if the drawing is very accurate , $\therefore 16\frac{1}{2}$ in.$=41^{\cdot}25$ cm.
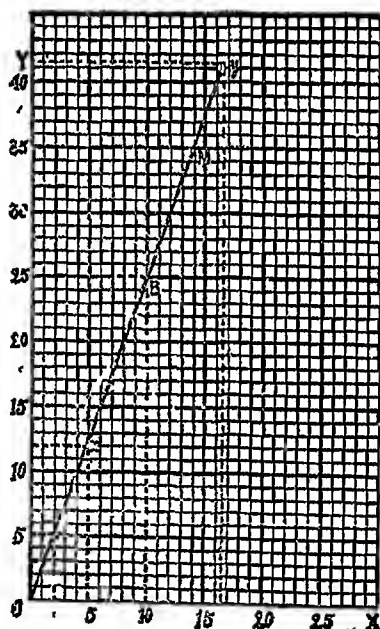
Thus a graph of this kind is very useful as it enables us to read off at once the equivalent of one table in terms of the other without calculation

The student should practicse several graphs of this kind, the chief point to be observed being the choice of a good scale when drawing the graph , in the above example one division of the squared paper denotes 1 inch or one centimetre, but this may not always be convenient, since the figure may be very close thereby making it difficult to read from the graph

Example 3. In a Reaumur Thermometer, the freezing point is $0°$ and the boiling point is $80°$, in a Centigrade thermometer the freezing yoint is $0°$ and the boiling point is $100°$. Draw a chart to convert Centigrade degrees into Reaumur degrees and *vice versa* and read off $15°$ C in R.
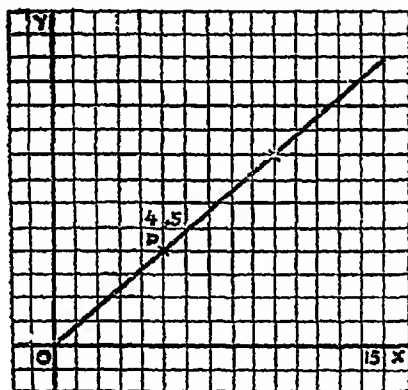
Sol　If $x°$ in Centigrade is equivalent to $y°$ in Reaumur, then $\dfrac{x}{100} = \dfrac{y}{80}$.

$y = \frac{4}{5}x$, therefore if $x = 0$, then $y = 0$, therefore the graph is a st line passing through the origin O.

Measure the centigrade degrees along the axis of x and the Reaumur degrees along the axis of y.

If $x = 5$, then $y = 4$, ∴ the point P $(5, 4)$ is also on the graph.

Join OP and produce it　This is the required graph.

∴ corresponding to the abscissa 15, we have the ordinate 12,　$15°$ C $= 12°$ R.

Example 4. *A* starts walking at the rate of 3 miles per hour, 30 minutes after, *B* starts from the same place at the rate of 4 miles per hour. Find *graphically* when and where *B* overtakes *A*
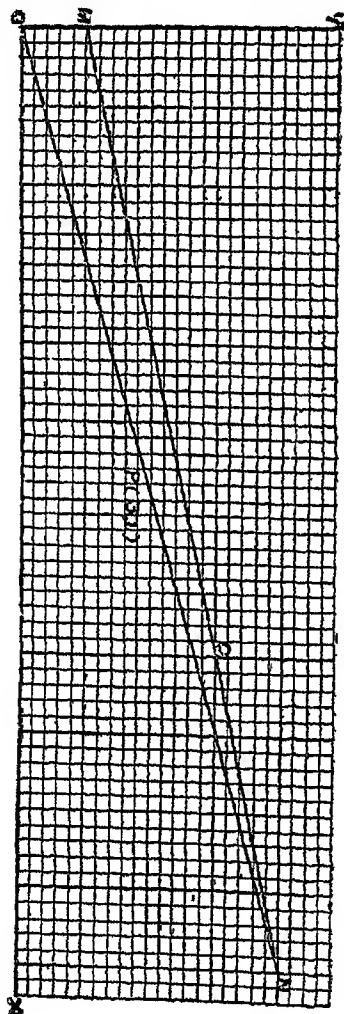
[Read this graph lengthwise]

Sol Measure distance along OX to the scale of 10 divisions to 1 mile and time along OY to the scale of 10 divisions to 1 hour.

     in one hour *A* goes 3 miles.

     plot point P whose co-ordinates are (3, 1),

     . OP produced represents the graph of *A*'s motion

∴ *B* starts 30 minutes after   M denotes the starting point of *B*. in one hour *B* goes 4 miles, plot point Q and hence, as before, MQ represents the graph of *B*'s motion

Then the intersection of MQ and OP *viz*, N will give the place and time when *B* overtakes *A* From the figure, it is clear that *B* overtakes *A* two hours after *A*'s starting and at a distance of 6 miles from the place where *A* started.

EXERCISE 169

1 The reading of a Centigrade thermometer in degrees and the corresponding readings on a Fahrenheit thermometer in degrees are given in the table below —

C	5	10	15	20	30	50	80
F	41	50	59	68	86	122	176

Illustrate *graphically* the connection between the two scales. Express 104° Fahrenheit in Centigrade.

2 If a man walks 8 miles east and then 5 miles north, how far is he from the starting point?

3 Represent *graphically* the increase of velocity of a train starting from rest as shown in the table below —

Distance travelled.	50	200	800	1800	3200	5000	feet.
Velocity	5	10	20	30	40	50	miles per hour

4. Given that 25 francs = £1, find from a *graph* the value in francs, as nearly as you can, of 8*s*, £1. 5*s* and 15*s* and find the English equivalent of 10 fr, 28 fr and 45 fr.

5. A man bicycles from A to B at 10 miles an hour and returns from B to A at 15 miles an hour. If he takes 5 hours to go there and back, find the distance from A to B. Find also his average speed per hour.

6 A starts from Ambala to walk to Ludhiana, a distance of 68 miles at 3 miles an hour, two hours later. B starts from Ludhiana for Ambala at 5 miles per hour When will A and B meet ? When will they be 20 miles apart.

7. Illustrate *graphically* the following variations in Indian Revenues in the years 1909 to 1914 from the following data

Years	1909	1910	1911	1912	1913	1914
Revenue in crores of Rs	225	229	227	321	307	351

8. A monkey climbing up a greased pole, ascends 5 ft and slips down 2 ft in alternate second, until he reaches the top of the pole. If the pole be 17 ft high how long will it take him to reach the top ?

9 Two taps A and B will fill a cistern in 10 and 20 hours respectively. Find *graphically* in what time they will fill it together.

10. A stone falling from rest goes through the following distance in the time shown in the table below—

No. of seconds	1	2	3	4	5	6
Distance in ft	16	64	144	256	400	576

Draw a graph to illustrate the above relation and read the graph to find the distance fallen in 3'8 seconds.

CHAPTER XXXIII.

CALENDAR

§1. The earth rotates round the sun in 365 days 5 hours 48 minutes and 48 seconds, but an ordinary year is taken as having 365 days, *i e*, nearly ¼th day too short. 5 hours 48 minutes and 48 seconds in 4 years' duration become 23 hours 15 minutes and 12 seconds, so an extra day is added once in every four years and that year is called a leap year. Again there is a difference of 44 minutes and 48 seconds *i. e*, the leap year is taken 44 minutes 48 seconds too long To rectify this error a century is considered an ordinary year. Thus we find that some hours are still left to be taken into consideration, so to make up this difference the year of the century divisible by 400 is considered a leap year

Briefly 1 An ordinary year is of 365 days' duration

2 Every fourth year is a leap year, *i e*, of 366 days

3 Year of a century is an ordinary year, but year of a century divisible by 400 is a leap year

OR, 1. In 100 years there are 24 leap years
2. In 200 years there are 48 leap years.
3. In 300 years there are 72 leap years.
4 In 400 years there are 97 leap years

§2 To find the day of the year

In an ordinary year there are 365 days, *i e.*, 52 weeks and one day As the same day is repeated after a week, so there will be a difference of only one day in one year, that is to say, if January 1, 1929 was Tuesday, January 1, 1930 will be Wednesday Similarly in 100 years there will be a difference of 100 + 24 days because there are 24 leap years in 100 years, 124 days make 17 weeks and 5 days, so in one hundred years there will be a difference of 5 days only If January 1, 1929 was Tuesday, January 1, 2029 will be Sunday

Now remember —
1. In 100 yrs. there are 5 odd days
2. In 200 yrs. there are 3 odd days   (cancelling 1 week).
3 In 300 yrs. there is 1 odd day   („   2 weeks).
4. In 400 yrs there are $5 \times 4 + 1$ or no odd day (cancelling 3 weeks).

From this we conclude that there is no odd day in 800, 1200, 1600, 2000 years and so on

Note   January 1 A D was Monday   If there is no odd day in a given period, then the day of the week is Sunday   For one odd day Monday, for 2 odd days Tuesday, and so on   Also the year in question should not be considered, $e\ g$, 3rd February 1918 means 1917 complete years and 1 month and 3 days

Example. What day of the week was 18th Feb 1925.
Sol   1600 years give no odd day
           300 years give 1 odd day
24 years give $(24 + 6)$ or   2 odd days (cancelling 4 wks).
January 1925 gives 31 or   3 odd days (cancelling 4 wks).
18 days of Feb give   4 odd days.
Total $=$ 10 days or 3 odd days (cancelling 1 wk).
          the day was Wednesday. Ans

EXERCISE 170.

What day of the week was $(1—6)$ —
1. February 5, 1896   2   March 15, 1921.
3   July 18, 1586   4. September 18, 1831.
5. October 26, 1925.   6. April 26, 1923

Find the day of the week on which the following events took place —
7   Death of King Edward VII, May 6, 1910
8   Death of Queen Victoria, January 22, 1901.
9. The battle of Waterloo, June 18, 1815
10. The battle of Trafalgar, October 21, 1805.
11 Delhi Darbar, December 12, 1911.
12. The declaration of independence of America, July 4, 1776
13 Opening of the Great Eng Railway, Sept 15, 1830.
14 Reform Bill III, December 18, 1831
15 Coronation of Aurangzeb, May 26, 1658.
16. Find the dates of March 1592 that fell on Monday
17. I have a calendar for 1927. Will it serve for 1938?

MISCELLANEOUS EXERCISES III.

I

1   Simplify —

$\frac{1664}{1408}$ of ($\frac{7}{10}$ of £$3\frac{4}{14}$ + $6\frac{2}{3}$ of £3. 9d —$4\frac{1}{3}\frac{3}{1}$ of £3 2s.)

2   Divide 2875963 by 5 × 3 × 11 by short division and find the true remainder

3   Multiply 3 720789426 by ˙0086341532 to five places of decimals by the contracted method

4   A person after paying 7d in the £ for income-tax on his income, has £1632 18s. 10d. What is his income ?

5   A certain number of men and women subscribe to a fund, the number of women being four times the number of men. Each man subscribes as many annas as there are men and each woman as many pies as there are women The total amount subscribed is Rs 756 Find the number of men and women

6.   If the income-tax be 6 pies in the rupee for the first half of the year and 3 per cent. in the second, what is the gross income of a gentleman, whose net annual receipts amount to Rs. 1454. 1a ?

7   The manufacturer of an article makes a profit of 25 per cent , the wholesale dealer makes a profit of 20 per cent and the retail dealer makes a profit of 28 per cent. What is the cost to the manufacturer of an article which is retailed for 16s. ?

8   A train, travelling at the rate of $18\frac{1}{2}$ miles an hour, started at 7 A. M on a journey of 148 miles   A second train started from the same station , its speed was to that of the former as 8 5 and arrived 15 minutes after the first train.   When did the second train start ?

9   If 48 oxen would consume a field of uniformly growing grass in 8 days and 64 oxen in 5 days, in how many days will 40 oxen consume it ?

10.   A person bought $2\frac{3}{4}$ per cent stock at 95, sold it and with the proceeds bought $3\frac{1}{2}$ per cent. stock , he got £900 less stock than before but the income remained the same.   How much money did he originally invest ?

II

1. Find the G. C. M. and L. C. M of 157 days 7 hrs 4 min 7 sec. and 243 days 2 hrs 11 min. 49 sec

2 The 15th of May, 1890 was Thursday What day of the week was the 27th April 1790 ?

3. Simplify $\dfrac{\frac{3}{5}-\frac{1}{6} \text{ of } \frac{7}{5}+\frac{8}{5}}{\frac{4}{9}-\frac{6}{7}-\frac{1}{5}+\frac{2}{3}}-\dfrac{\frac{6}{11}-\frac{4}{9}}{1-\frac{5}{9}-\frac{3}{5}}.$

4 (a) Find the value of ·0416 of £33 7s. 6d.−·03$\overset{.}{4}$5 of £32. 13s. 1$\frac{1}{2}$d

(b) Express Rs 371. 2a. 6p. as the decimal of a lakh of rupees.

5. Find the value of 2 03+1·3$\overset{.}{4}$5+27 34+16 231$\overset{.}{7}$ without reducing to vulgar fractions.

6. In the ten years from 1871 to 1881, the population of a country increased at the rate of 9 5 per cent. and in ten years from 1881 to 1891, the rate of increase was 10 5 per cent. If the population in 1891 was 31023759, find what it was in 1871

7 A man bequeaths his property amounting to Rs 49166 in such a manner that $\frac{1}{3}$rd of his wife's share, $\frac{2}{5}$th of his elder son's, $\frac{3}{5}$th of his younger son's and $\frac{1}{2}$ of his daughter's share may be all equal Find the share of the daughter.

8 Find the difference between the present values of £808 10s. due 2 years hence, according as simple or compound interest is reckoned, money being worth 5 p. c

9 A bought a horse which he afterwards sold to B and gained 5 per cent on his outlay , B sold the horse to C and gained 16$\frac{2}{3}$ per cent., C gave 140 guineas for the horse What price did A give ?

10. What sum must a person invest in the 3 per cents. at 90, in order that by selling out £1000 stock when they rise to 93$\frac{1}{2}$ and the remainder when they fall to 84$\frac{1}{4}$ and then investing the whole proceeds in the 4 per cents at par, he may increase his annual income by £9, 5s. ?

III

1   Divide the square root of 122·257249 by ·36856 and multiply the quotient by the square root of ·000625

2.   Simplify —

(a) $\dfrac{\frac{1}{3}+\frac{1}{4}-\frac{1}{14}}{\frac{1}{3}-\frac{1}{4}+\frac{1}{12}}$ of $\dfrac{\frac{1}{3}-(\frac{1}{4}+\frac{1}{12})}{\frac{1}{3}-(\frac{1}{4}-\frac{1}{13})}-\dfrac{\frac{1}{3}+\frac{1}{4}-\frac{1}{9}}{\frac{1}{3}\times(\frac{1}{4}-\frac{1}{13})}$ of $\dfrac{\frac{1}{2}-\frac{1}{4}\times\frac{1}{12}}{(\frac{1}{3}-\frac{1}{4})\times\frac{1}{12}}$

(b) Express $\frac{2}{3}$ of $\frac{6}{15}$ of £1. 10s. $+\frac{3}{4}$ of $\frac{5}{8}$ of 5s. 4d. $-8\frac{1}{2}$

of $\dfrac{1}{4\frac{1}{4}}$ of 5s $3\frac{3}{4}d$ as the fraction of 2s. $1\frac{1}{2}d$

3.   (a)   Divide Rs 38340 3a 6p. by 441 75.

(b)   Find the square root of $\dfrac{1\cdot7\times29\frac{4}{25}}{·000729}$.

4.   A man died on Thursday, the 7th August 1890 He lived for 21000 days (excluding the day of his death). Find the day and the date when be was born.

5.   Find the greatest and the least number of 6 digits that have 251 for their common measure   What is their G. C M ?

6.   A man used to pay Rs 15 as income-tax when the rate was 8 pies in the Rupee. Under the new rules, the tax is reduced by Rs 3. 12a , what is the new rate ?

7.   How much per cent must be added to the cost price of an article, so that a profit of 20 per cent. may be made after allowing a discount of 10 per cent. from the labelled price ?

8   At an examination $\frac{1}{5}$ of a class gets $\frac{7}{8}$ of the maximum number of marks , $\frac{1}{10}$ gets $\frac{3}{4}$, $\frac{3}{5}$ gets $\frac{1}{2}$, $\frac{1}{4}$ gets $\frac{1}{4}$ and the rest $\frac{1}{8}$ The average number of marks got by the whole class is 166.   Find the maximum marks.

9.   If the true discount on a bill of £14641 be £4641 at 10 per cent. Compound Interest, how many years has the bill to run ?

10   I find, on calculation, that if I invest my money in $3\frac{1}{4}$ per cents at 91, my income will be £25 more than if I invest it in 3 per cents at 88.   What is my capital ?

IV

1. Find the least sum of money that must be subtracted from £660 7s 4d. to make the remainder exactly divisible by 39.

2 (a) Convert $\dfrac{13}{20 \times 8}$ into a decimal Why is the result a terminating and not a recurring decimal

(b) Subtract ˙03 from ˙Ò3 and divide the remainder by ˙007.

3. Simplify without reducing to vulgar fraction
$(1˙25)^3 + 2˙25 \times (1˙25)^2 + 3˙75 \times (˙75)^2 + (˙75)^3$

4. The area of a rectangle is 2149908480 sq in Its length is 5 miles 7 fur. 5 po 1 ft. 6 in. , find its breadth.

5 20 men are employed to make a tank 40 ft long, 20 ft broad and 6 ft deep. They work for 30 days and just complete $\frac{1}{3}$ of the work, when it was decided to increase the length of the tank by 10 ft, the breadth by 4 ft and the depth by 2 ft How many additional men must be employed in order that the tank may be completed in 30 days more

6 Calculate correctly to 6 places of decimals the value of $\dfrac{2}{5} + \dfrac{4}{5.10} + \dfrac{8}{5.10.15} + \dfrac{16}{5.10.15.20} + \quad \cdots \quad \cdot \quad \cdot$

7. Rs. 49 was divided amongst 150 children, each girl getting 8a and each boy 4a How many boys are there?

8 A can beat B by 5 yds in 150 yds , and B can beat C by 10 yds in 200 yds By how much can A beat C in 600 yds ?

9. Three apples are worth as much as 4 plums, 4 pears as much as 9 apples, 13 apricots as much as 7 pears and 7 apples sell for 3d I wish to buy an equal number of each of the four fruits and to spend an exact number of pence Find the least sum I must spend.

10. A person has 200 shares in a railway company for each of which he paid Rs 1,000 when the dividend was at the rate of 2 per cent He sells them at Rs 460 per share and invests the proceeds in Government 3 per cents at 92. Find the change in his income.

V

1   Simplify $\frac{2}{3}$ of $\dfrac{\frac{1}{7}-\frac{1}{3}\text{ of }\frac{1}{1\frac{1}{4}}}{\frac{1}{3}-\frac{1}{5}\text{ of }\frac{1}{7}}$ of $\frac{1}{2}$ of $-\dfrac{\frac{7}{8}-\frac{3}{5}\text{ of }\frac{1}{3}\frac{7}{8}}{\frac{3}{5}\text{ of }\frac{3}{7}-\frac{1}{1\frac{1}{4}}}$

$\left\{\begin{array}{l}\frac{3}{5}\left(\frac{4}{7}\times\frac{1}{8}+\frac{3}{8}-\frac{2}{3}\right)-\dfrac{1\frac{2}{3}}{6\frac{2}{3}}\text{ of }\frac{10}{30}+\frac{3}{7}\text{ of }\dfrac{6\frac{5}{13}}{3\frac{2}{3}}\end{array}\right.$

2   Find by practice, the cost of 24 cwt 3 qr. 16 lb. 10 oz. at £2. 0s 8d a cwt

3   Find the value of
$\dfrac{3\frac{3}{3}}{6\ 0625}$ of $\dfrac{9\ 7}{2\cdot 42}-\dfrac{2\cdot 5}{1\ 09}(7\ 25+2\cdot 75)\times\dfrac{£3.\ 6s.\ 8d.}{£10\ 13s\ 8d}$

4   If a snail, on the average, creeps 2 ft 3 in up a pole during 12 hours in the night and slips down 1 ft 4 in. during 12 hours in the day, how many hours will it take to get to the top of the pole, if its height be 25 ft ?

5   A man buys wine at 4s a gallon, he mixes it with water and by selling the mixture at 3s a gallon gains 20 per cent on his outlay   How much water did each gallon of the mixture contain ?

6.   A debt of Rs. 700 is cleared by a payment of Rs. 180 in cash and a bill of Rs. 533 due 6 months hence At what rate is the discount calculated ?

7.   For two thirds of the distance up a ghat the rise is 1 foot in 24 (measured along the road) and for the remaining third the rise is 1 ft in 16   The top of the ghat is 14,000 feet above the bottom , what is its length ?

8.   If the daily wages of a labourer rise from $4\frac{3}{4}a$. to 6a., what percentage of the increase in the price of food and other necessities of life will make his position unaltered ?

9   A merchant sells tea to a tradesman at a profit of 60 per cent. but the tradesman becoming a bankrupt pays only 2s 6d in the £   How much per cent does the merchant gain or lose by the sale ?

10.   What must be the price of a £50 Railway share which pays a dividend of $2\frac{1}{2}$ per cent if the purchaser is to make 4 per cent on his outlay ?

VI

1　Divide 1028 5 by ·0000017 and $\dfrac{2\frac{3}{5}}{3\frac{1}{4}}$ by 0006· and multiply the difference of the quotients by ·00025.

2.　Find the least and the greatest numbers of six digits which being divided by 240, 275, 320 and 400, the respective remainders will be 210, 245, 290 and 370.

3.　Simplify

$$\left\{ \dfrac{2}{3-\dfrac{1}{1-\frac{1}{2}}} - \tfrac{1}{3} \text{ of } \left(5 - \dfrac{2}{\frac{3}{2}-\frac{1}{6}} \right) \right\} - \dfrac{\frac{1}{2}+\frac{3}{4}}{1\frac{1}{3}}.$$

4.　Find the value of $1\frac{3}{17}$ of ·01236 of Rs. 5　11a. 8p. and express the result as the decimal of one shilling, taking a rupee to be equivalent to 1s　4$\frac{1}{2}d$.

5　A man buys eggs at a certain price per　score and sells them at half that price per dozen , what is his gain or loss per cent ?

6　A room 20 ft. by 18 ft has a carpet 18 ft. by 16 ft. laid down so as to leave a margin of equal width all round　Find the cost of covering this margin with cloth at 4s. 3d a square yard

7.　A society has 3 classes of subscribers and each subscriber pays as many pies as there are subscribers in his class.　The rates of subscription in the classes are in the proportion 81　64　100　The total collections are Rs 510, 6a 8p. Find the number of subscribers in each class.

8　When the income-tax was 7d in a £, a person had to pay £63 more than when it was 4d in a £, although his income had diminished in the meantime by £225.　What was his income at first ?

9.　A crew, who can row 4 miles an hour in still water, rows down a certain distance and up again in 1 hour 36 minutes　Find the distance in miles, if the velocity of the current be $1\frac{1}{2}$ miles

10.　A man has Rs. 41000 which he invests in the 3 per cents. at 87 and 5 per cents. at 104　What sums must he invest in each stock that he may make $3\frac{1}{2}$ per cent. on the whole ?

VII

1   Divide Rs 240 among 8 men, 12 women and 16 boys so that each man receives Rs 2 more than each woman and each woman Rs 2 more than each boy.

2   Reduce $\dfrac{2\frac{1}{3}-1\frac{1}{4}}{2\frac{1}{3}+1\frac{1}{4}} \times 15\frac{15}{89} - \dfrac{3\frac{1}{3}\times3\frac{1}{3}\times3\frac{1}{3}-1}{3\frac{1}{3}\times3\frac{1}{3}-3\frac{1}{3}+1}$ of 1 cwt 3 qr 7 lb to the decimal of $2\frac{1}{2}$ tons

3   What is the least number, which being divided by 48, 64, 72, 80, 120 and 140 leaves the remainders 38, 54, 62, 70, 110, and 130 respectively ?

4   A reduction of 5% in the price of tea enables a man to buy 12 lbs of tea more for Rs 23 12a. Find the original price per lb

5   If Rs 450 amount to Rs 540 in 4 years at simple interest, what sum will amount to Rs 637 8a in 5 years at the same rate ?

6   A tank can be filled by a pipe in 6 hours and emptied by another in 4 hours   They are alternately opened and closed for $1\frac{1}{2}$ hours   find the time in which the tank when full, can be emptied

7   A person sells 40 horses at a gain of 10 per cent and 50 horses at a gain of 20 per cent.   If he had sold all of them at a uniform profit of 15 per cent , he would have got Rs. 40 less.   Find the cost price of each horse

8   If 9 lbs of rice cost as much as 4 lbs of sugar, 14 lbs. of sugar are worth as much as $1\frac{1}{2}$ lbs of tea and 2 lbs of tea worth 5 lbs of coffee, find the cost of 11 lbs of coffee, if $2\frac{3}{4}$ lbs of rice cost $6\frac{1}{4}d$

9   If the manufacturer makes a profit of 20 per cent , the wholesale dealer a profit of 25 per cent   and the shopkeeper a profit of 40 per cent, what is the cost of the manufacturer of an article bought from a shop at 17s. 6d ?

10   If 3 per cent stock be at such a price as to give $3\frac{1}{3}$ per cent interest, what rate of interest will it pay when the price of £100 stock has increased by £6 ?

VIII

1. I give away to each of my four children $\frac{1}{13}$, $\frac{7}{30}$, $\frac{5}{19}$ and $\frac{7}{23}$ of a basket of apples   I had only just enough to be able to do this without cutting an apple   How many had I ?

2 The difference between simple and compound interest on a certain sum of money for 2 years at 4 p. c. is Rs. 20   What is the sum ?

3 Express the difference between $\dot{3}78$ of $13s.\ 10\frac{1}{3}d.$ and $\cdot\dot{3}78$ of $16s.\ 6d.$ as the fraction of

$$\cdot426 \text{ of } \frac{3\frac{1}{3}}{08} \text{ of } \frac{3}{735} \text{ of } \frac{\cdot147 \times 4\frac{4}{0}}{11\frac{1}{70}} \text{ of } \pounds1.\ 17\frac{1}{5}s.$$

4 Simplify

$[(\frac{2}{19}+\frac{1}{8})-2\frac{2}{3}]$ of $\cdot5\dot3-[1\frac{1}{17}$ of $(1-\frac{6}{8}\frac{1}{1}+\frac{5}{11}$ of $\cdot1\dot6$ of $(\frac{3}{2}+\frac{5}{12})]$.

5 Find the product of $\cdot18988$ and $8\cdot08$, divide it by $\cdot0235$ and from the quotient thus obtained subtract the square of $8\cdot08$

6 What sum will amount to Rs. 15916. 9a. 5·8p. in 3 years at C. I., the rate of interest being 3, 2 and 1 p c. for the 1st, 2nd and 3rd year respectively.

7 A person pays an income-tax of $4d.$ in the $\pounds$ during the first half of the year and of $3d$ in the $\pounds$ during the second half.   He finds that owing to an increase in his income, he pays the same amount of tax for both the half years.   If his gross income for the year is $\pounds700$, find his net income.

8 I bought goods at $23s.\ 9d$ with 4 months' credit and sold them forthwith at $25s\ 6d$ with such allowance of credit as made my gain $6\frac{2}{3}$ per cent.   How long credit did I give, interest being reckoned at 4% ?

9. In a library $\frac{1}{9}$ of the books are literature, $\frac{1}{4}$ mathematics, $\frac{1}{6}$ history, $\frac{1}{8}$ philosophy and the rest novels.   What is the least number of novels that the library can have ?

10. A man calculates that if he invests his whole capital in the 4 per cent stock at 92, his income will be less by $\pounds21$ than if he invests it in the $4\frac{1}{4}$ per cent. at par   What is the capital ?

IX

1. Find the value of

$$\frac{6\ 27\times0\ 5}{(\frac{1}{2}\ \text{of}\ \frac{3}{4})\times8\ 36} \qquad \frac{(\frac{1}{5}\ \text{of}\ \cdot1)\times\ 75\ \text{of}\ 21\ \dot{3})}{(\cdot\dot{6}\ \text{of}\ \frac{5}{8})+1\ \dot{4}}$$

2 A's income is 150 per cent more than B's. How much per cent. is B's income less than A's ?

3 Two clocks begin to strike 8 together, one of them finishes in 14 seconds and the other in $10\frac{1}{2}$. What is the interval between their fifth strokes ?

4 A woman bought eggs at the rate of 8 for $5d$ and sold them so as to gain $1\frac{1}{2}d$ on a dozen. What did she charge for each and what was her gain per cent. ?

5 A postage stamp measures $\frac{8}{9}$ in by $\frac{3}{4}$ in How many such stamps will be required to cover the walls of a room whose height is 10 ft 6 in , length 15 ft and breadth 12 ft. 2 in , allowing a space of 50 sq ft. for the doors and windows ?

6 A wholesale dealer sells to a retailer at 10 per cent. profit, the retailer sells to the consumer at 50 per cent profit. What proportion of the price is paid by the consumer as profit ?

7 A man borrows two equal sums of money at the same time at 5 per cent and $3\frac{3}{4}$ per cent respectively. He finds that if he repays the former sum with interest just a year before the latter, he will have to pay in each case the same amount viz Rs 736. Find the amount borrowed

8 A person, who pays income-tax $5d$. in the £, finds that a rise of interest from 6 to $6\frac{1}{2}$ per cent increases his net income by £23 10s What is his capital ?

9 By selling 4 dozen mangoes for 13 rupees, it was found that $\frac{3}{10}$ of the outlay was gained , what ought the retail price per mango to have been in order to have gained 60 per cent ?

10 I invest equal sums in a 4 per cent stock and in a 3 per cent stock and get 5 per cent. for my money , the 4 per cents are at 90 What is the price of the 3 per cents. ?

X

1. Simplify

$$\dfrac{4\frac{3}{4}+\frac{7}{1\cdot7}}{8\ 16-3\ 7+\frac{6}{8}}\ \text{of}\ \dfrac{\frac{7}{8}-1\frac{4}{5}}{1\frac{5}{6}+3\frac{1}{18}}-\dfrac{0\cdot07+\frac{1}{1\cdot9}=-\frac{1}{9\cdot1}}{0\ 5\frac{2}{4}}\ \text{of}\ \dfrac{1\cdot\frac{637}{2\cdot9}}{1\frac{1}{4}+0\cdot2225}$$

and reduce £3 15s 4d. to the decimal of Rs 100. (£1= Rs 15)

2 What are the times between 3 and 4 when the hands of a watch are equidistant from Fig. III on its dial ?

3 In the centre of a room 21 ft square, there is a square carpet, the rest of the floor is covered with oil-cloth. The carpet and oil-cloth cost respectively Rs 8 4as and Rs 4 4a. per sq yd and the total cost of both is Rs 352. 4a., find the width of the oil-cloth border.

4. If Rs 31250 put out at compound interest amount in 3 years to Rs 43904, what is the rate per cent ?

5 A and B start at the same time from two places $3\frac{1}{2}$ miles apart and walk towards each other The lengths of their steps are as 15'14, but while A takes 10 steps B takes 11 Find who will reach the midway first and how many yards off the other will then be

6 In a half-mile race, A gives B 10 yds start and wins by $10\frac{1}{2}$ sec, but if A gives B 15 sec start B wins by 22 yds. Find the time each takes to run the half-mile

7 There are 3 pendulums The first makes 35 beats in 36 sec, the second 36 beats in 37 sec and the third 37 beats in 38 sec Supposing they begin together, find how many times they will beat simultaneously in 24 hours.

8 The average of 25 returns is 43 The first 9 average 52, the next 12 average 37. Find the average of the last 4

9. A person buys two kinds of tea at 5s. a lb and 6s. a lb respectively He mixes them and gains 17 per cent. by selling the mixture at 6s. 6d a lb. In what ratio does he mix them ?

10 A person invests a certain sum in the 3 per cents. when they are at $96\frac{3}{4}$, had he waited till they had fallen to $96\frac{1}{8}$ he would have got £16 more of stock. How much money did he invest, brokerage $\frac{1}{8}$ per cent. being charged in both transactions ?

LOGARITHMS

Note   This chapter is meant for students of the Bombay Presidency only

§1   The student is supposed to be familiar with the meanings of 'Index', 'Power' and 'Base' in Mathematics

In the equations $3^2=9$, the index 2 of the power 3^2 or 9 is called the *Logarithm* of 9 to the base 3 , in the equation $2^5=32$, 5 is called the *Logarithm* of 32 to the base 2 , in the equation $10^3=1000$, 3 is called the *Logarithm* of 1000 to the base 10   And in general if $a^x=N$, then x is called the *Logarithm* of N to the base a Thus the *Logarithm* of a certain number N to a base a is the index of the power to which the base must be raised in order to be equal to the given number N

Example 1   Find the logarithm of 625 to the base 5.
Sol.   Let x be the required logarithm   Then
$5^x=625=5^4$, $x=4$ Ans

Example 2   Find the logarithm of 216 to the base 6
Sol.   Let x be the required logarithm   Then
$6^x=216=6^3$, $x=3$ Ans

Example 3.   Find the logarithm of 3 to the base 81.
Sol.   Let x be the required logarithm.   Then
$81^x=3$ or $3^{4x}=3^1$, . $4x=1$,
$x=\frac{1}{4}$.   Ans

Example 4   Find the logarithm of 128 to the base $\sqrt[3]{4}$

Sol   Let x be the required logarithm.   Then
$(\sqrt[3]{4})^x=128$, or $4^{\frac{x}{3}}=128$, or $2^{\frac{2x}{3}}=2^7$, $\therefore \frac{2x}{3}=7$,

.. $x=\frac{21}{2}$ Ans

Example 5. Find the logarithm of 98 to the base 10.

Sol Let x be the required logarithm. Then $10^x = 98$.

We know that $10^1 = 10$ and $10^2 = 100$. Now 98 lies between 10 and 100, therefore in the equation $98 = 10^x$, x must lie between 1 and 2, $i.e.$, $x = 1 +$ a **Pure decimal.** What that pure decimal is, we cannot say for the present. As a matter of fact, the *exact* value of that pure decimal cannot be found out. We shall see later on how to know the *approximate* value of this decimal.

§2 The logarithm of N to any given base a is written as $\log_a$ N. Hence the two equations $a^x = N$ and $x = \log_a$ N have the same meaning. The student is advised to be perfectly familiar with this notation and to be able to readily derive one equation from the other.

Note 1   Since $a^0 = 1$, therefore the logarithm of unity to any finite base a is zero, i e, $\log_a 1 = 0$

Note 2   Since $a^1 = a$, therefore the logarithm of the base itself is unity, i e, $\log_a a = 1$

Note 3   The logarithm of zero to any base other than roze is *minus* infinity

Note 4   The logarithm of a negative number to any positive base is not real *imaginary*

§3. The student is already familiar with the following laws of indices —

$$(i)\quad a^m \times a^n = a^{m+n},$$

$$(ii)\quad a^m \div a^n = \frac{a^m}{a^n} = a^{m-n},$$

$$\text{and }\ (iii)\quad (a^m)^n = a^{mn}.$$

Corresponding to these laws we have three fundamental laws of logarithm, namely,

$$(i)\quad \log_a (mn) = \log_a m + \log_a n\ ;$$

$$(ii)\quad \log_a \left(\frac{m}{n} \right) = \log_a m - \log_a n\ ;$$

$$\text{and }\ (iii)\quad \log_a m^n = n \log_a m.$$

We shall now prove these laws in the three articles that follow.

§4. The logarithm of the product of two factors to any base is equal to the sum of the logarithms of the factors to the same base , i e ,

$$\log_a(mn) = \log_a m + \log_a n.$$

Proof — Let $\log_a m = x$, so that $m = a^x$,

Let $\log_a n = y$, so that $n = a^y$.

$$mn = a^x \ a^y = a^{x+y}$$

Hence by definition

$$\log_a(mn) = x + y$$
$$= \log_a m + \log_a n$$

Exercise   Since $\log_{10} 100 = 2$, $\log_{10} 1000 = 3$;

$$\log_{10} 100 \times 1000, \ i e \ , \ \log_{10} 100000 = 2 + 3 = 5.$$

Similarly we can prove that

$$\log_a(mnp) = \log_a m + \log_a n + \log_a p$$

Generally

$$\log_a(mnp \quad ..) = \log_a m + \log_a n + \log_a p + \ ..$$

Thus the logarithm of the product of several factors to any base a is equal to the sum of the logarithms of those factors to the same base.

§5   The logarithm of a quotient to any base is equal to the logarithm of the numerator to the same base diminish-ed by the logarithm of the denominator to the same base, i. e.,

$$\log_a \frac{m}{n} = \log_a m - \log_a n$$

Proof — Let $\log_a m = x$, so that $m = a^x$.

Let $\log_a n = y$, so that $n = a^y$.

Hence $\frac{m}{n} = \frac{a^x}{a^y} = a^{x-y}$.

$\therefore$ by definition $\log_a \frac{m}{n} = x - y$

$$= \log_a m - \log_a n.$$

Exercise   Since $\log_{10} 1000 = 3$, $\log_{10} 100 = 2$;

$$\log_{10} \frac{1000}{100}, \ i e \ , \ \log_{10} 10 = 3 - 2 = 1.$$

§6. The logarithm of any power of a number to any base is equal to the product of the index of the power and the logarithm of the number to the same base, $i\,e$,

$$\log_a m^n = n \log_a m.$$

Proof —Let $\log_a m = x$, so that $m = a^x$.

Hence $m^n = (a^x)^n = a^{nx}$

∴ by definition $\log_a m^n = nx = n \log_a m$

Exercise Since $\log_{10} 100 = 2$,

therefore $\log_{10} (100)^5 = 5 \log_{10} 100 = 5 \times 2 = 10$.

Caution. The student is advised to beware of the fact that $\log_a (m + n)$ is *not* equal to $\log_a m + \log_a n$.

§7. Any number may serve as the base of a system of logarithms, but we are chiefly concerned with the system used in practical calculations This has the number **10** for its base, and the logarithms to the base are called *common logarithms* Henceforth we shall exclusively be dealing with common logarithms. We need not therefore, as is usual, state the base 10 each time. It is customary to write log 43 for $\log_{10}$ 43, log 625 for $\log_{10}$ 625, log 12 9 for $\log_{10}$ 12 9 In numerical calculations, *when no base is mentioned, the base 10 is always understood.* In log 1501, the student must not understand that there is no base. The base is 10, which we have agreed not to write. It is to be understood to be there

EXFRCISE 171

Prove that :—

1. log 15 = log 3 + log 5.
2. log 55 = log 5 + log 11.
3. $\log 2\frac{2}{3} = 3 \log 2 - \log 3$.
4. $\log 81^{1\cdot25} = 5 \log 3$.
5. $\log (27)^2 = 6 \log 3$.
6. log 88 = 3 log 2 + log 11
7. $\log 33\frac{1}{3} = 2 \log 10 - \log 3$.
8. $\log 66\frac{2}{3} = 3 \log 2 + 2 \log 5 - \log 3$.
   $= 2 + \log 2 - \log 3$

9    $\log 105 = \log 3 + \log 5 + \log 7$

10    $\log 5\frac{2}{3} = \log 17 - \log 3$

11. $\log 10\frac{1}{2} = \log 3 + \log 7 - \log 2$

12.   Given $\log 3 = \cdot4771$, $\log 7 = 8451$ and $\log 11 = 1\cdot0414$ find the value of x from the equation $3^x \times 7^{2x+1} = 11^{x+5}$.

Sol   Taking logarithms of both sides, we have

$x \log 3 + (2x+1) \log 7 = (x+5) \log 11$

$\therefore x (\log 3 + 2 \log 7 - \log 11) = 5 \log 11 - \log 7$

$\therefore x = \dfrac{5 \log 11 - \log 7}{\log 3 + 2 \log 7 - \log 11} = \dfrac{5\cdot2070 - 8451}{4771 + 1\,6902 + 1\,0414}$

$= \dfrac{4\,3619}{1\,1259} = 3\,87$ nearly   Ans

13   Given $\log 2 = 3010$, find x from the equation $5^{7-4x} = 2^{x+5}$

14   Simplify $\log \dfrac{153}{13} + \log \dfrac{26}{51}$.

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§8   *The logarithm of a number is not always integral.* Thus since $10^2 = 100$ and $10^3 = 1000$, the logarithm of a number lying between 100 and 1000 lies between 2 and 3, and is therefore equal to 2 *plus* a positive proper fraction. Similarly since 00845 lies between ·001 and 01, *i. e.,* between 10^{-3} and 10^{-2}, the logarithm of 00845 is greater than -3 and less than -2, *i. e.,* it is equal to $-3 +$ a *positive* proper fraction Whenever a logarithm consists partly of an integer (positive or negative) and partly of a *positive* fraction, the integral portion is called the *characteristic* and the *positive* fractional portion is called the *mantissa* Thus if 5 234 be the logarithm of a certain number, then 5 is characteristic and 234 the mantissa   If $-4 + 1095$ be the logarithm of a certain number, then -4 is the characteristic and ·1095 is the mantissa   Note that $-4 + 1095 = -3\,8905$ But -3 is *not* the characteristic, nor $-$ 8905 is the mantissa of that number. A fractional portion, in order to be called a mantissa must be *positive* and *only then* the integral portion can be called characteristic *If a fractional portion is not positive, make it so before calling it a mantissa.*

Example. The logarithm of a number is $-8\cdot236$ Find the characteristic and the mantissa

Sol. $-8\,236 = -8 - \cdot236$
$= -8 - 1 + 1 - \cdot236$
$= -9 + \cdot764.$

$\therefore$ characteristic is -9 and the mantissa is $\cdot764$. **Ans.**

Note — In common logarithms, it is usual to write $-9 + 764$ as $\bar{9}\,764$ The student should note that in $\bar{9}\,764$ $\bar{9}$ *alone* is negative while 764 is positive, but in $-9\,764$, both 9 and 764 are negative. $\bar{9}$ is read as "nine bar"

§9   Advantages of the Common System. The common system of logarithms possesses the following *two very important advantages* —

(1)   The characteristic of the logarithms of any number can always be found by inspection.

(2)   The mantissæ of the logarithms of all numbers consisting of the same digits arranged in the same order (*i e*, of numbers which differ from each other only in the position of the decimal point) are always the same.

It is now proposed to prove these two statements in the next two articles

§10   To show that the characteristic of the logarithm of any number N can be written down by inspection

(*i*) Let the number N be greater than unity having n digits in its integral part.

Then since $10^0 = 1,$
$10^1 = 10,$
$10^2 = 100,$
$10^3 = 1000,$ and so on,

it follows that a number having one digit in its integral part lies between 10^0 and 10^1, a number having two digits in its integral part lies between 10^1 and 10^2; a number having 3 digits lies between 10^2 and 10^3, and so on.   Hence the given number N, having n digits in its integral part, lies between 10^{n-1} and 10^n.

Hence $N = 10^{(n-1)+k}$, where k is a positive proper fraction.

$\therefore \log N = (n-1) + k$.

Thus the characteristic is $n-1$.

Therefore the characteristic of the logarithm of any number greater than unity is one less than the number of digits in the integral part of the number.

(ii) Let the number N be positive and less than unity; also when converted to decimal form, let N have n cyphers *immediately* after the decimal point.

Since
$$10^0 = 1,$$
$$10^{-1} = \cdot 1,$$
$$10^{-2} = \cdot 01,$$
$$10^{-3} = \cdot 001, \text{ and so on },$$

it follows that a decimal fraction having no cypher immediately after the decimal point being greater than $\cdot 1$ and less than 1, lies between 10^{-1} and 10^0, a number having one cypher immediately after the decimal point being greater than $\cdot 01$ and less than $\cdot 1$, lies between 10^{-2} and 10^{-1}, a number having two cyphers immediately after the decimal point being greater than $\cdot 001$ and less than $\cdot 01$, lies between 10^{-3} and 10^{-2}, and so on. Hence the given number N, having n cyphers immediately after the decimal point, lies between $10^{-(n+1)}$ and 10^{-n}.

Hence $N = 10^{-(n+1)+k}$, where k is a positive proper fraction

Therefore, $\log N = -(n+1) + k$.

Thus the characteristic is $-(n+1)$

Therefore the characteristic of a logarithm of a decimal fraction is negative and numerically greater by one than the number of cyphers immediately *after the decimal point.*

Thus the characteristics of the logarithms of the numbers 5678, 56·72 and 587·2 are respectively 3, 1 and 2, and the characteristics of the logarithms of the numbers ·0025, ·02506 and ·50208 are -3, -2 and -1 respectively.

§11   The mantissa of the logarithms of all numbers consisting of the same digits arranged in the same order (i e, of numbers which differ from each other only in the position of the decimal point) are always the same.

Let N be a given number and let i be the characteristic and f the mantissa of its logarithm, so that the logarithm of N is $i + f$

Now in order to obtain a number which differs from N only in the position of the decimal point and consequently has the same digits arranged in the same order, we multiply N by 10^p, where p is an integer, positive or negative

But $\log (N \times 10^p) = \log N + \log 10^p$
$$= i + f + p$$

Hence since i and p are both integers and consequently $i + p$ is an integer, the mantissa f has not changed, it is the same for N as well as for $N \times 10^p$

Example   Given that $\log 2 = \cdot 3010$, find the number of digits in 2^{76}, and the position of the first significant figure in 2^{-35}

Sol.   We have $\log 2^{76} = 76 \log 2 = 76 \times 3010$
$$= 22 \cdot 8760.$$

Since the characteristic of the logarithm of 2^{76} is 22, it follows that in 2^{76} there are 23 digits
Again $\log 2^{-35} = -35 \log 2 = -35 \times 3010 = -10\ 5350$
$$= \overline{11}\ 4650.$$

Since the characteristic of the logarithm of 2^{-35} is -11, it follows that there are 10 cyphers following the decimal point $i\ e$, the first significant figure is the eleventh place of decimals.

EXERCISE 172 (Oral)

Write down the characteristics in the following cases :—

1	log 18.	2.	log 25.	3	log 426.
4	log 012.	5.	log 367 018.	6	log 52·567.
7	log 37000	8.	log 00123	9.	log 2·456.
10	log ·000016803				

§12   For assistance in practical work logarithms of all numbers consisting at the most of four digits have been calculated and tabulated in books known as Four-figure Log Tables.

The following is an extract from such a table —

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10

•

Similarly when the logarithm of a certain number is given up to four places of decimals, we use what is known as anti-logarithms in order to find that number   The following is an extract of such a table —

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
36	2291	2296	2301	3307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
37	2344	2350	2355	2360	2366	3371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
39	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5

The method of using these tables is given below —

§13   To find the logarithm of a given number.

Note   Only the mantissae are given in these tables, the characteristic in each case being found by the two well-known rules given before

Mantissæ of logs of all numbers from 1 to 9999, i e, of numbers consisting of four *significant* digits can be found   The following directions indicate the *method of using such a table* —

(i) The extreme left hand column, at the top of which there is a vacant square, corresponds to the first two significant figures of the numbers

(ii) The next 10 columns are headed 0, 1, 2, 9, they correspond to the third figure of the given number.

(iii) The small columns to the extereme right (generally called "difference columns") are similarly headed 1, 2, 9, and these figures corrrespond to the fourth significant figure in the given number.

The method of using the tables is illustrated in the following example.

Example. Find the logarithm of 4597.

In the first column look for 45 (first 2 figures in the given number) , in the same horizontal line as 45 and under 9 (the third figure in the given number) we get the number 6618 , under 7 (the fourth figure in the given number) in the small difference column and in the same row as 45 we find 7 This means that 6618 and 7 are to be added ; their sum being 6625, the mantissa in the log of 4597 is ˙6625 and the characteristic (not given in the tables) is evidently 3.

Hence   log   4597 = 3 6625. **Ans.**
Similarly   log   45 97 = 1̲ 6625.
and   log   ˙04597 = 2̄ 6625

§14   To find the number whose logarithm is given.

Tables of anti-logarithms are used in this case and they are used exactly in the same way as logarithm tables explained before

Example   Find the number whose logarithm is 2 9072.

Let x be the number . log $x = 2$ 9072

To find x we leave the characteristic 2 for the present and take the mantissa 9072 only

Turn to anti log tables , run down the first *column* till ˙90 (the first two figures in the given log) is reached , then in the *horizontal row* containing ˙90 and under the column headed by 7 (the third figure) is the number 8072 , and in the difference column headed by 2 (the fourth figure) and in the same *horizontal row* as 90 is found the number 4   This 4 is added to 8072 and the sum 8076 is the number corresponding to the mantissa ˙9072   Now since the given characteristic is 2, therefore x shall contain three figures in its integral part and hence combining the two facts, $x = 807$ 6. **Ans.**

Similarly the number whose log is 1˙9072 is 80˙76 and the number whose log is 2̄ 9072 is 08076.

EXERCISE 173.

Write down the logarithms of the following numbers —

1   1403.   2   41 03.   3.   340 1.
4   3041.   5   304.   6   403.
7.   20   8   2   9.   8.
10.   100

Write down the numbers whose logarithms are :—

11.   2 0204   12   3'1289.   13   3 0204.
14   $\bar{2}$ 1289 15 $\bar{5}$ 8906   16.   4 2935.
17   0'0061   18   3 0016   19 $\bar{2}$ 0108
20   $\bar{3}$ 5608.

Solved Examples

Example 1   Find the value of $\sqrt[3]{36}$

Sol   Let $x = \sqrt[3]{36} = 36^{\frac{1}{3}}$

      log $x = \frac{1}{3}$ log $36 = \frac{1}{3} \times 1\ 5563 = '518766 = 5188$
correct up to 4 places of decimals.
     $x = 3\ 302$ **Ans.**

Example 2.   Find the value of $\sqrt[7]{00002675}$.

Sol   Let $x = (00002675)^{\frac{1}{7}}$

    log $x = \frac{1}{7}$ log 00002675
        $= \frac{1}{7} \times \bar{5}\ 4273$
        $= \frac{1}{7} \times (-7 + 2'4273) = -1 + '3468 = \bar{1}\ 3468$
∴ $x = 2222$ **Ans,**

Example 3.   Find the value of $\dfrac{(435)^3 \times \sqrt{056}}{(380)^4}$

as accurately as you can

Sol.   Let $x = \dfrac{(435)^3 \times \sqrt{'056}}{(380)^4}$.

∴ log $x = 3$ log $435 + \frac{1}{2}$ log $056 - 4$ log 380
        $= 3 \times 2'6385 + \frac{1}{2} \times \bar{2}\ 7482 - 4 \times 2\ 5798$
        $= 7\ 9155 - 1 + 3741 - 10\ 3192$
        $= -3\ 0296 = -4 + 1 - 0296$
        $= \bar{4}\ 9704$
∴ $x = 0009342$.   **Ans.**

Example 4. Find the value of $\dfrac{(6\cdot45)^3 \sqrt[3]{\cdot00034}}{(9\,37)^2 \times \sqrt[4]{8\,93}}$

Sol Let $x = \dfrac{(6\cdot45)^3 \times \sqrt[3]{\cdot00034}}{(9\cdot37)^2 \times \sqrt[4]{8\,93}}$

$\log x = 3 \log 6\,45 + \tfrac{1}{3} \log \cdot00034$

$\qquad -2 \log 9\,37 - \tfrac{1}{4} \log 8\cdot93$

$\qquad = 3 \times 0\cdot8096 + \tfrac{1}{3} \times \bar{4}\cdot5315$

$\qquad -2 \times 0\cdot9717 - \tfrac{1}{4} \times 0\,9509$

$\qquad = 2\,4288 + \tfrac{1}{3}\,(-6 + 2\,5315)$

$\qquad -1\cdot9434 - 237725$

$\qquad = -\cdot908495 = \bar{1}\cdot091505 = \bar{1}\cdot0915$ approx.

$\therefore \qquad x = \cdot1234 \quad$ **Ans.**

Example 5. Find the value of $\dfrac{(3\cdot142)^3 \times (\cdot078)^{\frac{1}{8}}}{(\cdot005)^{\frac{1}{4}}}$

as accurately as you can.

Sol. Let $x = \dfrac{(3\cdot142)^3 \times (\,078)^{\frac{1}{8}}}{(\cdot005)^{\frac{1}{4}}}$

$\therefore \log x = 3 \log 3\cdot142 + \tfrac{1}{8} \log \cdot078 - \tfrac{1}{4} \log \cdot005$

$\qquad = 3 \times 0\,4972 + \tfrac{1}{8} \times \bar{2}\cdot8921 - \tfrac{1}{4} \times \bar{3}\cdot6990$

$\qquad = 1\,4916 + \tfrac{1}{8}(-3 + 1\,8921) - \tfrac{1}{4}(-4 + 1\cdot6990)$

$\qquad = 1\,4916 - 1 + \cdot6307 + 1 - \cdot42475$

$\qquad = 1\,69755$

$\qquad = 1\cdot6976$ correct to 4 places of decimals.

$\therefore \qquad x = 49\cdot84.$

Example 6. Find in how many years a sum of money lent at compound interest will double itself at 3% $p.\ a$

Sol Let $n =$ number of years required,
and $P =$ the original sum.

$\therefore$ the amount of P at the end of n years $= P\,(1\,03)^n$

.. by the question, $2P = P(1\ 03)^n$.

or $(1\ 03)^n = 2$

. $n \log 1\ 03 = \log 2$

or $n \times 0128 = 3010$

$n = \frac{3010}{128} = 23\frac{1}{2}$ years nearly.   Ans

Example 7   The edge of a cube is 4 83 ft , find the radius of a sphere whose volume is equal to that of the cube,

$\left(\pi = \frac{355}{113} \right)$.

Sol.   Let r be the radius required,

.   by the question $\frac{4}{3}\pi r^3 = (4 83)^3$,

$\therefore$ $r^3 = \frac{3 \times (4\ 83)^3}{4 \times \frac{355}{113}} = \frac{339 \times (4\ 83)^3}{1420}$

$\therefore$ $3 \log r = \log 339 + 3 \log 4\ 83 - \log 1420$

$= 2\ 5302 + 3 \times 6839 - 3\ 1523$

$= 4\ 5819 - 3\ 1523$

$= 1\ 4296$

$\therefore$ $\log r = 4765$

$\therefore$ $r = 2\ 995$ ft,

EXERCISE 174

1   Find by inspection, the characteristic of the logarithms of —

(i) 21735 , (ii) 28 8 , (iii) 592 , (iv) 871   (v) 875,

2   Write down the logarithms of —

(i) 7 623 , (ii) 762 3 , (iii) 007623 , (iv) 7623000.

3.   How many digits are there in the integral parts of numbers whose logarithms are —

4 3010 , 1 4771 , 3 6989 ; 5651

4.   Give the position of the first significant figure in the number whose logarithms are —

(i) $\bar{2}$ 8910 (ii)   7820 , (iii)   6 4871

Simplify by logarithms —

5. 673 × 38 54

6   3 73 × 8976.

7. 801˙7 × 2 546.

8   2˙106 × 3 006.

9. ˙301 × 2 303

10. 3957 × 3 142.

11   774 — 1393.

12. 10 — 1604.

13. 39 7 — 23 4.

14. 0 984 — 1884.

15   341˙6 — 88 15.

16. 62˙39 — 475 3.

17. 8 567 — 3 748.

18. $\dfrac{21˙3 × ˙67 2}{3˙6}$.

19   $\dfrac{19˙08 × ˙096}{1 73}$.

20. $\dfrac{15˙38 × 0472}{19˙82 × 0379}$.

21   $\dfrac{19˙72 × 7 81}{3 6}$.

22. $(1˙035)^5$.

23. $(25˙4)^3$.

24. $(13˙27)^6$.

25. $(6784)^{\frac{1}{6}}$

26. $\sqrt[5]{435\ 6}$.

27   $\sqrt[11]{39˙97}$.

28. $\sqrt[7]{3˙692}$.

29. $\dfrac{14˙7}{(2\ 205 × 144 × (2˙54)^2}$.

30. $(2˙7)^3 × (81)^{\frac{3}{3}} — (90)^{\frac{5}{4}}$.

31. $\sqrt[7]{˙00324}$

32. $\sqrt[11]{(39\ 2)^2}$.

33. $\sqrt[8]{\left(\dfrac{3^2 × 5^4}{\sqrt{2}}\right)^2}$

34. $\log \left(\sqrt[3]{48} × 108^{\frac{1}{4}} — \sqrt[12]{6}\right)$.

35. $\sqrt[3]{\left(\dfrac{294 × 125}{42 × 32}\right)^2}$

36   $(3{,}30 — 49)^4 — \sqrt[3]{22 × 70}$.

37, Find the number of digits in $(875)^{16}$.

38   $\dfrac{(˙41)^2 × (7˙801)^5 × \sqrt[3]{413}}{\sqrt[7]{876500} × \sqrt[4]{1˙035}}$

39   Find the value of $\frac{1}{2} r^2\ \theta$, when $r = 3˙125$ and $\theta = 1˙047$.

40   Find the amount of Rs. 567 in 25 years at $2\frac{1}{2}$ per cent. compound interest

ANSWERS

EXERCISE (oral).

1 Three thousand four hundred and sixty.

2 Fifteen thousand and seventy.

3 Two hundred thousand nine hundred and three

4. Ten million five thousand and eight.

5 Five hundred and six million seven hundred and twenty thousand eight hundred and ninety-one.

6 Four hundred and seven million eighty thousand two hundred and ninety-three

7. 200300 8   3005006. 9   406007005

10 400000000021   11   6000200027000500009.

12 40000, 7000, 10, 2

13. 1000000, 200000, 10000, 3000, 400, 50, 6.

14. 4000000, 400000, 40000, 6000, 600, 7

15 30000000000, 7000000000, 900000000, 60000000, 10000, 2000, 400, 50, 6

16 XXXVII , LXVII , LXXIX , LXXXIV , XCIX

17 CVII , CCLXXXIX , DCCII DCCCLXVIII

18 MCVXX MMLXX , $\overline{IV}$DCLIX Or MMMMDCLIX , $\overline{VII}$CC

19 $\overline{CC}$XVIII , $\overline{MX}$I 20   661032 21   1863

22 1880   23   1766. 24   2002610.

25 560227   26   99999 , 10000

27 98520 , 20589

28 548, 584, 485, 458, 854, 845

29 5000   30. 79993 31   899992

32 9543210 , 1023459

EXERCISE 1.

1. 136	2 187.	3. 226.	4. 288.
5 3165.	6 2025.	7 2845.	8. 2490.
9 17496	10. 30152.	11. 310074.	12 1231393.
13 1796091	14. 1688786.	15. 1933161	16 1103057€
17 90272260	18 5194290	19 144182	20. 1098354₺
21 1200.	22 1935 A. D.	23 1212.	24 271
25 366	26 1070.	27 9677550.	28. 9706702.
29. Rs 1188	30. 3001		

EXERCISE 2

1. 7221.	2. 5432	3 4311.	4. 6424.
5. 36236.	6. 13857	7 19172	8. 2647.
9. 11967	10. 211402	11. 22639	12. 400152.
13 75759	14 70002	15 180104.	16. 604534.
17 1075.	18 461.	19. 8798.	20. 379.

21 5132	22. 35256	23 83256	24. 19516
3762	29475	92639	9237
8894	64731.	175895	28753

25. 7895	26 2835	27 25329	28. 10000
3327	1396	9098	9935
4568.	1439	16231.	65

29 962615.	30 788461.	31. Rs. 596.	32. 298440.
33. 608717.	34. 470	35 Rs 298.	36. 1875 A. D
37 57 yrs	38. 3630098	39. Rs. 449 loss.	
40. 1936 A D. , 61 yrs		41 4548 ft.	42. Rs. 46.
43. 5538.		44. Rs 91790.	

EXERCISE 3.

1 6244.	2 44824.	3. 63624.	4 65299.
5. 99246.	6 134580.	7 100880	8. 151725.
9. 221670.	10 56050.	11. 126500	12. 732100.
13. 3164500	14. 53610000.	15. 172208000.	

EXERCISE 4.

1. 879060
2. 607068
3. 1000467
4. 6477056
5. 7530640
6. 2895420
7. 18720
8. 50400
9. 78204
10. 39375
11. 51072
12. 160875
13. 16887150
14. 879668400
15. 1512000000
16. 2331000000
17. 1250475045
18. 7340483975
19. 7956117875
20. 80407124282
21. 2025, 5041, 14641, 15625, 65536.
22. 1331, 9261, 15625, 46656, 64000.
23. 179
24. 121
25. 251
26. 71808
27. 2170671
28. Rs. 88592
29. 6095604
30. Rs. 33114
31. 3744 gallons
32. 130800
33. 924 miles
34. 83328000 miles.

EXERCISE 5

1. 4696
2. 2024
3. 5621
4. 536
5. Q=203, R=32
6. Q=856, R=1
7. Q=346, R=255
8. Q=1250, R=25
9. Q=748, R=414
10. Q=547, R=1081
11. Q=22, R=5853
12. Q=85, R=3784
13. Q=5653569, R=8, Q=56535, R=698, Q=565356, R=98, Q=5653, R=5698
14. Q=8930002, R=2, Q=89300, R=22, Q=893000, R=22, Q=8930, R=22
15. Q=483009, R=87, Q=4830, R=987, Q=48300, R=987, Q=483, R=987,
16. Q=10858, R=5
17. Q=16669, R=263
18. Q=220153, R=113
19. Q=28714, R=41
20. Q=26810, R=198
21. Q=57960, R=76
22. 163
23. 187
24. 137

EXERCISE 6.

1. 26
2. 11
3. 5
4. 100
5. 18
6. 54
7. 0
8. 3
9. 664
10. 10
11. 11
12. 53
13. 49
14. 638
15. 16

EXERCISE 7.

1. 5963400, 59634000
2. 2268720, 8318640
3. 425444502, 425062502
4. 1319455
5. 3958425
6. 9238450
7. 13263285

8. 37376925	**9.** 45196875	**10.** 27419625
11 17+739375	**12** 224616875	**13.** 56420892
14 378622998	**15** 4529556999	**16** 3783121650
17 6297585	**18** 959395162	**19** 1633771936
20. 4699309950	**21.** 3503991036	**22** 4813348118.
23 26304176	**24** 269267592.	**25** 236194245
26. 376548496	**27.** 1780316480.	**28** 4196545920.
29. 10253997444.	**30** 3138806016.	**31.** 5978588525.
32 12131254596.	**33.** 1737683904.	**34.** 1843953930.
35 92154273576	**36** 809962611032
37. 8211942220350	**38** 28046185413248.
39 101645725658288.	**40.** 633803630161104.

EXERCISE 8.

1 Q=30791,	R=1	**2.** Q=179126,	R=4.
3 Q=383755,	R=23	**4** Q=87825,	R=14.
5. Q=592906,	R=6	**6.** Q=592923,	R=9
7 Q=62733,	R=28	**8.** Q=180647,	R=25.
9 Q=1108894,	R=26	**10** Q=491318,	R=12.
11 Q=4552042,	R=113	**12** Q=3114855,	R=6.
13 Q=7517193	R=27	**14.** Q=2370724,	R=66
15 Q=15416003,	R=181	**16.** Q=10212209,	R=686.
17 Q=5706476,	R=382	**18** Q=5182291,	R=697.
19 Q=78580930,	R=765	**20.** Q=319112473	R=4327.

EXERCISE 9.

1. 9	**2** 72	**3** 28	**4** 11	**5** 255
6 490.	**7.** 417	**8.** 364.	**9.** 264	**10** 118.
11 5	**12** 34	**13** 104	**14** 40.	**15.** 15
16 1808	**17** 230

EXERCISE 10.

1. 112	**2** 96.	**3** 81	**4** 11
5 7	**6.** 239	**7.** 99905	**8** 999810.
9 100008	**10.** 10400	**11.** 22188	**12.** 26560
13. 2655	**14.** 119.	**15** 123934, 229, 45.
16 359348, 1119, 149.	**17.** 526, 328	**18** 826,721
19. 541	**20.** 1626	**21.** 4, 0, 3.	**22.** 1, 5, 4
23 6	**24** 356393.	**25.** 5177.	**26.** 3440.
27. Multipliers are (a) 556, (b) 543
28. 1911036, 8972.	**29.** 494424, 324.
30. A 10 , B 22 , C 15.	**31.** Rs. 55, 75, 95.

32	Rs 42, 31, 25.	33	Rs 35, 28, 23	
34.	Rs 740	35 Rs. 60	36.	Rs 50.
37	Rs. 2500, 2100, 1800	38	Rs 1244, 1460, 1796.	
39.	2220	40.	52 yrs	

41 *A* Rs 400, *B* Rs 600 and *C* Rs 800
42 *A* Rs 820, *B* Rs 1804, *C* Rs 2624.

43.	Rs 320	44	25
45	Rs 22500	46	7222 and 8456.
47.	(1) 119, (11) 2268	48.	1062. 49 8654.
50	5867	51	Gain Rs 3.

52. *C* is 23 years younger than *D*
53. Rs 3395, 3250, 3120, 3000

54	Rs. 8240	55	After 9 hours

EXERCISE 11.

1.	247	2	340	3.	598	4	569
5.	767	6	890.	7	4096	8	4497.
9	6801.	10.	5536	11	5061	12	5446
13.	2022	14.	2035	15	3301	16	1835
17	1845	18.	2687	19.	429	20	975
21	638	22	916	23	1139.	24	746
25	6666	26	23949	27.	5155	28.	10959
29	7618	30	33006	31.	28937.	32	30879.
33.	22650	34.	55002	35	57311.	36	50492
37	846	38	951	39	1815	40.	2107.
41	9500	42	8223	43	1401	44.	1145.
45	1676	46	3047	47	3167	48	787
49	443135	50	2311720	51.	1888530.	52	4105655.
53	3968325	54	2227536	55	17437	56	51394
57	51163	58	37251.	59.	79798	60	50844

61	Rs 150 14*a* 4*p*	62	Rs 122 12*a*. 7*p*.
63	Rs 71 12*a* 11*p*	64	Rs 1970 9*a* 8*p*.
65	Rs 1995 4*a* 4*p*	66	Rs 2518. 9*a* 5*p*.
67	Rs 5992 13*a* 6*p*	68	Rs 4973. 7*a* 6*p*.
69	Rs 5614 14*a*.	70	Rs 1718 12*a*. 3*p*.
71.	£897 6*s* 4*d*	72.	£1459. 3*s* 8*d*.
73	£3723 6*s* 4*d*	74.	£3466 15*s*. 9*d*.
75	£1295 15*s*. 3*d*	76.	£3705 12*s*. 10*d*.
77	£129 4*d* 3*q*	78.	£8773. 16*s*
79.	£5916. 15*s*.	80	£402

81. £1438. 10s.   82. £182 6s.
83   442 mds. 11 sr. 3 chk. 84   547 mds. 13 chk.
85. 747 mds. 14 sr. 5 chk 86. 838 mds. 23 sr 13 chk.
87. 971 mds 10 sr 10 chk.
88   1493 mds 36 sr. 5 chk. 1 tola
89. 56 mds 36 sr. 1 chk 8 mashas.
90. 124 mds. 19 sr. 7 chk 2 tolas 7 mashas.
91   10449 tons 3 cwt 3 qr
92. 1689 tons 2 cwt. 10 lb
93. 819 tons 10 cwt. 14 lb.
94. 1697 tons 11 cwt 1 qr. 24 lb.
95. 937 tons 10 cwt. 3 qr. 9 lb
96. 2099 tons 5 cwt. 1 qr. 8 lb.
97. 88593 weeks 2 days 8 hours 42 min.
98. 44663 weeks 3 days 16 hours 34 min.
99. 720 weeks 2 days 1 hour 59 min 2 sec
100 802 weeks 2 days 16 hours 59 min 51 sec
101 8886 miles 1110 yds 1 ft 3 in.
102 13516 miles 433 yds. 6 in
103 9201 miles 1126 yds 2 ft 5 in.
104. 6474 miles 369 yds. 11 in
105. 158 right angles 3 deg. 48 min.
106 39 right angles 73 deg. 48 min
107 1 right angle 41 deg. 33 min. 10 sec.
108 3 right angles 5 deg 4 min. 19 sec
109 1 ac. 1222 sq yds 4 sq ft. 9 sq in.
110. 1 ac 2876 sq. yds. 63 sq in
111. 1 ac. 1161 sq. yds 3 sq ft 49 sq. in.
112. 13 ac 4209 sq yds 5 sq ft. 97 sq in.

EXERCISE 12.

1. Rs 123 10a. 10p   2   Rs 153 12a 4p.
3. Rs 122. 12a   4   Rs 913 8a 5p.
5   Rs. 449 12a 10p.   6   Rs. 1183. 13a. 5p.
7. £7844 3s 8d.   8. £10993. 18s. 8d
9. £3163 8s 6d   10. £9655. 9s. 2d.
11. £2600. 1s 6d.   12. £2284 19s. 9d.
13   9188 mds 26 sr. 5 ch   14. 8470 mds 18 sr. 14 ch.
15   5873 mds. 5 ch.   16. 2055 yds. 1 ft 9 in.
17. 627 yds. 1 ft   18   951 yds 1 ft. 7 in
19. 147 tolas 11 mashas 7 ratis

20   362 tolas 2 mashas 2 ratis.
21   265 tolas 8 mashas 3 ratis.
22   171 miles 3 fur 18 p   23.   379 miles 7 fur 21 p.
24   479 miles 7 fur 34 p   25   99 hrs 44min. 51 sec.
26.   190 hrs 36min 38sec 27.   276hrs. 26 min. 42sec.
28.   263 sq yds 3 sq ft 53 sq in.
29.   493 sq yds 3 sq. ft. 74 sq in
30.   516 sq yds 7 sq ft 120 sq. in
31   216 ac 3 rood 6 sq p. 32   424 ac. 2rood 39 sq. p.
33   339 ac 2 rood 7 sq p. 34.   56 wks 5 days 9 hrs.
35   82 weeks 6days 16hrs. 36.   138 wks. 2 days 9 hrs.
37.   93deg. 11 min 20sec 38   138 deg 8 min 26 sec.
39   128 rt. ang. 54 deg 1 min

EXERCISE 13

1   Rs 4 1a 4p   2.   Rs. 5 1a 2p
3   Rs 26 13a 6p   4   Rs 5 11a. 8p
5   Rs. 38 14a. 11p   6   Rs 58 10a 8p.
7   £70 10s 11d.   8   £8 7s 8d.
9   £51. 7s 5d   10   17s 9d
11.   £3 12s 10d   12.   10s. 8d
13   2 tons 16 cwt 3 qr   14   67 tons 5 cwt. 3 qr
15   31 tons 6 cwt 1 qr   16   15 mds 29srs 15 ch.
17.   15mds 17 sr. 14 ch   18   37 mds. 20 sr 13 ch.
19.   6 yds 5 in   20   3 yds 1 ft. 11 in
21   1 ft 8 in   22.   Rs 1326. 11a 7p
23   Rs 1399 14a 7p.   24   £550 18s. 8d.
25   £2381 14s 9d   26   110 mds 32 sr 13 ch.
27.   189 mds 38 sr 13ch 28.   35 yds. 6 in.
29   64 tons 14 cwt. 3 qr. 30.   30 tons 7 cwt 3 qr.
31.   Rs 27 13a. 5p   32   £31 4s 9d
33   Rs 5 6a 11p   34.   £14 3s. 2d
35   25 mds   33 sr 7 chk

EXERCISE 14.

1   Rs. 100   2.   Rs 325
3   Rs 219.   4.   Rs. 241. 5a. 4p.
5   Rs. 448. 3a 6p   6   Rs 657 4a 8p.
7   £424.   8   £463 10s.
9   £995 6s 8d   10   £713 13s.
11.   £2403. 16s.   12.   £3752 10s. 8d

13. £20895 9s 8d   14. £20707. 10s. 6d.
15. 1864 mds 27 sr. 8 ch. and 1896 mds 22 sr. 8 ch.
16 2780 mds. and 2454 mds. 8 sr. 12 ch.
17. 1646 tons 7 cwt. 16 lb, 1472 tons 4 cwt 1qr 24 lb.
18 6162 tons 6 cwt. 2qr 7 lb , 6801 tons 7cwt 2qr. 21lb.
19. 4253 sq. yds. 3 sq ft 68 sq. in. and 4641 sq. yds
     1 sq. ft. 132 sq in   20   4571 sq yds 8 sq ft.
     48 sq. in and 5198 sq. yds   6 sq. ft. 72. sq. in
21. Rs 3748 13a 8p   22   6042 9a.
23 Rs 6538. 10a   24. Rs 1682. 5a 6p.
25. £7115 13s. 9d.

EXERCISE 15.

1. Rs 47. 13a 8p   2. Rs 56 13a 1p
3 Ps 80 11a. 2p.   4 Rs. 85 15a 10p
5 £135 4s 10d   6. £121. 11s. 11d.
7. £175 7s 7d   8 £140 2s. 1d.
9 Rs. 157. 10a 2p   10 Rs. 217. 13a 5p.
11 £175. 6s 8d   12 £215. 12s 10d
13 Rs 143 12a 3p.   14 Rs 322. 4a. 7p.
15 £226. 16s 5d.   16. £635. 8s. 3d.
17 Rs 38 9a. 1p.   18. Rs 51 9a 7p.
19 £48 8s   20 £51. 8s
21. £15 7s. 10d   22. £49. 11s 2d
23. Rs 38 8a 6p , rem 17p
24. Rs. 123 13a 8p. , rem 57p
25. Rs 52 8a. 10p. , rem 19p.
26 Rs. 44. 10p , rem 112p
27 £33 1s. 5d., rem. 35d. 28. £48. 6s , rem 153d
29 £30. 12s. 6d , rem 86d 30 £36 17s., rem. 34d.
31. £65 14s 11d , rem 63d
32 £27. 2s. 2d., rem 11d. 33 Rs 774 9a 3p
34 Rs 721 5a 4p   35 £351 3s 8d
36 £301. 14s 3d   37. 417 mds 21 sr 7ch.
38 375 mds 39 sr. 5 ch.   39 96 tons 3 cwt 24 lb.
40 360 tons 17 cwt 3 qr 9 lb.
41. 15.   42. 16 43. 21   44. 45.
45. 25.   46. 41. 47 5.   48. 8.
49 Rs. 5. 5a 8p. 50. 13 51 14.   52 21
53 12222   54. 298 mi 2f. 28pole 55 56.

EXERCISE 16.

1 Rs 501. 4a.   2. Rs. 495. 9a.
3. A Rs 356 3a. , B Rs 428. 2a , C Rs 512. 9a.
4   A Rs 728 12a 9p , B Rs 536 8a. 6p.
    C Rs 321 7a 3p
5   A Rs 39 12a , B Rs. 52 8a , C Rs 24 12a
6. A Rs 49 8a. , B Rs 58 8a , C Rs 75. 6a.
7   37 yards   8   40 sheep.   9. 240 times.
10. Carriage Rs 64. 4a , Horse Rs 257
11 Furniture Rs 432 7a. , House Rs 3459 8a
12. A Rs 33 , B Rs 16 8a. , C Rs. 8 4a
13   A Rs. 25. 4a. , B Rs 101 , C Rs 404
14. 11 pies   15   1a. 9p   16   1651¾ times
17. 957   18 Rs. 9a. 6a 3p. 19. Rs 23 11a 10p.
20   Rs 35 4a   21 Rs 8 8a 6p 22. 15 days
23. 20 days.   24. 7   25   20
26 10 sov., 20 cr , 50 half cr , 80s., 120 six-pences
27. 80   28 45.   29   30
30   1s 6d.   31   Rs. 12. 11a 8p.
32   4 sr   33   11 gallons
34   Rs 728 9a., Rs 684. 7a. , Rs 576 8a
35. Rs. 832 7a , Rs. 356 9a   Rs 348 11a.
36   Man £5 7s 6d , wom £3. 11s 8d , child £1. 15s 10d.
37   10. 38   10   39. Rs 56. 14a   40   Rs 62 8a
41. Rs. 2322.   42. Rs   100000.

EXERCISE 17.

1. 5×3.   2. 5×5.   3. 13×3.   4. 7×6
5   11×5   6   13×5   7. 13×6.   8   17×5.
9   7×7.   10   35×2 or 14×5 or 10×7.
11. No   12. No   13 Yes. 14   Yes   15 No
16. No   17 Yes   18. No   19. No   20 Yes.
21 No.   22. Yes.

EXERCISE 18

1   Yes   2   Yes   3   No.   4   No   5   No
6   Yes   7   Yes   8   No.   9. Yes 10. No
11. Yes   12. Yes 13. No. 14   No. 15   No
16. Yes   17. Yes. 18   Yes. 19   No 20. No
21   Yes. 22   No   23   Yes. 24   No. 25   No
26. Yes. 27   Yes 28   No. 29   Yes 30   Yes.

31	No	32	Yes.	33	Yes.	34	No	35	Yes.
36.	Yes	37	No	38.	No	39	Yes	40	Yes
41.	No	42	Yes.	43	Yes.	44	No.	45	Yes
46	Yes	47.	4	48	6	49	9.	50	3or7
51.	1,5or9	52.	2 or 6.	53	2.	54	4	55.	2.
56	4.	57	6	58	8	59.	3	60.	6
61	1	62.	2	63	2	64	8	65.	3,3.
66.	5,2	67.	5,8.	68	6,8.	69	4,9	70.	3,9.

EXERCISE 19.

1   $2\times2\times2\times7\times13$.   2. $2\times2\times2\times67$.   3 3×109.

4   $3\times5\times7\times7\times7$ 5. $2\times2\times7\times139$

6. $3\times3\times4\times11\times13$ 7 $3\times3\times5\times11\times11$.

8   $2\times2\times3\times3\times151$.   9 $2\times11\times307$. 10. $2\times3\times5\times293$

11. $3\times3\times11\times59$ 12 $5\times11\times11\times13$

13   $2\times2\times2\times2\times2\times2\times2\times2\times2\times3\times5$. 14 $2\times3\times5\times281$.

15   $5\times7\times11\times19$ 16 $2\times2\times2\times3\times3\times5\times7\times11$.

17   $3\times3\times3\times7\times11\times13\times37$

18. $2\times2\times2\times2\times2\times2\times2\times2\times3\times3\times3\times3\times5\times7$.

19   $3\times3\times7\times5\times11\times13\times37$

20   $2\times2\times2\times3\times5\times7\times7\times11\times37$

21   $3\times3\times3\times3\times3\times7\times7\times7\times11\times13$

22.   17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79.

23. $x=2, y=2$.   24 $x=0, y=4$

25   $x=9, y=3$ 26   909090909090909090909.

27.   3, 5, 7, 11.   28.   3,5,7,11,19.   29   9,11,17,19.

31.   29.   32   615825.   33   72   34. $a=9, b=5$.

EXERCISE 20

1	5.	2.	8.	3.	5	4.	3.	5	7.	6	10.
7.	16.	8.	13.	9	28.	10.	8	11	6.	12.	4
13	4.	14.	5.	15.	17.	16.	7.	17	12.	18.	16.
19	24.	20.	25	21.	144.	22.	50.	23	42.	24.	74.
25.	160.	26.	760.								

EXERCISE 21.

1.	18.	2.	16	3.	25.	4	4	5	3	6	13.
7.	13.	8	2.	9	25	10.	15	11.	125	12	25.
13.	4.	14.	7.	15.	45.	16	180.	17.	180	18.	25.
19.	24.	20	5.	21.	61.	22.	20.	23.	51.	24.	30.
25.	30.										

EXERCISE 22

1	223	2	4a.	3.	5a. 4b	4	5 tolas
5.	25 tolas	6	Rs 1215.	7	14.	8	21
9.	15	10	27	11.	18.	12	11.
13.	No	14	No.	15	Yes	16	12.

17. 21. 18 15, 165 and 75, 105
19 24, 168 and 72, 120
20. 18, 144, and 36, 126 and 72, 90.
21 15, 180 and 45, 60   22. 24, 360 and 72, 120.
23. 45   24 71.   25 35   28 9971,10140
29 9936, 10080.   30 99960, 10030, 170
31 4032   32. 179,96. 33. 6660 , 5696
34. 143, 199.   35 132, 428 36 945, 2925
37. 1625, 16725   38 13080, 12765 , 1, 10
39. 999984, 100149 , G C M =753.   42 No.
43 33 , 8 oranges and 21 mangoes

EXERCISE 23

1.	12	2.	18	3	36.	4.	60	5.	48	6.	80
7.	60	8.	30.	9	36	10	40	11	20.	12	24.

EXERCISE 24

1	144.	2	216.	3.	375.	4	1815.	5	2816
6	450	7	6400.	8.	672	9	666	10	2500.
11.	1584	12.	15730.	13.	1344.	14.	2520.	15	2760.
16	5460	17.	3720	18	4875.				

EXERCISE 25

1.	36.	2.	48.	3.	189	4.	180	5.	72.
6.	756	7.	540	8.	1260	9	1080	10.	1815
11.	2040	12	900	13.	1134.	14	360.	15.	1344.
16	18876	17.	1232	18.	2376.	19	1512	20.	2016.
21	504.								

EXERCISE 26.

1.	36	2	120.	3	60.	4.	300	5	600.
6	96	7	120.	8	60.	9	60.	10	630.
11.	1440.	12.	720.	13.	2520.	14.	3150	15	600
16	2520.	17	1680.	18	4504	19.	4200.	20	1008.
21.	360.	22.	2940.	23.	17160.	24.	1008	25.	51480.

EXERCISE 27

1 144.   **2.** 840.   **3.** After 6 min.   **4.** 115.
5. 1893.   **6** 1885.   **7** 1263   **8.** 2518 **9.** 1796.
10 933   **11.** 10080.   **12.** 1075   **13.** 999846.
14. 99988. **15** 214   **16.** 1012.   **17.** 119 (*b*) 5882.
18 30   **19** 99960.   **20.** 67054   **21.** 875.
22 842.   **23** 60 min.   **24** 22 ft   **25** 2 md. 10 sr.
26 420, 480, 540.   **27** 342, 390   **28** 533, 713.
29 1999, 998   **31** 480.   **32.** 904
33. 175,225 **34** 2880   **35.** 7   **36** 10080
37 90090 **38.** 100801.   **39** 60, 120, 360.
40 85, 170, 255, 340, 425, 510.
41 15, 1260 , 45, 420 , 60, 315 , 105, 180
42 21, 2520 , 63, 840 , 105, 504 , 168, 315.
43 1125, 2100 or 1500, 1575.
44 11940, 16915.   **45** 288, 432, 576 **46** 24, 36.
47. 90, 126   **48.** 12, 144 , 36, 48
49. 14, 168 ; 42, 56. **50** 1155, 385, 231.

MISCELLANEOUS EXERCISES I.

I.

1 651000218400.   **2** 127.   **3.** 11*a* 8*p*
4 5 P M. on the 10th day after leaving New York.
5. He copied 8 instead of 3   **6** 10080
7. 42 half crowns, 126 florins, 168 shillings.   **8.** Rs. 900.

II

1. 998130.   **2.** Rs. 937 8*a*   **3.** 121.
4 Rs 97. 3*a*., Rs 48 9*a* 6*p*.   **5.** 262.
6 600.   **7.** 16047229956480
8. 10 Sovereigns, 40 shillings, 60 four pennies.

III

1 5040.   **2.** 4266833663232   **3** 89356, 36.
4 288.   **5.** *A* Rs. 80 , *B* Rs 90 , *C* Rs 120.
6. Horse, Rs. 1093 12*a* , Saddle, Rs 156 4*a*
7. 3, 4, 3.   **8.** (*a*) 9920, 10044, (*b*) Seventy seven billion six hundred and sixty five thousand five hundred and forty four million three hundred and thirty two thousand two hundred and eleven.

IV.

1· 94. 2. (2852, 3565) or (3865, 4278). 3. £980 2s,
4 71955 5. Dividend=153296, Q =652, R. =76.
6 A Rs 2636, B Rs. 329. 8a , C Rs. 988. 8a.
7. £76. 17s· 3d.   8 5

V.

1. 713.   2 £168. 7s. 6d   3. 625.
4. 3×13×7×101   5. 6124.   6 Rs. 325,
7. 12.   8 24720 and 4155.

VI

1 25. 2. 4s. 2d. gain   3 210   4 5, 3, 3. 5. 2153,
6 Rs. 156. 7a.   7. 1353. 8 (a) 231 (b) 124

VII.

1. 214072620945792   2 2. 3 15.   4 6
5 A £21 10s , B £18. 3s 4d. , C £17. 6 5, 5, 4
7 15   8 99900, 10200.

VIII

1 180.   2 105 persons and one pie will be left
3 A Rs. 220. 10a , B Rs 150 8a , C Rs 316 4a
4. 301.   5 6 minutes. 6. 12, 72 7. Rs 52. 8a 8 15.

IX

1. (a) 75349394307 (b) Q=263779, R=9472.
2 1389787967265584+ 3 Rs 50. 4. £5 , 25s. , 40d.
5 704.   6 5, 140 or 20, 35
7. Dividend=2439883, R=19.   8. 54 gallons

X.

1 Rs 1600.   2. 450   3. £249. 4s 4d.
4. 15, 120 and 30, 105 and 60, 75   5. Rs 66 10a.
6. Rs. 15 8a   7 Multipliers are (a) 615 (b) 423,
8. (a) 731) 52781 (72.   (b) 1301,-

                  5117   ι
                 ─────
                  1611
                  1462
                 ─────
                  149

EXERCISE 29

1. $\frac{5}{7}, \frac{3}{4}$. 2. $\frac{1}{3}, \frac{3}{8}$. 3. $\frac{3}{2}, \frac{4}{5}$ 4. $\frac{2}{5}, \frac{6}{5}$ 5. $\frac{7}{9}, \frac{5}{11}$. 6. $\frac{3}{5}, \frac{7}{9}$.

7. $\frac{11}{16}, \frac{9}{15}$ 8. $\frac{1}{21}, \frac{16}{21}$ 9. $\frac{12}{15}, \frac{13}{15}$ 10. $\frac{16}{21}, \frac{2}{3}$. 11. $\frac{24}{45}, \frac{8}{9}$ 12. $\frac{10}{11}, \frac{7}{5}$.

13. $\frac{8}{11}, \frac{5}{6}$ 14. $\frac{7}{9}, \frac{5}{10}$. 15. $\frac{11}{15}, \frac{5}{7}$ 16. $\frac{3}{4}, \frac{9}{13}$ 17. $\frac{16}{25}, \frac{9}{17}$. 18. $\frac{10}{19}, \frac{3}{4}$.

19. $\frac{9}{10}, \frac{11}{81}$ 20. $\frac{59}{85}, \frac{5}{17}$. 21. $\frac{11}{41}, \frac{35}{28}$ 22. $\frac{24}{41}, \frac{13}{85}$ 23. $\frac{15}{16}, \frac{25}{24}$. 24. $\frac{7}{13}, \frac{5}{4}$.

25. $\frac{751}{689}$ 26. $\frac{15}{41}$. 27. $\frac{29}{85}$. 28. $\frac{2}{15}$. 29. $\frac{192}{343}$ 30. $\frac{1}{5}$.

31. $\frac{1}{86}$. 32. $\frac{40}{189}$

EXERCISE 30.

1. $7\frac{1}{2}$. 2. $\frac{13}{6}$ 3. $\frac{17}{4}$ 4. $\frac{57}{8}$ 5. $\frac{69}{4}$.

6. $\frac{103}{8}$ 7. $7\frac{1}{6}$. 8. $\frac{101}{8}$ 9. $\frac{135}{8}$ 10. $\frac{240}{11}$

11. $\frac{825}{16}$. 12. $\frac{231}{8}$. 13. $\frac{609}{5}$. 14. $\frac{999}{8}$ 15. $\frac{5493}{20}$

16. $\frac{6257}{12}$ 17. $\frac{20606}{21}$ 18. $\frac{16753}{21}$ 19. $\frac{207824}{35}$ 20. $\frac{140646}{25}$,

21. $1\frac{1}{7}$ 22. $3\frac{1}{4}$ 23. $4\frac{1}{5}$ 24. $5\frac{3}{5}$. 25. $8\frac{5}{9}$

26. $9\frac{3}{11}$ 27. $9\frac{7}{15}$. 28. $9\frac{2}{15}$ 29. $20\frac{2}{15}$. 30. $58\frac{3}{13}$

31. $68\frac{2}{15}$ 32. $67\frac{17}{24}$. 33. $248\frac{86}{90}$ 34. $245\frac{30}{51}$. 35. $366\frac{13}{53}$

36. $342\frac{28}{49}$ 37. $407\frac{14}{45}$ 38. $552\frac{1}{90}$ 39. $384\frac{14}{45}$ 40. $426\frac{7}{51}$.

EXERCISE 31

1. 3 2. 8 3. 12 4. 6 5. 15. 6. 8

7. 8 8. 25 9. 35 10. 25. 11. 24 12. 208.

13. 12. 14. 36. 15. 72. 16. 65 17. 80. 18. 100.

19. 125 20. 216 21. 126 22. $\frac{156}{13}, \frac{195}{13}, \frac{208}{13}$

23. $\frac{220}{40}, \frac{460}{40}, \frac{500}{40}, \frac{200}{40}$. 24. $\frac{64}{96}, \frac{60}{96}, \frac{64}{96}, \frac{54}{96}$.

25. $\frac{80}{128}, \frac{56}{128}, \frac{34}{128}$ 26. $\frac{9}{12}, \frac{10}{12}$.

27. $\frac{20}{48}, \frac{21}{48}$ 28. $\frac{30}{36}, \frac{21}{36}$ 29. $\frac{24}{24}, \frac{23}{24}$.

30. $\frac{15}{40}, \frac{22}{40}$ 31. $\frac{12}{27}, \frac{27}{27}$ 32. $\frac{16}{24}, \frac{20}{24}, \frac{21}{24}$

33. $\frac{16}{30}, \frac{27}{30}$ 34. $\frac{25}{30}, \frac{16}{30}, \frac{7}{30}$ 35. $\frac{16}{180}, \frac{75}{180}, \frac{70}{180}$.

36. $\frac{60}{228}, \frac{18}{228}, \frac{27}{228}$ 37. $\frac{150}{420}, \frac{72}{420}, \frac{90}{420}$.

38. $\frac{40}{96}, \frac{42}{96}, \frac{27}{96}, \frac{56}{96}$ 39. $\frac{162}{378}, \frac{546}{378}, \frac{135}{378}, \frac{112}{378}$

40. $\frac{80}{120}, \frac{168}{120}, \frac{105}{120}, \frac{246}{120}$

EXERCISE 32.

1. $\frac{5}{24}$ 2. $\frac{5}{13}$. 3. $\frac{11}{12}$ 4. $\frac{8}{33}$ 5. $\frac{4}{13}$.

6. $\frac{5}{7}$. 7. $\frac{9}{10}, \frac{7}{8}$. 8. $\frac{16}{13}, \frac{5}{21}$ 9. $\frac{9}{11}, \frac{5}{28}$ 12. $\frac{7}{15}, \frac{13}{30}, \frac{5}{12}$.

10. $\frac{5}{9}, \frac{3}{4}$. 11. $\frac{11}{13}, \frac{7}{9}, \frac{5}{6}$ 15. $\frac{5}{8}, \frac{11}{18}, \frac{9}{16}$

13. $\frac{6}{11}, \frac{16}{33}, \frac{10}{22}$ 14. $\frac{8}{7}, \frac{7}{13}, \frac{16}{16}$ 17. $\frac{4}{7}, \frac{7}{11}, \frac{17}{21}, \frac{5}{8}$ 18. $\frac{3}{7}, \frac{15}{35}, \frac{13}{28}, \frac{13}{21}$

16. $\frac{7}{8}, \frac{10}{27}, \frac{7}{15}, \frac{15}{48}$ 20. $\frac{11}{24}, \frac{189}{720}$

19. $\frac{15}{76}, \frac{9}{95}, \frac{15}{57}, \frac{7}{16}$

EXERCISE 33.

1 $1\frac{7}{8}$ 2 $1\frac{11}{12}$ 3. $2\frac{11}{24}$ 4 $1\frac{13}{23}$ 5. $\frac{68}{75}$
6 $1\frac{1}{48}$. 7 $1\frac{59}{385}$ 8 $1\frac{7}{12}$ 9. $1\frac{11}{88}$. 10. $1\frac{11}{72}$
11 $10\frac{3}{4}$ 12 $17\frac{1}{4}$ 13 $26\frac{233}{330}$ 14 $9\frac{87}{140}$. 15 $\frac{22}{33}$
16 $\frac{7}{8}$. 17. $24\frac{23}{18}$. 18. $26\frac{2}{27}$ 19. $18\frac{7}{15}$ 20 $39\frac{3}{4}$

EXERCISE 34

1 $\frac{1}{2}$ 2 $\frac{1}{5}$. 3 $\frac{1}{6}$ 4 $\frac{1}{8}$ 5 $\frac{3}{30}$ 6 $\frac{13}{58}$
7 $4\frac{1}{94}$ 8 $2\frac{1}{8}$ 9 $2\frac{11}{12}$ 10 $3\frac{38}{81}$ 11 $8\frac{1}{2}$ 12 $11\frac{1}{17}$
13 $10\frac{11}{72}$ 14 $1\frac{97}{100}$ 15. $\frac{1}{4}$ 16 $1\frac{7}{13}$ 17 $16\frac{2}{3}$ 18 $\frac{413}{450}$
19 $1\frac{3}{4}$ 20. $2\frac{181}{225}$ 21 $\frac{16}{78}$ 22 $\frac{2}{3}$ 23 $3\frac{31}{30}$

EXERCISE 35.

1 48 2 399 3 $192\frac{1}{2}$ 4 $238\frac{1}{2}$ 5. $603\frac{1}{2}$
6 762 7 $1119\frac{101}{111}$ 8 $99999\frac{899}{999}$ 9 $\frac{3}{10}$ 10. $\frac{42}{80}$
11. $7\frac{83}{95}$ 12 $49\frac{2}{27}$. 13 $23\frac{5}{18}$ 14 $93\frac{1}{8}$ 15 120
16 $765\frac{1}{3}$ 17 82 18 1 19 $5\frac{1}{7}$ 20 $247\frac{1}{2}$.

EXERCISE 36.

1. $\frac{1}{34}$ 2 $\frac{3}{80}$. 3 $\frac{1}{4}$ 4 $\frac{5}{11}$ 5 $\frac{29}{73}$ 6 $\frac{101}{352}$
7 $2\frac{7}{10}$ 8 $4\frac{7}{13}$ 9 14 10 $1\frac{1}{3}$ 11 $2\frac{1}{8}$ 12 $\frac{9}{18}$
13 $\frac{11}{16}$ 14 $\frac{5}{9}$ 15 $1\frac{5}{18}$ 16 $\frac{91}{99}$ 17 $1\frac{11}{16}$ 18 $1\frac{1}{5}$.
19 8 20. Rs 7 $8a$ 21 $1\frac{5}{18}$ 22. $\frac{95}{148}$

EXERCISE 37

1. 5 2 7 3 $11\frac{1}{3}$ 4 $1\frac{7}{11}$ 5 $2\frac{5}{8}$.
6 $16\frac{1}{8}$ 7 $\frac{1}{4}$. 8 $\frac{3}{50}$ 9 $5\frac{1}{2}$ 10 $7\frac{5}{7}$.
11. $1\frac{1}{4}$ 12 $4\frac{1}{8}$ 13 $7\frac{1}{2}$ 14 $3\frac{3}{4}$ 15 1
16 $1\frac{1}{3}$ 17 $7\frac{1}{2}$ 18 $\frac{3}{5}$ 19 1 20 1
21, $1\frac{1}{8}$ 22 $2\frac{1}{18}$ 23 $+\frac{11}{12}$ 24 $3\frac{5}{13}$ 25 $\frac{1}{8}$.

EXERCISE 38

1 $\frac{1}{2}$ 2 $\frac{5}{7}$ 3 2 4 $1\frac{1}{2}$ 5 $\frac{1}{2}$. 6 $\frac{5}{6}$.
7 1 8 $\frac{6}{11}$ 9 1 10 $2\frac{1}{4}$ 11. $11\frac{16}{67}$
12 4 13 5 14 3 15 5 16 1 17 $\frac{1}{5}$.
18 1 19 $\frac{1}{2}$ 20 $\frac{1}{5}$ 21 $\frac{7}{8}$ 22. 2 23 2.
23.(b)$18\frac{7}{18}$ 24 2 25 $\frac{1}{4}$ 26 $1\frac{1}{2}$ 27 1 28 $\frac{1}{4}$
29 1 30 $\frac{1}{2}$ 31 $\frac{1}{4}$ 32 1. 33 $\frac{1}{4}$. 34 $\frac{241}{281}$
35 $\frac{1}{4}$ 36 $\frac{3}{4}$ 37 9 38 3 39 1 40 14.

EXERCISE 39.

1. $2\frac{3}{17}$. 2. $\frac{81}{640}$. 3. $\frac{9}{4}$. 4. $\frac{88}{157}$. 5. $1\frac{71}{115}$

6. $9\frac{43}{40}$. 7 $\frac{1}{3}$. 8 $\frac{103}{193}$. 9. $\frac{29}{70}$. 10. $17\frac{3}{7}$.

11. $1\frac{4}{101}$. 12. $\frac{129}{140}$. 13. $\frac{1}{2}$ 14. $7\frac{3}{4}$. 15. 3.

16. (ı) $4\frac{3}{6}$, (ıı) $1\frac{47}{65}$ 17.

$$\cfrac{1}{7+\cfrac{1}{2+\cfrac{1}{\frac{1}{4}}}}$$

EXERCISE 40

1 £49 13s $11\frac{1}{12}d$	2 £51 17s $1\frac{11}{10}$d.
3. Rs 37 9a $2\frac{2}{31}p$.	4. 113 mds $9\frac{12}{21}$ ch.
5 123 tons 10 cwt. $1\frac{1}{18}$ qr	6. 75 mi. 6 fur $5\frac{84}{125}$ po.
7. £28. 18s. $10\frac{13}{18}d$	8 Rs 5 13a, $7\frac{17}{16}p$
9. 18 yds 1 ft $9\frac{17}{16}$ in	10. 11 mds. 11sr $10\frac{13}{16}$ ch
11 Rs. 3. 13a. 10p.	12 Rs 8 9a 8p.
13 Rs. 26 12a.	14. Rs. 31 7a. 6p.
15 17 mds 33 sr. 9 chk.	16 21 mds. 28 sr $3\frac{1}{4}$ ch.
17. 7 tons 19 dwt $3\frac{8}{9}$qr	18. 23 tons 9 cwt $2\frac{3}{4}$qr.
19. Rs 17 11a $7\frac{7}{12}p$.	20 Rs 87. 13a $1\frac{1}{2}p$.
21 152 mds. 7sr. $13\frac{1}{3}$ chk.	22 155 tons 6 cwt. $3\frac{1}{4}$ qr.
23. 162 tons 10 cwt. $3\frac{3}{4}$qr.	24. 52 yds 1 ft $7\frac{3}{4}$ in.
25. 55 yds $7\frac{1}{2}$ in	26. Rs 1766 12a. 11p
27. Rs 25. 17a 6p	28. Rs 8 5a 4p.
29 7 mds 5 sr. $7\frac{1}{2}$ chk	30 2 mds. 12 sr. $13\frac{3}{5}\frac{3}{5}$ ch
31 3 tons 10 cwt. $3\frac{1}{3}$qr.	32. 1 ton 5 cwt. 2qr. $8\frac{1}{4}$ lb.
33 Rs. 67. 9a $8\frac{3}{5}p$.	34. Rs. 56 4a.
35. £50.	36. 6 mds 18 sr
37. Rs 5 10a. 7p.	38 £2 8s $7\frac{1}{2}d$.
39 £2. 4s 5d.	40 Rs. 217. 15a. 6p

EXERCISE 41

1 $\frac{1}{5}$. 2. $\frac{1}{6}$. 3 $\frac{1}{4}$. 4 $\frac{3}{4}$. 5 $\frac{1}{4}$. 6. $\frac{5}{14}$.

7. $\frac{3}{5}$ 8. $\frac{5}{8}$ 9 $\frac{5}{6}$ 10 $\frac{1}{4}$ 11 $\frac{1}{4}$ 12. $\frac{344}{513}$.

13 $\frac{73}{335}$. 14 $\frac{77}{118}$. 15. $\frac{2}{37}$ 16 $\frac{2}{15}$. 17 $\frac{1}{48}$. 18. $\frac{1}{50}$

19 $\frac{17}{18}$. 20. $\frac{3}{5}$ 21 $\frac{8}{15}$ 22. $\frac{1}{7}$ 23 $\frac{5}{6}$. 24. $\frac{12}{19}$.

25 1 26 $1\frac{11}{16}$ 27. $\frac{13}{320}$ 28 $\frac{13}{85}$ 29 7 cwt. 11 lb.

30 $\frac{3}{5}$, 31 $\frac{5}{18}$ 32. $\frac{49}{400}$

EXERCISE 42.

1. $\frac{1}{13}$, $8\frac{3}{4}$. 2 $\frac{7}{10}$, $1\frac{1}{10}$. 3. $\frac{3}{89}$, $8\frac{11}{89}$. 4. $\frac{1}{73}$, $2\frac{1}{2}$.
5. $\frac{1}{120}$, $8\frac{3}{8}$. 6. $\frac{1}{181}$, 15. 7 $\frac{1}{73}$, $178\frac{3}{4}$. 8. $\frac{1}{204}$, $80\frac{5}{17}$.
9. $\frac{1}{89}$, 60. 10. 57. 11. 14s. 8d. 12. 54.
13. $1\frac{1}{3}$ hours. 14. 6336 pounds.

EXERCISE 43

1. Rs. 2672. 2. Rs. 139. 4a. $\frac{2}{3}$p. 3. Rs. 2528
4. Rs. 33. 10a. 4p. 5. Rs. 6 4a.
6. Rs. 3. 13a. $10\frac{2}{15}$p. 7. 45.
8. 5a , Re. 1. 9. $857\frac{1}{7}$. 10. Rs. 24000.
11. 100. 12. 50. 13. Rs. 25000.
14 Rs. 429. 8a 15. $39\frac{2}{3}$ 16 Re $\frac{3}{4}$
17. 7 in. 18 630 sec. 19 Rs 6750.
20 400 in 21 Re. 1. 5a $5\frac{1}{2}$p. 22. 1 furlong.
23 Rs. 10 , $\frac{375}{28}$ 24. 5. 25 240.
26. 945. 27 Rs. 563. 5a 4p.

EXERCISE 44.

6. $7 + \frac{1}{10} + \frac{2}{100} + \frac{5}{1000}$, $8 + \frac{3}{100} + \frac{4}{1000}$.
7 $12 + \frac{3}{1000}$, $16 + \frac{2}{10} + \frac{3}{1000}$
8. $20 + \frac{2}{1000} + \frac{2}{10000}$, $15 + \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{4}{100000}$.
9. $7 + \frac{1}{100} + \frac{5}{10000}$, $21 + \frac{1}{10} + \frac{2}{100} + \frac{5}{10000}$.
10. 6. 11. '06. 12. 006. 13 '0006.
14. 5 3. 15 15'906 16. 35 17 357
18 0959. 19 8079 20 5 0387. 21. 8 0097.
22 15 3905. 23 21 0046 24. 15 0709. 25 15 8007

EXERCISE 45.

1 $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$. 2 $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}$. 3. $\frac{1}{20}, \frac{1}{40}, \frac{1}{60}$
4 $\frac{31}{250}, \frac{75}{125}, \frac{01}{125}$. 5. $\frac{63}{125}, \frac{27}{500}, \frac{63}{1250}$.
6. $\frac{51}{10}, \frac{5028}{1000}, \frac{45}{8}$ 7 $\frac{65}{4}, \frac{75}{4}, \frac{151}{8}$. 8 $\frac{191}{3}, \frac{1901}{30}, \frac{18011}{30}$.
9 7, 17, 1 77 10. '75, '185, '1825.
11 '9, 09, '009 12 '375, '075, 005
13 5 07. 8 009, 15'0175. 14. '05, '004, '054
15. 789 35, 756'356, 7 5001 16 '015, 0007, '00101.

EXERCISE 46

1.	1, ˜1, ˙01, ˙001	2 10 1, 10 01, 10˙001
3.	25, 25˙5, 252˙5, 250 02	4. 1˙5, 1˙15, 1˙015
5	˙61, 825, 1˙0123	6 1˙53, 1˙503, 1˙2156
7.	˙0153, ˙01057, ˙01425	8. ˙001, 0001, ˙00001
9.	100 1, 10˙01, 1˙001	10. 1602 3, 1512˙5, 2113˙07
11	15425, 12310, 15802	12. 10300˙2, 10300˙02
13.	7 2813, 12˙2602	14 12 0209, 120˙209
15	˙001, ˙0001, 0000t, ˙000001	
16	˙02001, ˜20001, 2˙0001	17. 20, 10, 1
18	3, 4, 504	19. 4030, 15003, 21007
20	100˙2, 1 2, ˙25	21. 000003, ˙000516, 004123
22	˙016123, 1˙880116	23. 56˙21002, 70˙23105
24	˙21231, ˜72345, 5˙5605	

EXERCISE 47.

1. 18˙0022	2. 18˙9255	3. 198˙111
4. 136˙301	5. 242˙243	6. 154˙2185
7 86 404	8. 5510˙6207	9. 557˙45001
10 317˙16035		

EXERCISE 48.

1. ˙25	2 ˙06	3. ˙075
4 ˜123	5. 108˙9	6. 1114
7 13 86965	8. ˙88698	9. 10 9765
10 0644	11. ˙99765	12. 9˙019235
13. 14˙886	14 22˙01995	15. 3˙09955

EXERCISE 49.

1. 21	2. 244	3. 450˙65	4. 420˙46
5. 1816˙78	6. 1427˙081	7. 30˙625	8. 42 93315
9. 290 602852	10. 21˙05300125	11. 6˙4016001	
12 375˙145008	13. 141˙4333	14. 5248˙2136	
15. 1432˙3113	16. 4277˙5295	17. ˙0000000001	
18. ˙000016	19. ˙008004	20. 655 36	

EXERCISE 50.

1 1˙6 ; ˙54 ; 22˙43   2. 59˙514 , 9˙919 , 4˙578 , ˙763
3. ˙00001108 , ˙00000554 , ˙0000006925

4	5'12 , 2'417.		5	17945 ; 281500.
6.	24172 , '0000746		7	10 , 10'01 , 110000.
8.	281 315 , 039			
9.	1'092 , 1092 , 1092 , '0001092			
10.	'0032 ; 602400.	11. 1 644	12	1 353. 13 0005.
14.	5'570	15 2'770.	16	'001 17 2.
18	00007	19. 25	20	'00025 21. 05
22.	4.			

EXERCISE 51.

1.	'25, 125, '04.	2	625, '35, 275
3	8, 375, '28	4	'256, '1953125, 128
5	'0512, 084, 128125	6	'484375, 6875, 1171875
7.	3 94615, 5 20714, 1'62179		
8	56 89873, 59'85333, 56'23003		
9.	$\frac{2}{4}, \frac{7}{18}, \frac{5}{13}$	10	$\frac{7}{8}, \frac{5}{8}, \frac{1}{2}, \frac{3}{4}.$
11.	8'75 , 2'59375.	12	6, 1 37 13 1 5.
14	007080078125, 0029.		
15.	greatest $\frac{6}{15}$, least $\frac{13}{144}$.	16	greatest $\frac{39}{70}$, least $\frac{16}{38}$.

EXERCISE 52.

1. '07, 1'68.	2. '11, 1'32.	3. '05, 2 25	4. '13, 1'56
5 14, 1 68.	6. '005, '5.	7. 0014, 1'4	8. 009, 8 1.
9. 012, 10'8.		10. 00011, 1'21.	

EXERCISE 53.

1.	4'75.	2. 1.	3 241.	4. '85.
5.	267.	6. 2'925.	7. 1.	8 10 9975.
9	3.	10. 1.	11. 1001.	12. 112
13.	1.	14 2.	15. 2.	16 9'45.

EXERCISE 54.

1.	No	2. No.	3. Yes.	4. No.	5 Yes.
6.	Yes.	7. No.	8. Yes	9. No.	10. No.
11.	83	-12 '4	13. '90.		14. '857142.
15.	'384615.	16. '07	17 '194.		18. 074.
19	'160714285.	20 '081.		21. 5 571428 .	
22.	3'1538461	23. 9'27.	24. 5 296		
25	14 136	26. 416.	27. 29 629.		
28	'846153	29. 5'714285	30. 6		

EXERCISE 55.

1. $\frac{4}{9}$. 2. $\frac{25}{99}$. 3. $\frac{9}{11}$. 4. $\frac{5}{11}$. 5. $\frac{1}{37}$.
6 $\frac{18}{111}$. 7. $\frac{1}{3}$ 8 $\frac{64}{111}$. 9. $2\frac{2}{111}$. 10. $5\frac{4}{9}$.
11. $7\frac{4}{27}$. 12. $10\frac{8}{27}$. 13. 8. 14. 13. 15 $8\frac{3}{11}$.
16. $15\frac{6}{11}$. 17. $\frac{8}{19}$. 18. $\frac{9}{7}$. 19 $3\frac{2}{15}$. 20. $\frac{5}{7}$.
21. $7\frac{3}{90}$. 22. $\frac{7}{85}$. 23. $\frac{127}{445}$. 24. $\frac{4}{15}$. 25. $\frac{1}{2}$.
26. 6 27. $\frac{3}{10}$. 28. $\frac{21}{80}$. 29. $25\frac{502}{555}$. 30. $21\frac{808}{880}$.
31. $2\frac{10712}{11828}$.   36. ·035, ·531, ·026, ·0023, ·0141.

EXERCISE 56.

1 ·523423̇   2. ·54686.   3. ·567567̇
4. ·21565̇.   5 235635.   6. ·2543854̇
7. ·2564256   8 ·123567123̇.
9. ·2444444, ·576576, ·575757̇   10. ·255, ·565.
11. ·3566, 7878̇   12. 3456, ·3783̇ 13. ·0232, ·5678̇.
14. ·5555555, ·0050505, ·0005005.
15. ·06565655, ·08555555, ·08508508̇
16 2252252252, 0225252525, ·0022555555̇
17. ·5765765765, 0035656565, 0004325325
18 3 125612561256125, 5 012356356356356,
                              2·123523523523523̇
19 ·2343434343434, 2345345345345, ·234523452345

EXERCISE 57

1. 1 85.   2. 25 523401674. 3. 8·568986804̇
4. 40·18250̇.   5. 23·731039312̇. 6 40·92643̇.
7. 40 463536697.   8. 54·529016.
9. 29·318401219̇.   10. 92·8602̇.
11. 6·32608699.   12 16 48014924̇. 13 6·60788̇.
14. 16 00631722.   15 6·9999998908.

EXERCISE 58.

1. 3·072̇   2. 5·030   3. ·92.
4. 36·396.   5. 91 418̇.   6. 1361·3405̇.
7. ·5893217̇.   8 ·263047138. 9. ·4100.
10. 1·0001̇.   11. ·1105.   12 03.
13. 009.   14 ·1   15. ·02.
16. ·106.   17. 1·3.   18. ·285714̇

19. 14·7.   20. 230769̇.   21   3·27̇
22   47·3   23   076923̇.   24   ·68921875.
25   ·5   26   142857̇.   27   21.
28   ·571428̇.   29   92 5   30. +   31. 65.

EXERCISE 59.

1. 1224.   2   2916   3. 259·2.   4. 37·5.
5. 509·4.   6   13716   7   234000.   8. 1384
9   16906.   10. 9234 4. 11. 26·25   12   2232.
13. 0078125. 14. 609375 15. ·734375.   16. 51 3375.
17   2·1875.   18. ·06875. 19. 24·406875. 20. ·6875

EXERCISE 60.

1. Rs. 12. 2a. 84p.   2. Rs. 16. 3a. 7 92p
3   16 mds. 11 sr 13 6 chk.   4   24 mds 36 sr 8 chk.
5   £16. 3s 3d 6 £21 9s 9d. 7. 12 yds. 6 741 in.
8   12 tons 3 cwt 1 qr 15 12 lb   9   Rs 15 53125.
10   Rs. 21 859375 11 £18·6375 12   £12·875
13   3 778125 tons   14   16 81875 tons
15   12 75 yards   16. 12 70625 mds
17   Rs. 3 3a 5 25p.   18   Re 1. 5a 4p.
19. Rs 2 11a. ·75p   20   Rs. 6. 5a
21   1 yd 1 ft 8 25 in. 22   £4 15 1 875d
23. £1 9s 1·47d.   24   9s. 3·248d 25. 5½d
26   £1 15s. 3·3709d.   27. 1a 5 76p
28   Rs 6. 1a 2p.   29   19s. 1½d.
30. 19 mds. 15 sr.   31. £1 18s 7d.
32   Rs 3 15a. 4·8p.   33. £14. 10s 8 5d
34   £4. 0s. 9d.

EXERCISE 61.

1   ·5.   2   ·25. 3   2   4. ·25.   5 ·125
6   26   7   5 8. ·2.   9. ·25 10. 25.
11. ·125   12   ·25 13   ·125. 14   ·25   15. ·25.
16. 001136̇.   17. ·083̇. 18. $\frac{1}{25}$, ·036̇.
19   1a. 4p , ·114583̇. 20   ·0203125

EXERCISE 62.

1   ·009̇.   2   142857̇.   3. ·583̇.
4. 25.   5. 1 125.   6. ·12̇.

7. 20. 8 49 9. ·5.
10. ·2$\frac{1}{3}$$\frac{1}{6}$$\frac{1}{6}$. 11. Rs. 516. 12 1.
13 0345. , 14. 5·90625. 15. 9
16. 6·3. 17. 350·6. 18. 16s.
19 Re. 1. 8a 20. 1·916. 21. 14$\frac{1}{1}$$\frac{4}{7}$.
22. $\frac{1}{7}$. 23. Rs. 7. 14a. 1p
24. 3.· 25. 1. 26. ·01.
27. 814; Rem. 003.

EXERCISE 63.

1. ·641. 2. 488000. 3. 9·07. 4. ·0008.
5 8 48. 6. ·008. 7. 2500 8. 215.
9. 330. 10. 1·04. 11. 860.
12. 45·8195338. 13. 194·049. 14. 000906.
15 2·14. 16. 1293·47. 17. ·33. 18. 575 56
19. ·086. 20. 1·911. 21. ·524. 22 ·003.
23. ·95421. 24. 69·6. 25 ·000408 26 ·00534.
27. ·07. 28. 1·58. 29. 79. 30. ·3183
31. ·146. 32 ·017. 33 1·0298212.
34 1 4106861. 35 ·202733. 36. ·333. 37. 1·297.
38 ·1667. 39 23077. 40. ·0684931. 41. 2 98.
42. ·04. 43. 2 9522. 44. 11157. 45. ·2554.

EXERCISE 64.

1. Rs. 4. 2. Rs. 13. 5a. 4p. 3. Rs. 18. 12a.
4. Rs 40. 10a. 5. Rs. 62. 8a. 6. Rs 63. 12a.
7. Rs. 287. 6a. 8. Rs. 573. 8a. 9. Rs. 398. 7a.
10. Rs. 674. 11a. 11 Rs. 1078 7a. 12. Rs. 1176. 3a
13. Rs. 1484. 5a. 14. Rs. 1768.
15. Rs 2679 ·11a. 16. Rs. 3776. 1a.
17. Rs. 4021. 14a. 18. Rs. 6660. 15a. .
19. Rs. 2015. 20. Rs. 1078.
21. Rs. 1445. 8a. 22. Rs. 443. 10a. 8p.
23. Rs. 1417. 15a. 6p. 24. Rs. 2701. 13a. 4p.
25. Rs. 1537. 8a. 26. Rs. 2114. 1a.
27. Rs. 5467 28 Rs. 7115. 1a.
29. Rs. 5923 14a. 6p. 30. Rs. 10142. 13a. 8p.
31. Rs. 5435. 14a. 2$\frac{2}{3}$p. 32 Rs. 17060. 1a. 3p.

33	Rs 4859. 2a 8p.	34	Rs 12733. 9a. 10⅗p
35	Rs. 8847. 2a. 2p	36.	Rs. 11272. 4a. 6¼p
37.	£38718 12s 6d	38.	£19316 13s. 4d
39.	£66021 7s. 9½d.	40	£105268. 0s 5d
41	£419 13s 1½d	42	£1838 3s. 1 86d.
43	Rs 546 14a.	44	Rs 6. 15a 1½p
45.	Rs 3 10a 1½p.	46	Rs. 7299. 11a 6¾p.
47.	Rs 80606. 15a. 1⅛p.		

EXERCISE 65.

1	Rs. 10 13a. 4p.	2.	Rs. 24. 12a. 8p.
3.	Rs 32 0a 4p.	4.	Rs 50. 3a. 7⅛p.
5.	Rs 92. 0a. 11p	6.	Rs 53 5a. 6½p.
7	Rs 55 3a. 10p	8.	Rs. 1116. 13a. 10p.
9.	Rs 1212 14a. 3p.	10	Rs 39 14a. 10¾p
11.	Rs 50 14a. 5p.	12.	Rs. 352 7a. 3⁷⁄₁₀p.
13	Rs 93 12a.	14	Rs 95. 0a 11⅝p.
15	£87 3s. 8d.	16	£32 17s. 1d
17	Rs 110 9a 11p	18.	Rs 253. 6a. 9⅞p
19.	Rs. 401. 6a 2⅔p.	20	Rs. 746. 0a 4⅛p.
21.	Rs 628 13a 11½p.	22.	£2400 15s. ⁶⁸⁄₈₄d.
23	£96. 16s 9⅞d	24.	£934. 18s. 2d.
25.	Rs 5267 11a 11¼p.	26.	Rs 3955. 3a. 11p
27.	Rs 268 12a.	28.	Rs. 1635. 15a. 10p.
29	Rs 4781 4a	30.	Rs 765. 15a 10p
31.	£240. 10s. 9¾d.	32.	Rs. 12137. 15a. 9⁷⁄₁₀p
33	£513 6s 6¼d.	34	£36 17s. 6d.
35	£17. 12s. 2¼d	36.	£1034. 14s 4¹³⁄₂₈d.
37	£5247. 2s 6³⁹⁄₈₄d	38.	Rs 1968. 7a 7¼p.
39	Rs. 606. 11a 9¾p.	40	£77 7s. 7¼d.
41.	£97. 11s 5¾d	42.	£1455 19s. 4½d.
43.	Rs 409. 3a 4p	44	£57 19s. 11d. nearly.

EXERCISE 66.

1	Rs. 91. 13a	2.	Rs. 99 14a.
3	Rs. 392. 4a.	4.	Rs 85. 4a.
5	Rs 108. 10a	6	£4. 19s 8¼d.
7.	Rs. 46. 15a 6p.	8	Rs. 95 1a 6p.
9	Rs 1265 7a. 3p.	10.	£6. 14s. 8½d.
11.	Rs 190 4a. 6¾p.		

EXERCISE 67.

1. 1936	2. 3249	3. 1296	4. 2304
5 2401	6 2116	7 3481	8 7744
9 2809	10 2209	11. 9801	12 3136
13 16384	14 24336	15 27556	16. 11236
17 4225	18. 5625	19 21025	20. 24025
21. 27225	22. 42025	23. 60025	24. 93025
25. 390625	26 863041	27. 1755625	28. 4515625

EXERCISE 68.

1. 20	2. 21	3. 66	4. 130	5. 35
6 45	7 96	8 132	9. 140	10. 144
11 252	12. 286	13 3, 5, 91, 30030.	14. 810000	

EXERCISE 69

1 160	2. 170	3. 210	4. 225
5. 271	6. 293	7. 916	8. 905 .
9 989	10 1516	11. 2125	12 3105
13. 8972	14. 8513	15. 9009	16. 15631
17. 15321	18. 71556	19. 162550	20 921986
21. 1500025	22. 7000025	23 56	24. 115
25. 125	26 232	27 526	28. 501
29. 136, 41	30. 126	31. 14400	

EXERCISE 70.

1. 3	2. ·08	3 09	4. 2·7
5. 4·1	6. 5 9	7. 3·05	8. 9·21
9. 6 09	10 12 25	11. 21·85	12. 15 02
13 7·256	14. ·125	15 902	16 501·95
17 15 093	18 ·3162	19 ·0031	20. 0707
21. 3 8746	22. 11 0165	23. 16·0032	24. 3·31662
25. 4 35889	26. 5·56776	27. 10·04987	28. 22 38302
29. 24 59674	30. 10·81665	31. ·073	32. 0231
33. ·0007008	34. 200 001	35. ·125, 625	36. 2·1, 3 4, 5 6

EXERCISE 71.

1. $\frac{8}{9}$	2. $\frac{11}{10}$	3. $\frac{15}{10}$	4. $\frac{18}{35}$	5. $2\frac{1}{4}$
6 $2\frac{3}{4}$	7. $2\frac{1}{4}$	8. $2\frac{3}{5}$	9. $5\frac{8}{9}$	10 $9\frac{2}{11}$
11. $9\frac{3}{7}$	12. $5\frac{3}{7}$	13. $3\frac{4}{15}$	14. $4\frac{4}{15}$	15 $\frac{13}{15}$
16. $\frac{13}{35}$	17. ·559	18. ·692	19 ·440	20. ·553

21 1·452 22   2·397 23. 1 820 24. 2 291
25. ·912 26   935   27.   645   28   1·443
29 16 959 30. 19·164 31   845   32. 5 606
33. 6⅗, 7½ 34. 2⅓, 3¼, 5⅙

EXERCISE 72.

1. ·1ḃ   2   5 3   3   68 83   4.   36   5. ·2
6. 1·5367 7   2659 8. 2 007+ 9. ·0750

EXERCISE 73.

1   11   2   28   3   127   4.   267
5. +73   6   8888 7. 23·+   8   +7
9. 08   10. 0231 11. ·957 12. ⅘
13. 7¾ 14 1·275 15 2 65 16. ·101
17. ·3   18 11 6 19   893 20. 2 648
21 ·144 22. ·46+ 23. 1 959 24. 1 759
25. 111111111

EXERCISE 74.

1. 36   2. 6·3   3. 9   4. 2 6   5   11   6. 12
7. 5   8. 8.

EXERCISE 75

1   1 dm 7 cm 9 mm   2.   1 Dm. 2 m 1 dm. 5 cm.
3. 16 m 7 dm. 9 cm.   4. 1500 mm.
5. 15500000 mm.   6. 5 m. 7 cm. 5 mm
7   9 Km 7 Hm 1 m 8 dm 8   5 m. 1 dm 5 cm 5 mm
9   8 m 9 dm. 7 cm 2 mm. 10. 195 m. 8 dm 5 cm. 5 mm.
11. 7 m. 3 cm 3 mm.   12. 33 m. 9 dm. 4 cm 9 mm.
13   345 mm   14 3 m 2 dm 8 cm 7 5 mm
15   166812000 mm   16.   12104
17   1 hr. 46 min 40 sec.

EXERCISE 76.

1. 3700 kilos. 2   127   3 270 grammes 4. 7·92
5. 1·+52   6   (1) 6c (2) 1s 2¼d nearly 7. 9/17
8   102+   9   +2 min. 40 sec   10. 6 +51+8

EXERCISE 77

1. 180 sq. ft   2. 133 sq. ft.   3 143 sq. ft.
4 28 sq yds. 6 sq ft. 48 sq. in
5. 36 sq yds 6 sq. ft. 112 sq in.
6 144 sq yds 8 sq ft 48 sq in.
7. 20 acres   8 $62\frac{1}{2}$ 9. 25
10. $91\frac{2}{3}$ 11 12 ft.   12 24 ft. 4 in.
13. 440 yds   14 220 yds. 15. 239'197 ..
16 2 fr. 3 po 4 yds. 17 220 sq. ft 72 sq in
18. 18 ft.   19. 50 min   20 2288 yds.

EXERCISE 78.

1 30 yds   2. 42 yds   3. 37 yds.
4 $38\frac{1}{4}$ yds   5 14   6 10
7 375   8. 666   9 800.
10 Rs 54 10a 11. Rs. 5 8a 12 Rs 5 9a 5p
13 Rs. 22   14 Rs 38.   15 Rs. 352
16 30 ft.   17. 40 ft   18. 25 ft.
19. 36, 12 ft.   20 45, 30 ft   21. 50, 40 ft.
22 2 ft. 1 in.   23. $4\frac{1}{2}a$ 24 Rs 32. 8a.
25 Rs 32   26 15 in. square , 744 tiles.
27 18 in. square , 525 tiles
28 Rs. 15326 10a 8p Rs. 7340   29 £39 3s 9d
30 10 yds , 22 yds 31. 16 ft   32. Rs 24
33. Rs. 51 9a 34 Rs 31. 4a 35 25, 16 ft

EXERCISE 79

1 2624 sq. ft , Rs 95. 10a 8p 2 Rs 136 7a 4p
3 Rs 4. 1a 4. Rs. 595   5. Rs. 610.
6. Rs 34 15a.   7. Rs 124 7a. 1$\frac{1}{3}p$
8 5226 sq ft   9 110400.   10 210, 150 yds
11 5 ft   12 £491 8s 13 4 ft.
14 5 ft   15 4 ft.   16 30 yds , $2\frac{1}{2}$ ft
17 11 ac 2929 sq. yds , Rs 877. 10a 3p.
18. Rs. 65.   19 Rs 666 12a. 20. Rs 312 12a.
21. Rs. 458.

EXERCISE 80

1 792 sq ft   2 1170 sq ft   3 1170 sq ft
4 972 sq ft   5 120 yds   6 136 yds

ANSWERS

7. Rs 52. 12a. 5½p.   8. Rs. 10 6a 8p
9. £55 10s 3 17/13 d.   10. Rs 4 8a. 4p.   13. 12 ft.
11. 29 in   12. 8a per yard   16. Rs. 17 2a
14. 15 ft.   15. 702 sq. ft   18. Rs 20 4a.
17. width 18½ ft , height 14½ ft.
19. 25, 20, 16 ft.   20. 40, 32, 24 ft.

EXERCISE 81.

1 Rs 5 7a 1½p   2. Rs 10 15a. 6p   5   16 ft.
3 Rs. 19 12a. 9¾p. 4 Rs 17 8a.   8   18 ft
6 19 ft   7   24 ft.   11. 3024.
9 24, 16 ft.   10 20 ft
12 32 ft   13. 1066 yds. 2 ft   16. 25 ft 20 ft.
14 Rs. 20 4a   15 4 ft   19 20 ft.
17. 25 ft , 20 ft.   18. 15 ft   22. Rs 54.
20. 15 ft   21. Rs 24.   25. 20 ft
23 Rs. 81. 4a.   24 Rs. 20   28 Rs 237 8a.
26 Rs 6 4a   27. Rs 20 13a
29 8000 sq ft , Rs 673 12a   31. Rs. 66. 10a 8p
30 301 ac 2424 sq yds , Rs 7340
32. Rs 15326 10a 8p   34 16 ft , 10 ft   37 £3. 5s 10¾d.
33 12 ft 7½ in.   36 10 ft.   40. 12¾ ft
35 5 ft   39. Rs 1120.
38 14 ft
41 £3 3s 9d.

EXERCISE 82

                                               3   75 sq. ft.
1. 24 sq yds.   2   10½ yds   6   13½ sq ft.
4   44 sq yds.   5   90 sq ft.   9   154 sq. ft
7   28 ft   8   24½ ft   12. 11 ft
10 6 928 sq ft.   11 10 ft 8 in   15 217 sq ft
13. 96 ft   14. 187 083 sq ft.
16. 40 ft   17 22⅗ , 14 14/30 , 10 ft
18 1082·5 sq ft   19 25 ft   20 10 ft. 9 9 in.
21. 52½ sq ft   22 70 sq ft   23 11 ft. 3 in.
24 160 sq ft   25 15 ft 4 in.   26 17½ sq. ft.
27. 15 ft.   28. 33 ft   29 44 ft
30 14 ft   31 14 ft   32 38½ sq. ft
33. 3850 sq yds   34 5 ft   35 60, 80 ft
36 32 ft   37 18⁴⁄₇ sq. ft.   38 3 55 4
39 3 sq. ft 63 sq in   40 210 sq. lks.

EXERCISE 83

1   18 cu. ft.   2   6 cu. ft.   3   108 cu. in.
4   75 cu in.   5.   11¼ cu ft.   6   240 cu. cm.
7   1 cu ft.   8   1 cu yd   9.   10 cu. yds.
10   48 cu ft   11   (a) 64 cu in. (b) ⅓ cu. ft
     (c) 27 cu. yds   12   27 13 ft.
13.   15 588 ft   14   746 sq in.   15   41½ sq. ft.
16.   28⅞ sq ft   17   108 sq. ft 54 sq in
18.   1 cu yds. 10 cu. ft 64 cu. in   19.   72 cu. yds.
20   68 cu yds 14 cu ft   21.   17280.
22   20412   23.   10530   24   Rs 2 12a 5p.
25   Rs. 1 13a. 6p   26   Rs 8 1a
27.   52 tons 14 cwt 2 qr 21 lb
28   4 tons 16 cwt. 1 qr. 20 lb.   29   234.
30   Rs 46   31.   10 cu. ft
32.   1 cwt 1 qr. 11⅘lb   33.   2970
34.   4 hrs. 3 min.   35   4½ miles   36   630.   37.   60.

EXERCISE 84.

1.   2½ ft   2.   2 ft.   3   2 ft
4   1 ft 8 in.   5   4 ft   6.   1 ft. 8 in.
7.   4 ft. 6 in   8   5 ft 6 in.   9   2 ft. 6 in
10   6 ft.   11   1½ ft.   12.   1¾ ft.   13.   18 ft.
14.   1½ in.   15.   2¼ ft   16   11 ft.   17.   6 in.
18.   2 in   19   3 ft 9 in 20   1¼ in.

EXERCISE 85.

1   105 cu ft   2   960 cu ft   3.   150 cu. in.
4.   1540 cu in.   5   44 cu in.   6.   25 cu. in.
7   8 cu ft 9 cu in. 8   756 cu ft   9.   240 cu in.
10   30 cu. ft.   11   594 cu. in.   12   792 cu. in.
13.   4620 cu ft   14   1925 cu ft.   15   35 ft
16   3½ ft.   17   704 cu. ft.   18.   14080
19.   11 cu. ft. 1188 cu in   20   18 cu. ft. 576 cu in
21.   7 in   22   8 in   23   2420.
24.   1⅔.   25   3⅓ ft   26.   100   153.

EXERCISE 86.

1   Re 1. 15a   2   11a.   3.   1 min. 48 sec.
4.   Rs. 26 4a   5.   Rs. 196. 8a

Rs 2 1*a*, Rs 30 15*a*   7 Rs 3 10*a* 6*p*
Rs 6   9 Rs. 45   10 6035 metres nearly
Rs 97 2*a*   12 127½   13 Rs 3. 1*a*
26   15 304   16 £1 15*s*
Rs 21 14*a* 18 2*s* 2¼*d*   19 Rs 17 8*a*
1 min 22½ sec   21 Rs. 5 14*a* 7½*p*
24 miles 4 fur 28 poles   23 60
Rs 403 9*a*   25 $\frac{15}{34}$ 26 Rs 101 4*a*
Rs 1009 5*a* 28. Rs 407. 11*a* 4*p*
Rs 984 15*a* 4*p*   30 3 hrs 10 min
9   32 Rs 517 8*a*   33 183⅓ miles.
7½   35 3   36 Rs 30   37. Rs 60

EXERCISE 87

20.   2 28⅓   3 38⅖   4. 42 days
10 days   6 8⅜ days   7 25⅘ days. 8 11¼ days
28   10 700   11 38¼ days 12 28 days
48 men   14 6 months 15 5 hrs 12 min
30   17 24   18 64
£3 7*s* 10*d*   20 2⅔ chk   21 Rs 87 7*a* 8 5/6*s*3*p*
101   23. 4 miles   24. 14 days
202½ miles   26 11⅓
1600   28 13¼ days   29 26⅚ days

EXERCISE 88

8⅓ days   2 21   3 9
8⅘   5. Rs. 480   6 £545 6*s* 3*d*.
Rs 33. 5*a* 4*p* 8 6   9 12⅓
60   11. Rs 37 8*a*   12 100.
⅓ hr a day 14 6 days   15 9
36 days   17 2   18 7
Rs. 34 6*a* 20 63 days   21 48 oz
15   23 15   24 18 days
Re 1. 8*a* 26 9 months   27. 2 days
540   29 48 days   30 25⅞
32   32 14   33 45
7⅓   35 27 days   36 125
1250   38 Rs. 5096.   39. 84. 40 32 men.

EXERCISE 89

1. Rs. 72.   2. 48   3 36.   4. Rs 16.
5. 10.   6 24   7 Rs. 68. 12a. 8. 9 days.
9. $7\frac{1}{2}$ days.   10 82 fr 50 c.

EXERCISE 90.

1. 3 1.   2 3 4.   3 4 5.   4. 5 : 9.
5. 12 13.   6 3 1.   7 11 4.   8 4 1.
9. 1 · 15.   10. 8 · 9   11. 5 6.   12. 15 20.
13. 16 27.   14 81 64   15. 216 343 16. 9 16.
17. (a) $4\frac{1}{2}$ (b) $3\frac{3}{5}$. 18. 25 39. 19 91 · 81. 20 9 . 16.
21 21 32 , 105 126 144 . 160
22 12 . 18 20 25   23. 20, 25.   24 6, 10.
25. 21, 42, 35 26. 5 4   27 3 28   28. 15 : 25.
29. 25 24.   30. 15 30 40 48, 5 16.

EXERCISE 91

1. 12.   2. 9   3. 30.   4 18   5 16.
6. 84   7. 42   8 54   9 120.   10. $3\frac{3}{4}$.
11. 16   12. 12.   13 No   14. Yes.
15 217 216   16 50 , 3rd term $= \frac{8}{\overline{5\cdot8}0}$.
17. 1 3. 18. 217 865   19 £3.   20. $\frac{2}{5}$ lb.

EXERCISE 92.

1. 6 days   2 $3\frac{7}{11}$ days.   3 60 days.
4 $9\frac{2}{3}$ days.   5 $10\frac{4}{7}$ days.   6. $13\frac{1}{3}$ days.
7. 8, 10, 12 days 8 42 days   9 4 days.
10 $7\frac{1}{2}$ days.   11. 60 days   12. 15 days.
13. $16\frac{2}{3}$ days.   14 16 days   15. 3 days
16. $33\frac{3}{5}$ days   17. 6 days.   18. 5 days.
19. 8 days   20 $11\frac{8}{13}$ days.   21. 72, 36 days
22. $93\frac{1}{3}$, 40 days.   23. 24, 72, 36 days.
24 20, 25, 30 days.   25 20, 30 days
26 $43\frac{3}{4}$, $58\frac{1}{3}$ days.   27. 40, $18\frac{2}{11}$, 50 days.
28 $28\frac{4}{5}$, 36, 48 days.   29 5 days
30 $10\frac{10}{11}$ days.   31 16 days.   32. 2 1.
33 $21\frac{8}{17}$ days   34. 8 hours   35. 10 days
36. 16 days.   37. $7\frac{7}{50}$ days.   38. 60 days
39. 480, 600 days   40. 8 days.   41. 51 %.

EXERCISE 145.

1 10. **2.** 5. **3.** 4 **4** 5. **5** 4. **6** $3\frac{1}{3}$.

EXERCISE 146.

1. £2000 **2** £800 **3.** Rs 2000 , 10 p c.
4. £250, 4 p. c **5** £4600 **6** Rs. 32000
7 £315. **8.** £13. 12s. **9** £1000.
10 £1156 **11** 4 **12** Rs. 4+10 **13** £4394.
14 Rs 420, Rs. 400. **15** Rs 3200, Rs. 3889 9a. 11'04p
16. Rs 16400 **17** £3125, 3250. **18** £500
19. £2205, £2100, £2000. **20** £3497 5s $4\frac{16}{61}d$
21 A Rs. 23409 , B Rs. 22500.

EXERCISE 147.

1 Rs 70 **2** £51 5s **3.** £70 17s 6d.
4 Rs. 19 4a. **5.** £296 13s $4\frac{1}{4}d$. **6.** £200.
7. £800 **8.** Rs 216. 10a. 8p **9** £576. 18s. 9d
10. £575. **11.** £8000 **12.** Rs 504 8a 11p.
13 £188 13s $5\frac{1}{4}d$ **14.** Rs. 260. 15a. **15** Rs. 8000.
16 £237. 10s. **17.** Nothing. **18.** £963 19s. $10\frac{454}{35}d$.
19. Rs 6250.

EXERCISE 148

1. Rs 41204 4a 8p **2** £520. 17s 6d.
3. £621 3s 4d **4.** £505 5s
5 Rs. 872 5a **6.** £652 11s
7. £450. 13s. 4d. **8.** £10780. 14s 8d.

EXERCISE 149

1 2 years **2.** 9 months **3.** 146 days.
4 $3\frac{4}{5}$ years **5.** 4 months. **6** $3\frac{1}{2}$ yrs **7.** 20 months.

EXERCISE 150

1. 4 p. c. **2** 4 p c. **3** $5\frac{1}{4}$ p. c.
4 4 p c **5.** $4\frac{1}{3}$ p c. **6** $3\frac{1}{4}$ p. c.

EXERCISE 151

1. 4 years. **2** 4 months. **3.** Rs 3218. 12a.
4 2 years. **5** $4\frac{1}{2}$ **6.** 15 years.
7 £3750 , $4\frac{1}{2}$ p c. **8.** $6\frac{1}{4}$
9 Rs 180. **10.** 12 months.
11 (i) Rs 45. 13a 4p (ii) Rs 13 1a $6\frac{2}{7}p$.

2   80 83, Rs 320   13 $9\frac{1}{11}$
4   10 months   15   £130   16   Rs 9 9a $10\frac{3}{13}p$
17 £2500   18. $1\frac{1}{2}$ p c gain. 19 $8\frac{5}{3}$
20 25 p c   21 $£17\frac{13}{41}$.   22   £720 , $4\frac{1}{6}$ p c
23 £375. 10s.   24   17s. 6d   25   Rs. 4556. 4a

EXERCISE 152

1   5 per cent   2 $6\frac{1}{4}$ p c.   3   Rs. 108000
4   £1 2s 10d.   5   £515   6   £65 13s $6\frac{1}{2}d$
7   $4\frac{3}{4}$ 8. £508 15s , 3% 9   Rs 9.
10 Rs 255   11 £6020   12. £59 7s. 6d
13 31st May   14 $11\frac{1}{9}$ per cent 15. $4\frac{1}{8}$ %.
16 73 days   17 $2\frac{1}{2}$ per cent.
18. Rs. 20150 , 3 p c.   19. £17 8s. $5\frac{3}{8}d$, £1742 5s.
20 $8\frac{1}{3}$ % , £300

EXERCISE 153.

1   8a   2   12a   3   5a. 4p.
4. 10a 8p   5   12a , Rs 5346
6   5a. 4p., Rs 8840, Rs 15446, Rs 5038 2a 8p
7   10 8p , Rs 5981 5a 4p   8 Rs 25200
9   Rs 13900.   10 £900   11 Rs 26891.
12 Rs 245 1a   13 Rs. 14400, Rs 12000
14. Rs 3000, Rs. 2500.   15. £2760.
16 £7500.   17. 10a. 8p   18   15s
19 £320   20 £267

EXERCISE 154

1   Rs 1473. 8a.   2   Rs 1144. 13a 8p.
3   Rs. 39 1a.   4   Rs 2697. 14a 8p
5   Rs. 4700. 6. Rs. 1750 8a. 6p 7. Rs 1460. 15a.
     Rs $1600\frac{17}{31}$ 9. £2500   10 $£2252\frac{55}{473}$
     Rs. 1259. 6a 12. £1800.   13   Rs 1844.
.. 4d   15. Rs 4   16. £20   17 £5 18 Rs 855

EXERCISE 155.

1   Rs. 4. 0a. $9\frac{3}{5}p$.   2. Rs   528. 13a. $\frac{12}{25}p$.
3   2   4   Rs 3700.   5   Rs 600
6   Rs. 15000   7 $3\frac{1}{2}$.   8   Rs 90
9. Rs. 73. 7a.   10. Rs 7525.   11. Rs. 25000.
12. Rs. 14550.   13. £1900.   14. Rs 700
15. Rs. 766. 10. 8p.

EXERCISE 155 (A)

1 Rs 23400   2 £1867 10s
3 Rs 52800   4 10⅔d
5 7s 6¾d   6 1˙5 frank per rupee
7. £85 17s. 4d   8 Rs 3972
9 ⅕   10 Rs 200.
12 1s 2d.   13. Rs 120000
14 Rs. 10020 5a.   15 ²⁵⁄₁₃₃, Rs 64=£5.
16 Rs 13⅔ Rs 960, Rs 28₁⁴₇. 17. 15¹⁰₁₁, guinea
18. 2 5 frank   19 1s 10¼d
20. Rs 62 9a ¹²⁄₁₃p.

EXERCISE 156

1 Rs 1840   2 Rs 2472.   3 Rs 3960
4 £5418 3s 9d   5 £588 5s
6 Rs 3078 13a 6p   7 £9175 12s 6d
8 Rs 1225 1a 6p   9 Rs 1881. 4a
10 £2412. 5s 7¼d

EXERCISE 157

1 Rs 14100   2 £15573 8s 10¼d
3 £91004 6s 8d   4 Rs 10793 12a   5 Rs 6825.

EXERCISE 158

1. Rs 9000   2 £1000   2 Rs. 4500   4 £7440
5 110   6. 3419 fr 58 c   7. £5300

EXERCISE 159

1 £238⅞   2 Rs 50 13a 4p   3 112⁷⁄₃₋
4 Rs 70   5 Rs 6250   6. Rs 7500.
7 Loss £5   8 Loss Rs 75

EXERCISE 160

1 Rs 1368 8a   2 £2 1s. 8d   3 Rs 43 8a.
4. £83 6s 8d   5 £5750

EXERCISE 161

1 Rs 44072 1a 4p 2 Rs 9000   3 Rs 16800
4 Rs 11250   5 £1942 10s 6 Rs 122169.

EXERCISE 162

1 84   2 94½   3 Rs 104 4a
4 77⅓   5 90   6 105

EXERCISE 163.

1	Increase Rs. 20.	2.	£26. 13s 4d.
3	Increase Rs 34. 8a	4	No change.
5.	Rs. 1250, Rs. 2½ less	6	£ 60 increase.
7.	Rs. 47 increase.	8	Rs. 100 increase.
9.	Rs 31 4a.	10	Increase Rs. 14. 8a
11	Increase £6 13s. 4d.	12.	Rs. 355.
13.	Increase Rs 300	14.	Decrease 7s
15.	Increase Rs 310	16	£53 6s 8d increase
17	Rs. 3800 , Rs 11 decrease		
18	£3$\frac{3}{14}$	19.	Rs. 150 increase.
20.	£1. 12s.	21.	Rs 19 13a 4p. decrease

EXERCISE 164

1	£122. 3s 4d	2.	80	3	The latter.
4	1st , £1342. 10s , 3$\frac{6}{17}\frac{3}{0}$ p. c.			5	£100
6.	Equal.	7.	£78	8.	Rs 19992
9	110$\frac{15}{16}$.	10	4½.	11.	The latter, Rs. 60
12	4½.	13.	£4725.	14	Rs. 23400
15	£3429 , £135 , £139. 14s			16.	86$\frac{31}{41}$
17	Rs 34 decrease.	18.	None.	19	£2500.
20	Rs 1638.	21	Rs. 21735	22.	106$\frac{11}{10}$.
23.	Rs 49700.	24	4%	25	£25000
26	£818 8s.	27	Rs. 25200	28.	Rs. 151710
29	Rs 27000 , Rs 185 more			30.	£24 increase.
31.	Rs 16000 , Rs 24000			32	£397 increase.
33	Rs. 13200	34	(i) £4 16s (ii) 35 34		
35	Rs 39333 5a. 4p			36	Rs 38200.
37	88$\frac{7}{8}$	38	£215.	39	Increase £47.
40.	Rs 30000	41.	85$\frac{1}{4}$.	42.	94$\frac{7}{8}$
43	£6000	44	£257. 5s. 5d.		
45	Rs 4800 , Rs 5200			46	82$\frac{1}{4}$.
57	Rs 30780	48	156	49	10 p. c.
40.	Rs 945 , Rs. 35190			51	£2852 , £25935.
42	Rs 12960 , 11220			53	Rs. 480000
55.	£25935.	55	Rs. 136$\frac{1}{8}$		

EXERCISE 165.

1.	11344000.	2.	1 526.	3.	9.	4.	810000.
5.	1.	6.	·0625.	7.	100	8	17$\frac{13}{36}$
9	·5.	10.	1.	11.	·2.	12.	13$\frac{5}{6}$.

| 13 | 100 | 14 | 1 | 15 | '1 | 16 | ½ |
| 17 | 11/30 | 18 | 1 | 19 | 06 | 20 | 1. |

EXERCISE 166

1	102	2	100	3	17a 8p
4	Rs 15	5	Rs 40	6	Rs 7 , 10.
7	60 days	8	18 days	9	25 days
10	8 days	11	2⅔ days	12	266⅔ days.
13	Man 7½ hours, boy 18 hours, together 5¼ hours

EXERCISE 167

1	309	2	3 146	3	3 968	4	17 796
5	1 366	6	3 827	7	434.	8	4 051.
9	1	10	972	11	0	12	√3.
13	∜15	14	294 151	15	9√77

EXERCISE 168

| 1 | 153 miles. | 2 | 35 ft. | 3 | 5 ft 7½ in | 4 | 32⅓ in. |

EXERCISE 169

1	60°C	2	94 miles	5	30 m , 12 m
6	9¾ hrs from A's start, 7½ hrs and 12¼ hrs from A's start
8	9 sec.	9	67 hrs

EXERCISE 170

1	Wednesday	2	Tuesday	3	Friday
4	Sunday	5	Monday.	6	Tursday
7	Friday.	8	Tuesday	9	Sunday
10.	Monday	11	Tuesday.	12	Thursday
13	Wednesday	14	Sunday.	15	Sunday
16	2, 9, 16, 23, 30	17	Yes

MISCELLANEOUS EXERCISES III

I

1	£10 8s	2	Quotient 17430, Rem, 13
3	'03212	4	£1682	5	72 men , 288 women
6	Rs 1500	7	8s 4d	8	10. 15 a. m.
9	11¾.	10	£3990

II.

1. 14 days 7 hrs. 11 min. 17 sec., 2674 days 9 min. 59 sec
2. Tuesday   3. $\frac{1}{4}$.   4. 5s. 3d , 0037115625
5. 46·947177.   6 25640000.   7. Rs. 9672.
8. £1. 13s. 4d.   9. £120   10. £1350.

III.

1. 30, ·75   2. $\frac{1}{151}$ and $\frac{1}{3}$ 3. Rs. 86. 12a. 8p , 266·6
4. Tursday 7th February 1833.
5. 999984, 100149, 753.   6. 6 pies in a Re.
7. $33\frac{1}{3}$ 8 320.   9. 4 years. 10. £15400.

IV.

1. 2s 7d   2 ·08125, ·0003, ·038961.   3. 8.
4 160 yards.   5 80   6 ·+91824.   7. 104.
8. 49. yds.   9. £3. 7s. 9d., 364 of each
10 Decrease Rs. 1000.

V.

1 $1\frac{1}{4}$.   2. £50. 12s. $6\frac{7}{16}d$.   3. ·03.
4. $611\frac{1}{9}$.   5. $\frac{3}{8}$ gallon.   6 5 p. c.
7 96000 yds   8. $26\frac{6}{19}$ 9. Loss 80 p. c.
10 £31. 5s.

VI.

1. 151249 7.   2 105570 , 950370. 3 1.
4 1a 4p., 114583   5 Loss $16\frac{2}{3}$ p. c.   6 £1 14s.
7. 180 , 160 , 200   8 £5565   9. $2\frac{3}{4}$ miles.
10 Rs. 39440 , Rs. 1560.

VII

1. Men Rs 72. 14a $2\frac{3}{8}p$. , Women Rs 85 5a 4p.
    Boys. Rs. 81 12a. $5\frac{1}{8}p$.
2. ·0725.   3 20150   4. 10a.
5 Rs. 510.   6. $16\frac{1}{3}$ hours.   7. Rs. 80.
8. 19s. 3d.   9. 8s 4d.   10. $3\frac{1}{3}$ p. c.

VIII.

1 720   2. Rs 12500   3. $\frac{1}{16}$
4 $\frac{5}{19}$ 5. 0.   6 Rs 15000
7. £690   8. 6 months   9 3.
10 £13800.

IX.

1 5   2 60   3 2 seconds.   4 ¾d. , 20
5 112428.   6 65 p c   7 Rs 640.   8 £4800
9 5a +p   10. 54.

X

1 0 , 565   2 13 11/13 min and 16 4/13 min past 3.
3 1½ ft   4 12   5. B , A will be 80 yds off.
6 A 120 sec , B 132 sec   7 Three
8 40 75   9 4 of inferior with 5 of superior 10 £2387.

EXERCISE 171.

13   1 09   14 $\log 2 + \log 3$

EXERCISE 173

1   3 1470.   2   1 6131   3   2 5316.
4   3 4830   5   2 4829   6   2 6053.
7   1 3010   8   0 3010   9   0 9031.
10   2   11   104 8   12   1346
13   001048   14   01346   15   00007773
16   19650   17   1 014   18   1003.
19   01025   20   003638

EXERCISE 174

1   (ı) 4, (ıı) 1, (ııı) 2, (ıv) 1, (v) 1
2   (ı) 8821, (ıı) 2 8821, 3 8821, 5 8821
3   5,   2,   4,   1,
4.   (ı) Second, (ıı) unity place, (ııı) sixth decimal place
5   25940   6   33480   7   2041
8.   6 330   9.   6933   10   12440
11   5555   12   62 34   13   1 696
14   0 5223   15   3 877   16   0 1312.
17   2 287   18   397 7   19   1 069
20   9663.   21   42 79.   22   1'187
23. 16390   24   2337.   25   5 838
26   3 371   27   1 395   28.   1 205
29   007176   30.   05999   31   4409
32   1 9484   33   15 84   34   1 0039
35   9 076.   36   178'1   37   48
38   4467   39   5 113   40   Rs 1050

MATRICULATION EXAMINATION PAPERS.

1 (a) Express 7009056700 in words

(b) State in figures the number

Five thousand million, seven hundred thousand and twenty eight

2 (a) Simplify $\dfrac{3\frac{5}{9}-1\frac{7}{4}\text{ of }\frac{9}{3}\frac{4}{5}}{11\frac{2}{3}\text{ of }\frac{9}{14}\text{ of }\frac{2}{5}}-\dfrac{4\frac{1}{4}-7\frac{5}{4}+3\frac{2}{3}}{\frac{5}{9}\text{ of }12}$

and express the result as a recurring decimal fraction

(b) Get the product of 1+416 and 16515 in two lines only

3. (a) Find the square root of $\frac{7}{8}$ to three places of decimals.

(b) What must be the least number of soldiers in a regiment that will allow it to be drawn up 10 15 or 25 deep, and also to be formed into a solid square ?

4 (a) The difference between simple and compound interest on a certain sum of money for 2 years at 4 p c. per annum is one rupee   Find the sum

(b) A bill is drawn for Rs 5050 on June 12th at five months   It is discounted on Sept 3rd at 5 per cent per annum   How much does the holder of the bill receive and what is the gain of the banker in the transaction (Bankers discount being allowed)

5 (a) A man's net income, after paying income-tax at the rate of $8p$ in the rupee, is Rs 950 $10a$ $8p$ What will be his net income when the tax is reduced by 2 pies in the rupee ?

(b) Find by practice the value of 61 maunds 37 seers 8 chk of goods at Rs 17 $5a$. $4p$ per maund.

1936

1   (a) Write down the local value in fraction of each of the figures in the number 0 0103

Or,

Simplify $\dfrac{2\frac{3}{4}+5\frac{7}{9}}{1\frac{1}{2}-\frac{4}{9}}-\left(\ \frac{5}{9}\text{ of }\ \dfrac{3\frac{1}{2}}{4}\right)\times\dfrac{2\frac{3}{16}}{32}.$

(b) What decimal of an hour is a second ?

2   (a) Find, by Practice or otherwise, the price of 3 cwt. 2 qrs 16 lbs. at £3 7s. 8d. per cwt.

(b) Define the L C M. of two or more integers. The mint price of gold is £3 17s 10½d. per ounce   Find the smallest exact number of ounces that can be coined in to an exact number of sovereigns.

3   (a) By selling goods at £31 a merchant loses 7 per cent on his outlay. Find his percentage profit on his outlay when he sells the same goods at £35

Or,

(a) Find, correct to the nearest hundred weight, the weight, of lead required to cover a flat roof 40 ft long and 32 ft wide with lead 0 2 in. thick, a cubic foot of lead being taken to weigh 12,000 ounces.

(b) In what proportion must tea worth 4s per lb be mixed with tea worth 2s 10d per lb. that the mixture may be worth 2s 6d per lb

4   (a) A certain sum of money at simple interest amounts to Rs 505 2 in 2 years and to Rs 589 4 in 4 years Find the principal and the rate of interest.

(b) Find the banker's discount on a bill for £ 734 15s. drawn on January 17th, 1896 at, 3 months and discounted on February 7th, at 2½% per annum.

5   A man invests £20,420 partly in shares at 125 bearing a dividend (free from income-tax) of 7 percent and partly in a mortgage at 5 per cent interest, on which he pays income-tax at 4s in the £. His net income from each investment is the same.   What is his whole income ?

1937

1 (*a*) Find the square root of $\dfrac{16\ 9}{25\ 6} \times \dfrac{0\ 678 \times 9\ 01}{0\ 234}$

(*b*) Find the greatest number of 4 digits which, when divided by 7, 10, 15, 21 and 28 leaves 5, 8, 13, 19 and 26 respectively as remainders Express the answer in Roman numerals.

2 (*a*) Find, by Practice or otherwise, the cost of 25 bales at Rs 9 10*a*. 7*p* per maund if a bale contains 13 mds 24 srs 12 chhataks

(*b*) Express $\frac{5}{8}$ of 12*s*. 6*d*.+0·625 of 7*s* 6*d* −0 505 of 16*s* 6*d* as the decimal fraction of £1 Write the local value of significant digits in the answer

3 (*a*) Two equal sums are lent at the same time at 4 per cent and 3 per cent. simple interest respectively. The former is recovered 2 years earlier than the latter, and the amount in each case is Rs 620 Find the sums and the time.

(*b*) On what sum does the difference between the simple and compound interest for 2 years at 5 per cent. amounts to Rs. 15 ?

4 (*a*) A man undertakes to do a certain work in 150 days He employs 200 men. He finds that only a quarter of the work is done in 50 days How many additional men should he employ so that the whole work may be finished in time ?

(*b*) A reduction of 20 per cent in the price of oranges would enable one to purchase 48 more for Rs 2 8*a* Find the rate per dozen.

5 (*a*) What is the face value of 3 months' bill when banker's discount at 3 per cent per annum is Rs 18 ?

Which is the better investment $2\frac{1}{2}$ per cent. consols at 55, or 3 per cent stock at 63 ?

(*b*) A garden roller 16 in wide, with a girth of 44 in. is made of cast iron $\frac{1}{2}$ in. thick Find its weight if one cubic foot of cast iron weighs 450 lbs

Or

Find the day of the week on 9th March, 1592.

1938.

1. (a) Find correct to 6 decimal places the value of

$$\frac{1}{1.2} + \frac{1}{2.4} + \frac{1}{2\,4\,6} + \frac{1}{2\,4\,6.8} +$$

(b) Express $\frac{3}{4}$ of 7s 6d +1 25 of 5s —·545 of 9s 2d as a decimal fraction of £10

(c) Find the least multiple of 17 which leaves a remainder 2 when divided by any of the first 6 even natural integers.

2 (a) Find, by Practice, or otherwise, the value of 31 cans of mercury each containing 5 mds 25 srs., 10 chks., at the rate of Rs 5 10a 8p. per maund

(b) The cost of carpeting a room is Rs 72, and papering the walls at 1a, 8p per square foot is Rs 106 4a The length of the room is 18 ft. If the width had been 4 ft less, the cost of the carpet would have been Rs 18 less. Find the height of the room.

3 (a) Three men borrow Rs 2550 in all from a money-lender at 5 per cent, interest and pay back equal amounts in full settlement of their respective debts after 4, 10 and 12 years respectively Find the sum borrowed by each.

(b) A man borrows a certain sum and pays back in 2 years in 2 equal instalments. If compound interest is reckoned at 5 per cent and if he pays back annually Rs 441, what sum did he borrow ?

4. (a) A can do a piece of work in 16 days, B in $12\frac{4}{5}$ days, and C in 32 days All begin to do it together but A leaves after 4 days, and B leaves 3 days before the completion of the work. How long did the work last ?

(b) A man sells an article at 20 per cent. profit If he had bought it at 20 per cent. less and sold it for 10s. less, he would have gained 25 per cent Find the cost price.

5 (a) The banker's discount on a bill due 9 months hence is Rs. 21, and the true discount on the same at the same rate is Rs. 20. Find the sum and the rate

(b) Which is the better investment, 3 per cent consols at 66 or 4 per cent. London Stock at 84 ? What equal sums of money must be invested in each to make a difference of £100 in the yearly dividends ?

6 (a) Two open cylindrical metal pipes have equal internal volumes. The external and internal diameters of one are 13 and 12 inches, and the corresponding diameters of the other are $6\frac{3}{4}$ and 6 inches   Compare the quantities of metal in each pipe

(b) What day of the week was 20th June, 1836 ?

1939.

1 (a) Simplify $5\frac{1}{3} + 7\frac{3}{7} \times 4\frac{7}{13} - 2\frac{3}{5} - \frac{4}{7}$ of $\frac{8}{3}\frac{1}{2}$

(b) Which is the greater of $\frac{455}{400}$ and $\sqrt{\frac{9}{7}}$ and   by how much ?

(The answer should be correct to two significant figures.)

(c) Find, by Practice, the cost of 9 sacks of wheat each weighing 2 maunds, 20 seers at Rs. 6. 4as 6ps per maund

2 (a) The G C M of two numbers is 26 and their L C. M is 4284   If one of them is 204, find the other

(b) The difference between the Simple and Compound interest on a certain sum is Rs 2, 8as for 2 years at 5 per cent   Find the sum

3 (a) What day of the week was 27th July, 1922.

(b) The difference between the interest and discount on a certain sum of 6 months at 4 per cent. is Rs 2.   Find the sum

4 (a) A, B, C and D enter into a partnership   A subscribes $\frac{1}{3}$ of the capital, B $\frac{1}{4}$, C $\frac{1}{5}$, and D the rest. How should they divide a profit of Rs 6640, 10as ?

(b) In what ratio must I mix two teas worth Re 1, 2as and Re. 1, 12as per lb respectively so that by selling the mixture at Rs 2 per lb   I may gain 20 per cent. ?

5 (*a*) A medical student has to secure 40 per cent. marks to pass   He gets 40 and fails by 40 marks   Find the maximum marks

(*b*) How much should a person invest in $3\frac{1}{2}$ per cent stock at 25 in order to secure an annual income of £665 after paying an income tax of one shilling in the pound ?

6. (*a*) If by selling milk at Rs. 7 8*as* per maund a milkman loses 10 per cent. ? at what price per seer must he sell it to gain 10 per cent. ?

(*b*) How many cubic feet of earth must be dug out to make a well 21 feet deep and 4 feet in diameter ?   What will it cost to plaster its inner curved surface at 4*as* per square yard ?

Or,

(*b*) A student walks from his house at 4 miles per hour and reaches his school 5 minutes too late   If his speed had been 5 miles per hour he would have reached 10 minutes too early.   How far is the school from his house ?

1940

1 (*a*) Find correct to three decimal places the value of

$$\frac{1}{8} + \frac{1}{8^2} + \frac{1}{8^3} + \frac{1}{8^4} + \frac{1}{8^5} +$$

(*b*) Find the square root of

$$\frac{081}{0064} \times \frac{484}{6\ 25}$$

2 (*a*) Find by practice the price of 31,250 bricks at the rate of Rs 12. 5*as*. 6*p* per 1,000 bricks.

(*b*) The difference between the interest and the true discount on a certain sum of money for six months, at 4 per cent. is Rs. 2.   What is the sum ?

3. (*a*) Find what day of the week was the Armistice Day, 11th November, 1918.

(b) A grocer bought one kind of tea at Re. 1 0a 6p per lb , and the second kind of tea at 15as. per lb He mixed them and sold the mixture at Re 1 2as per lb Find the ratio in which he mixed them if his gain per cent. was $15\frac{5}{13}$.

4 (a) If I buy eggs at the rate of 1s 8d. a dozen and sell them at the rate of 5 for a shilling, what is my gain per cent ?

Or,

(a) A house is worth £2,500 and its contents £4,000 How much must be paid annually to insure the house and its contents against fire for 80% of their value at a premium of 7s. 6d per cent ?

(b) A boat moves downstream at the rate of one mile in 6 minutes, and upstream at the rate of 6 miles an hour What is the velocity of the current ?

5 (a) A man had two sons To the elder he left $\frac{5}{11}$ of his property, to the younger $\frac{5}{11}$ of the remainder, the rest to the widow Find the share of the sons if the widow gets Rs. 3,600

(b) What sum of money should be invested in 4 p c stock at 105 to produce a net income of £146. 5s. after paying an income-tax of 6d in the £ ?

6 The length of a rectangular plot of ground is three times its breadth and its area is 30 acres How long twould it take a man to walk across it along a diagonal at he rate of $3\frac{3}{4}$ miles per hour ?

Or,

A cylindrical cistern whose diameter is 1 ft. 9 in is partly filled with water If a rectangular block of iron measuring 1 ft 2 in in length, 11 in, in breadth, and 1 ft. in thickness is wholly immersed in the water, by how many *inches* will the water level rise

$$(\pi = \tfrac{22}{7})$$?

II. RAJPUTANA BOARD.
1935

1. (*a*) Find the greatest and the least number of six digits that are divisible by 251. Obtain their prime factors, and find their G C M.

(*b*) How many times can ·053 be subtracted from 14·578, and what will be the magnitude of the remainder ?

2 (*a*) The circumference of a wheel is 48 decimetres and it makes $2\frac{1}{2}$ revolutions per second   How long will it take to travel 45 miles, if 1 kilometre$=\frac{5}{8}$ of a mile ?

(*b*) Calculate the square root of $1+(·07)^3$ correct to four decimal places

3 (*a*) A person bought 15 maunds of wheat at Rs 5 a maund, and mixes 5 maunds of barley with it By selling the mixture at Rs 5 per maund he gains $6\frac{2}{3}\%$, find the cost of barley per maund.

(*b*) The average age of the boys in a school of 480 boys was 15 7 , 40 boys left the school, thereby diminishing the average to 15 2 Find the average age of those who left.

4 (*a*) A room 20 ft. by 15 ft has a carpet laid down so as to leave a margin 1 ft. 6 in wide all round   If the length of the carpet be 17 feet, find its width   Also find to the nearest penny, the cost of covering the above margin with cloth at 3*s*. 4*d* per square yard

(*b*) A garrison of 1,500 soldiers has provisions for 48 days. At the end of 13 days, a reinforcement arrives and it is found that now the provisions will last only for 25 days more   What is the reinforcement ?

5 (*a*) A person invested Rs 1600 for 3 years, and Rs 1,100 for 4 years at the same rate of interest (simple). The total interest from these investments was Rs 506 Find the rate per cent.

(*b*) If Rs 2,652 4*a* be due three years hence, allowing compound interest at 3 per cent per annum. What sum will be due at the end of the first year

1936.

1 (a) Simplify $\dfrac{£4\ \ 3s\ \ 4d.}{£4\ \ 6s.\ \ 8d} \times \dfrac{(\frac{1}{2}-\frac{1}{4})\text{ of }(\frac{1}{4}-\frac{1}{5})}{(\frac{1}{3}-\frac{1}{3})\text{ of }(\frac{1}{7}-\frac{1}{3})} \times (\ 4\dot{5}-\ 45)$

(b) 378 oranges and 462 mangoes are to be distributed among some girls so that each girl may get as many mangoes and as many oranges as another girl   Find the largest possible number of girls and the least possible number of fruits of each kind which a girl gets

2 (a) In an examination 20 per cent of the candidates fail in English, 25 per cent in Mathematics, and 10 per cent in both the subjects   Find the percentage of those who pass in both the subjects

(b) By selling a horse for £79 15s a man lost 8 per cent   For how much should he have sold it to gain 15 per cent ?

3 (a) The floor of a room is 50 ft long and 40 ft wide   Find the cost of supplying it with carpet 2 ft wide at Rs 4 2a per yd and oil-cloth 4 ft wide at Re 1 4a per yd , the oil cloth is to be laid along the sides and ends 5 ft wide the carpet to extend one foot over the oil-cloth

(b) Two pipes A and B can fill a cistern in 3 hours and 4 hours respectively , a waste pipe C can empty it in 2 hours   If these pipes be opened in order at 7 A M , 8 A M and 9 A M , find when the cistern will be filled.

4 (a) A person invests £5,740 in $3\frac{1}{2}$ per cent stock at $71\frac{3}{4}$ What is his yearly income, and how much per cent does he obtain for his money ?

(b) If the difference between the interest and discount on a sum of money for 2 months at $4\frac{1}{2}$ per cent be 2s 3d , find the sum

5 (a) A merchant gained annually at the rate of 20 per cent   At the end of 3 years he was worth Rs 5,400. What was his capital at the beginning ?

(b) How much a grocer mix two kinds of tea which cost Re 1 8a and Re 1 12a per lb respectively to sell the mixture at Re 1. 13a 4p. per lb in order to gain 10 per cent. ?

1937

1. (*a*) Express $\frac{3}{8}$ of 7*s*. 6*d* + 1˙25 of 5*s* — 545 of 9*s*. 2*d* s a decimal of

$$\frac{2}{2+\dfrac{1}{2-\frac{1}{3}}} \text{ of } £13.$$

(*b*) The records of a dairy of a certain gentleman show that his average daily expenditure for the first four months in the year 1936 was as follows —

		Re.	a	p		
January		1	5	4	per day	
February		1	4	0	,,	,
March	,	1	10	8	,,	,,
April	..	1	6	0	,,	,,

Find to nearest pie his average daily expenditure for these months taken together

2. (*a*) The length of a hall is 3 times the breadth. The cost of white-washing the ceiling at $5\frac{1}{2}d$ per sq. yd is £4 + 12*s*. 7 1*d*. and the cost of papering the walls at 1*s* 9*d* per sq yd is £35. Find the height.

(*b*) *A* and *B* can do a piece of work in 24 days. Had *A* been absent for the first 6 days, the work would have been completed in $27\frac{1}{5}$ days In what time could *A* alone do it ?

3. (*a*) A father wishing to divide a sum of £12750 between his two sons, David and Solomon who are 23 and 24 years old, divides it in such a way that if their shares are laid out at compound interest at 4 per cent they will receive equal amounts on attaining the age of 26 years. Find how much each will receive now and when 26 years old.

(*b*) In an examination, *A* gets 10 per cent. less than the minimum number of marks required for passing, *B* obtains $11\frac{1}{9}$ per cent. less than *A*, and *C* $41\frac{3}{17}$ per cent. less than the number of marks obtained by *A* and *B* together. What per cent of the minimum marks does *C* get ? Does he pass or fail ?

4 (a) A tongawala sold his horse for Rs 230 and his carriage for Rs. 85, thus gaining 5 per cent on his outlay Had he sold the carriage for its prime cost and the horse for Rs 235, he would have lost 5 per cent. on the whole Find the original cost of each

(b) A person invested £2500 at a certain rate per cent and £1500 at 1 per cent higher rate, and the total interest from these investments in 3 years was £525 Find the rates.

5 (a) A gardener had a number of shrubs to plant in rows At first he tried to plant 2 in each row, then 3, then 4, then 5, and then 6, but he had always one shrub left. On trying 7, he had none left What is the smallest number of shrubs he could have had ?

(b) If Rs 5 is allowed as discount off a bill of Rs 125 due a certain time hence What should be the discount allowed off if the bill had twice as long as to run ?

1938

1. (a) What annual payment will discharge a debt of Rs 440 due in 5 years, simple interest being reckoned at 5 per cent ?

(b) The true discount on a certain bill is ⅔ of the Trade discount and the rate is 4 per cent Find the time

2 (a) Divide Rs 5854, 8a between A and B, so that A's share at the end of 7 years be equal to B's share at the end of nine years, compound interest at 4% being allowed

(b) A person invests £4000 partly in the 2 per cents at 80 and partly in the 3 per cents at 90. The total income derived is £123⅓ , find the amount invested in each stock.

3 (a) A trader sells goods at a discount of 25% from the marked price and still makes a profit of 25% on the cost , at what per cent above the cost price did he mark the goods ?

(b) I sell an article at a loss of 20% Had I sold it for Rs 45 more I should have gained 4 per cent Find the cost of the article

4. (a) Two trains 88 yards and 44 yards long respectively are running on parallel rails at the rate of 15 and 20 miles an hour. In what time will they pass one another if they are running in opposite directions ?

(b) In a one-mile race A can beat B by 40 yards and B can beat C also by 40 yards By how many yards can A beat C.

5 (a) The length and the breadth of a room are 5 4. The cost of carpeting the floor, at 12a a square foot, amounts to Rs. 540 and the cost of painting the walls at 3a. a square foot is Rs. 162. Find the dimensions of the room.

(b) A rectangular cistern is 12 ft. by 8 ft. by 6 ft. Water flows into it through a rectangular pipe whose mouth is 2 in. by $1\frac{1}{2}$ in. If the water is flowing with a velocity of 6 ft per second, in what time will the cistern be full ?

1939.

1. (a) A owes B Rs. 1,200 , he pays Rs. 100 at the end of every month What amount will he still have to pay at the end of the year if the rate of the interest is 6 per cent simple interest ?

(b) A sum of money placed out at compound interest amounts to Rs 2,420 in 2 years and to Rs 2,662 in 3 years Find the sum and the rate of interest .

2 (a) How much stock must be transferred from the $3\frac{1}{2}$ per cents at $97\frac{5}{8}$ to 4 per cents at $101\frac{7}{8}$ in order to produce an increase in income of £33 a year ? (Brokerage $\frac{1}{8}$ per cent.)

(b) Find the banker's discount on a bill of £637. 10s drawn on March 16th, 1909, at 10 months and discounted August 26th, 1909, at 3 per cent

3. (a) Milk and water are mixed in a vessel A in the the proportion of 4 1 , and in a vessel B in the proportion of 5 2. What quantities should be taken from the vessels to have a new mixture consisting of seven seers of milk and two seers of water ?

(b) I bought two horses for £60   I sold one at a loss of 15 per cent and the other at a gain of 19 per cent , and then I found that each horse was sold for the same price   Find the cost price of each horse

4. (a) The length, breadth, and height of a rectangular room are in the ratio of 8   6   5 , if each of the dimensions be increased by a foot, the area of its four walls would be 1,408 sq ft   Find the dimensions of the room

(b) The external length, breadth, and height of a box of wood are 18, 10, and 6 inches respectively and the thickness of the   wood is half an inch   When the box is empty, it weighs 15 ℔ and when filled with sand 100 ℔. Compare the weights of equal bulks of wood and sand.

5 (a) A person finds that it takes him 5 times as long to row up a river as to row down it.   If the speed of the stream be 6 miles an hour, find how many miles per hour the person can row in still water

(b) A does half as much work as B, and C does half as much as A and B together   If all three work together, how many days will they require to do a piece of work which takes C alone 40 days ?

1940

1 (a) At an election, A expected to beat his only rival B by 300 votes , but 50 of his supporters did not vote at all and an equal number changed over to the other side. If A still wins by a margin of $15°/_o$ of the votes finally cast in his favour, find the votes for his rival.

(b)   A batsman has a certain average of runs for 11 innings. In the 12th inning he makes a score of 90 runs, thereby increasing his average by 5.   What is his average after the 12th inning ?

2. (a) A merchant marks prices on his articles, so that after allowing $10°/_o$ commission to the customers, on marked prices, he may still have a profit of $10°/_o$ on his outlay. What is the cost price of an article marked Rs 10 ?

(b) A grocer buys two kinds of tea at 3s 9d. and 3s per lb. respectively   In what proportion does he mix them, if by selling the mixture at 4s a lb he gains $20°/_o$?

3 (a) From a distance, I observe a railway train entering a bridge, a mile in length. If the train is half of the bridge in length, and clears it in 5 minutes, find the speed of the train

(b) From my place I walked to my school at 3 miles an hour and reached 20 minutes late   Next day, starting at the same hour I byked at 8 miles an hour and arrived 5 minutes before school time. How far is my school from my place ?

4 (a) The cost of whitewashing the walls and the ceiling of a room at $3p$ per square foot is Rs. 19. 12 *as* If its length, breadth, and height be proportional to 5, 3, 4, find the area of its floor

(b) Owing to scarcity of water, the municipality wants to reduce the supply of water by half, by replacing one inch diameter pipes by narrower ones. Calculate, correct to two decimal places, the diameter of the narrower pipes.

5 (a) A man has Rs. 400 in the post office on January 1   He deposits Rs 100 on the 1st of every subsequent month, except on August 1, when he withdraw Rs 500. What will be his final balance at the end of the year, if he stops deposits after the withdrawal ? Post office pays 3% p a simple interest added to the principal at the end of June and December

(b) A Weekly paper offers a prize of Rs. 5,000 down or Rs 2,000 down plus two instalments of Rs 1,800 each payable at intervals of six months   If the market interest be 10% p a , and interest be payable every six months , by how much is the one offer better than the other ?

6. (a) The true discount on a certain bill is five sixths of the trade discount and the rate is 4%. Find the time.

(b) A man sells £6,000 of $3\frac{1}{8}\%$ stock and buys 3% stock at 75, thereby increasing his annual income by £18. At what price did he sell his $3\frac{1}{2}\%$ stock ?

1 (a) Extract the square root of $\frac{3}{7}$ to four places of decimals

(b) A broker bought 425 bags of Juar at Rs 4 8a per bag He got his commission at the rate of of Rs $2\frac{1}{4}\%$ Find out how much amount the buyer had to spend

2. A box is externally 3 feet long, 2 feet broad, and $1\frac{1}{2}$ feet high   This box is made of planks one inch thick. How many sq feet of planks are used in the box and what is the price of the planks, at Rs 3 per cubic foot ?

3   A bill for Rs 900   due after 9 months was written on the 1st January and was cashed on 11 May at $5\frac{1}{2}$ per cent per annum interest. Find out the banker's gain, and also the amount received by the person who got it cashed.

4   Find out the ratio in which charcoal worth Rs. 12 +a. per ton he mixed with charcoal worth Rs 13 +a per ton so that by selling the mixture at 12 annas per cwt. there should be a gain of 20 per cent

5   When floor was sold at Rs. 3 per md 6 men could live for 8 days on Rs 10 , if the price of floor increases by Rs 2. 8a   per maund, for how many days can 4 men live on Rs 15

6   A train left A for B, and at the same time another train left B for A   The trains crossed each other after 4 hours   If the train coming from B to A travelled 16 miles per hour faster than the first and the distance between A and B is 216 miles, find out the speed of the trains

Show the motions of the trains on a graph, and find from your graph   their distances from A five hours after the start.

1936

1 The length of a room is 21 ft. The cost of paperng the walls at $2a$. $6p$ per sq. ft. is Rs. 118. $2a$ and the ost of carpeting the floor at Re 1 $12a$. per sq. ft. is Rs 551. $4a$. Find the breadth and the height of the room.

2 A man leaves a town A at 9 A. M and walks at 4 miles an hour towards town B which is 20 miles from A. At 10 A. M his friend leaves B on a bicycle and comes to meet him, cycling at 8 miles an hour. At 10-30 A. M. the walker rests for 15 minutes and then proceeds to walk as before. Draw a graph to show when and where they meet, and verify your results by calculation.

3 (a) A bank advances £1,500 to a person on an agreement that interest at the rate of 9 per cent. per annum shall be paid half yearly for its use The person fails to make any interest payment and at the end of eighteen months, the Bank obtains judgment against him for the principal and compound interest at the rate and on terms agreed to. Find to the nearest pence how much he has to pay.

(b) A bill for Rs. 750 was drawn on the 10th April for 8 months and was discounted in a bank on the 20th of July at 5 per cent. Find (i) the banker's discount (ii) the banker's gain

4. How much per cent. more than the purchase price hould a shopkeeper charge for his goods so that after paying a discount of one anna in the rupee he should have a gain of 5% on his outlay ?

5. A person engaged a workman for 35 days on 2 shillings 9 pence and food every day. It was agreed that for the days on which he did no work he would receive no pay and would have to pay 1s. 6d for the cost of his food At the end, the workman received £3 6s 6d. Find out the number of days for which he worked.

6. A invests Rs 3000 and B Rs 4000 in a shop. After 5 months A adds to his share $\frac{7}{9}$ of his capital, while after 6 months B takes out $\frac{3}{8}$ of his capital Find the share of each in a profit of Rs. 714 at the end of twelve months.

1937

1   A rectangular garden of breadth 200 ft. has two types of road within it, one of breadth 6 ft going inside along its border, is paved with stone, and another of breadth 4 ft goes right across the centre parallel to the shorter edge of the garden and is asphalted   The cost of paving with stone is Rs 25 per 100 sq ft and that of asphalting is Rs 40 per 100 sq. ft. If the total cost of paving and asphalting the roads be Rs 2,064 8, find the length of the garden

2 (a) A banker borrows money at 4 per cent. per annum and pays interest at the end of the year   He lends it at 8 per cent. per annum payable half yearly and receives interest at the end of the year. By this means he gains Rs 624 a year.   How much money has he borrowed ?

   (b) Find the present worth of Rs 1,829 4a. due 4 months 10 days hence at $4\frac{1}{2}$°/₀ per annum simple interest

3   A tradesman uses false weights both in buying as well as selling, and gains 25 per cent each time. If the sale price be the same as the purchase price, how much per cent does he gain by his dishonesty ?

4   Two passengers have together 4 maunds of luggage, and they have to pay respectively Rs 2. 4a and Rs 2 12a for the extra luggage with them   If all the luggage had been with one passenger, he would have had to pay Rs. 6 8a. Find how much luggage each person can carry free of charge. What is the rate at which excess luggage is charged ? Verify your answer.

5   A train passes two persons who are travelling in the same direction at 2 miles and 4 miles per hour in 9 and 10 seconds respectively. Find the length of the train and its speed per hour   Verify your answer

1938

1   The length of a room is twice its breadth. Its flooring at the rate of 2 annas per square yard costs Rs 2 4as and the papering of the walls at the rate of one anna per square yard costs Rs 4 8as Find the length, breadth and height of the room.

2. The interest on a certain sum of money for one year is £75, and the Compound Interest for 2 years is ℗154 13s 9d   Find the principal and the rate of interest per cent per annum.

What will be the total amount at Compound Interest after three years ?

3 A motor-car runs from A to B by one road in $37\frac{1}{2}$ minutes. Another car goes by a different road which is five miles longer and travelling 10 miles an hour quicker reaches B in 35 minutes   Find the rates and the distance travelled by each. Verify your answer

4. (a) A person fills a glass with medicine and drinks a quarter of it. He then fills up the glass with water and drinks a third of it. Again he fills it up with water and drinks a half. How much medicine does he drink altogether and how much each time ?

(b) Two positive numbers differ by 2 and their reciprocals differ by $\frac{1}{40}$. Find them,

IV. DELHI UNIVERSITY

1935

1 (a) Find the value of $\dfrac{1\cdot8}{\sqrt{2}+\sqrt{2}}$ correc to four places of decimals

(b) Find the least number which when divided by 8, 9, 12 and 14 leaves 4 as remainder in each case, but when divided by 11, leaves no remainder.

2. (a) Simplify . $\dfrac{2\cdot6\times2\cdot6-1\cdot4\times1\cdot4}{2\cdot6-1\cdot4} - \dfrac{\frac{1}{8}+\frac{1}{5}-\frac{1}{7}}{\frac{1}{3}-(\frac{1}{8}+\frac{1}{13})}$

(b) Express $\frac{4}{5}$ of $\frac{3}{37}$ of £2 15s. 6d + $\frac{4}{15}$ of £2 8s. 9d. as the decimal of £29.

3 A man sells 150 articles at Rs 2 10a 8p each Find, by Practice, the selling price of these articles

Find also his gain per cent. if the cost price of the articles be Rs. 350,

4. Find the cost of papering the walls of a room 17 ft 3 in. long 12 ft. 9 in. broad and 10 ft 4 in high at Re 1. 8*a* per square yard, allowing for a door 5 ft by 3 ft. and two windows, each 3 ft by $1\frac{1}{2}$ ft.

5 On what sum is the simple merest for 3 years at 3 per cent per annum equal to Rs 22 8*a*. ? Find the compound interest on the same sum for the same period at the same rate of interest

6 *A* and *B* can do a piece of work in 10 days, *B* and *C* in 12 days, and *C* and *A* in 15 days If *B* alone works for 15 days and is then joined by *A* and *C* find the number of days in which the whole work will be finished.

1936

1 (*a*) Simplify —

$$\frac{\frac{1}{3}+\frac{2}{11}+\frac{7}{24}-\frac{1}{3} \text{ of } \frac{2}{11} \text{ of } \frac{7}{24}}{1-\frac{1}{3} \text{ of } \frac{1}{11}-\frac{2}{11} \text{ of } \frac{7}{24}-\frac{7}{24} \text{ of } \frac{1}{3}}$$

(*b*) In an examination 9 candidates received 97 marks each, 25 received 79 each, 33, received 57 each and 69 received 40 each Find the average marks of the whole number examined, to the nearest whole number

2 (*a*) What are prime factors of 45090045, and what is the smallest number by which it must be multiplied in order to make it a perfect square ?

(*b*) What fraction of $\dfrac{10\frac{3}{4}}{16}$ of Rs. 10 10*a*. 8*p* must be added to $\{\frac{2}{3}-\frac{3}{5}(\frac{5}{13}-\frac{7}{8} \text{ of } \frac{20}{31})+\frac{2}{13}\}$ of Rs. 9 5*a*. 4*p* that the sum may be equal to Rs 10 ?

3 A rectangular metal plate of uniform thickness is 7'5 inches long and 3 6 inches broad, its weight is 2 4 oz From it is cut a circular plate of diameter 3'6 inches Find the weight of the circular plate.

4 Divide Rs 145 among *A*, *B* and *C* so that $\frac{1}{2}$ of *A*'s share may be equal to $\frac{2}{3}$ of *B*'s and $\frac{2}{3}$ of *B*'s may be equal to $\frac{1}{4}$ of *C*'s.

5 The present value of a bill of Rs. 442. 12*a*. is Rs 385. Find how long the bill has to run at $4\frac{1}{2}$ per cent per annum simple interest

6 A debtor can pay 14*as.* in the rupee, but if his creditors would take 20 per cent off his debt, he could pay them and have Rs 45 left. What are his debts and what are his assets ?

1937.

1 (*a*) Simplify $1\frac{1}{11} - \frac{1-3\frac{7}{2}}{2-\frac{1}{3}} + \frac{1\frac{2}{5}}{3\frac{1}{3}} - \frac{5\frac{3}{8}}{6\frac{1}{4}}$ of $\left(\frac{1}{3} - \frac{\frac{1}{3}-\frac{1}{4}}{4\frac{3}{4}-3\frac{3}{8}} \right)$

(*b*) The circumferences of the wheels of a carriage are $6\frac{3}{14}$ ft and $8\frac{1}{18}$ ft. What is the least distance in which both wheels simultaneously complete an integral number of revolutions ? Find also the number of revolutions completed by each wheel when the carriage has moved through this distance.

2. (*a*) When a rupee is worth 1*s.* $4\frac{3}{16}d$, how many rupees can be bought for £13 9*s.* $9\frac{1}{3}d$.?

(*b*) Find by the method of Simple Practice, the cost of $5354\frac{3}{4}$ cwt. of soap at £4 4*s* 8*d.* per cwt.

3. A man bought a number of oranges at 4 for 3 annas and sold them at 5 for 4 annas, and thus gained Re. 1. What was the number of oranges bought and what did he gain per cent. ?

4 *A, B* and *C* can separately do a work in 10, 15 and 20 days respectively In what time will the work be done if only *A* works whole time while *B* works three-fourths and *C* one-third of the working hours each day ?

5 The difference between the simple interest and true discount on a sum of money for 9 months at $2\frac{1}{2}$ per cent is Re 1. 6*a.* 6*p* Find the sum.

6. A man sells 4 per cent Rs 4,500 stock at $96\frac{1}{8}$ and invests in $3\frac{1}{4}$ per cents. Government Securities at $89\frac{7}{8}$. Find the change in his income, if $\frac{1}{8}$ per cent. brokerage is charged on each transaction.

1938.

1. (*a*) Simplify $\frac{9}{11}$ of $\frac{44}{45} + \frac{7}{10} - 1\frac{2}{5}$ of $\frac{3}{4}$ of $\frac{1}{2} + \frac{5}{6} - \frac{1}{8} - \frac{16}{45}$.

(*b*) Reduce 3*s.* 9*d.* to the decimal of £5.

2. (*a*) Find by practice, the cost of 435 maunds 38 seers 12 chhataks of rice at Rs 6. 8*a.* per maund.

(b)   A man in India wishes to send to his son in England £300 a year in monthly instalments   How much will he have to pay monthly in rupees, the value of 1 rupee in English money being 1s. $4\frac{7}{3}\frac{.}{.}d.$?

3   A shopkeeper sells an article so as to make 25 per cent on the cost price. If he had sold it for 6a more he would have made 30 per cent   What is the selling price ?

4.   The length of a room is twice its width.   The cost of carpeting it at Rs 3. 12a. per square yard is Rs 163. 5a 4b and the cost of papering the walls at 1a 9p. per square foot is Rs. 110 4a.   Find the height of the room

5.   On what sum of money will the compound interest (payable annually) for 2 years be the same as the simple interest on Rs 943 for 10 years, reckoning interest at the rate of 5 per cent per annum ?

6.   A man holding Rs. 45,000 of a 3 per cent. stock sells out when the stock is standing at 95, and invests the proceeds in a 5 per cent. stock at 114. What is the change in his income ?

1939

1   (a) Simplify — $\frac{163}{300} + \frac{3}{-} - \frac{5}{6}$ of $\frac{3}{-} \times 1\frac{1}{4} - \frac{1}{10}(10 + \frac{13}{30})$.

(b)   Show that to four places of decimals $\frac{181}{-.}$ is a correct approximation to the square root of 5

2.   (a) Find by Practice, the cost of 6 tons 7 cwts. 21 lb at £ 2 6s 8d. per ton.

(b)   If 46 per cent of a regiment containing 1150 men are recruits, how many recruits must be taken out in order that 25 per cent. of the remainder may be recruits

3   A tradesman's prices are 25 per cent above cost price, but he allows a customer a discount of 12 per cent on the amount of his bill.   What percentage of profit does the tradesman make ?

4   A rectangle 110 ft by 90 ft has a grass-plot 55 ft. by 45 ft in the centre   Find the cost of gravelling the rest of it to a depth of six inches at Rs. 4. 8a per cubic yard

5. The compound interest on a certain sum of money at 4 per cent. for 2 years is Rs 15 more than the simple interest for the same time at the same rate. Find the sum.

6. A person invests Rs. 12000 partly in the 3 per cents at 72 and partly in the 4 per cents. at 84. He sells the former at 64 and the latter at 96, and thus realizes the sum invested. How much does he invest in each stock ?

1940.

1 (a) I have to spend $\frac{1}{10}$ of my income on house rent, $\frac{1}{10}$ of the remainder on conveyance, $\frac{1}{3}$ of the further remainder on children's education, after which I have Rs. 648 left over. What is my income ?

(b) Find the square root of 7, correct to 3 decimal places

2. (a) Find by practice the cost of 72 yards 1 foot, 4 inches of cloth at Rs. 3. 14a. 6p. per yard, correct to the nearest anna

(b) In an examination, a candidate must get 40 p.c. marks to pass. A candidate who gets 210 marks fails by 40 marks. Find the maximum number of marks.

3. The simple interest on a certain sum for 8 months at 6 per cent. per annum is Rs. 100 less than the simple interest on the same sum for 19 months at 3 per cent. per annum. Find the principal

4 The difference between the interest and discount on a certain sum for 15 months at 3 per cent. amounts to 9d. Find the sum.

5. The length of a rectangular court is to its breadth as 5 . 3. If the cost of matting the floor at 10 a. per square foot be Rs 150, find its length and breadth.

6 A fruiterer buys a certain number of mangoes at 20 for 3a. and an equal number at 30 for 3a. He mixes them and sells them at 25 for 3a. What is his gain or loss per cent. ?

ANSWERS

1935

1. (a) Seven thousand and nine million, fifty-six thousand and seven hundred (b) 5000700028.
2. (a) $\frac{79}{198}$; ·5851   (b) 2380802+0
3. (a) 1 183.   (b) 900.
4. (a) Rs 625.   (b) Rs. 4999. 8a , 8a.
5 (a) Rs. 961.   (b) Rs 1073. 9a. 4p.

1936.

1. (a) Local value of 0 is always zero, $\frac{1}{100}$, $\frac{3}{10000}$, 1.
   (a) 00027
2 (a) £12 6s 6d   (b) 160 oz
3. (a) 5% or, (a) 143 cwt.   (b) 2 3
4. (a) Rs 421 , 10%.   (b) £3 13s $5\frac{7}{10}d$
5 £816 16s

1937

1 (a) 4·151 nearly.   (b) 9658, $\overline{\text{IX}}$DCLVIII·
2. (a) Rs 3289 6a. $9\frac{17}{31}p$
   (b) 20795, $\frac{7}{10}$, $\frac{7}{1000}$, $\frac{9}{10000}$, $\frac{5}{100000}$ etc.
3. (a) Rs 500 each, 6 yrs. , 8 yrs.
   (b) Rs 6000.
4. (a) 100 men , (b) Original Rate 3a 6p per dozen
5. (a) Rs 2+00 , 2nd (b) 279$\frac{31}{385}$ lbs or, Monday

1938

1. (a) 648721. (b) 0203125   (c) 1802.
2 (a) 990. 13a 11p   (b) 15 ft
3 (a) Rs 1000, Rs 800, Rs. 750 (o) Rs 820.
4 (a) 9 days   (b) £2. 10s
5 (a) Rs 420 , $6\frac{3}{4}$%   (b) 2nd , £69300.
6 (a) 100 153   (b) Monday

1939

1 (a) $30\frac{1}{3}$ (b) $\frac{455}{1000}$is greater by ·0036
   (c) Rs 1+1, 5a. 3 p
2. (a) 546   (b) Rs. 1000.

3. (a) Thursday   (b) Rs 5100.
4   (a) A Rs. 2213. 8a 8p., B Rs. 1660. 2a 6p., C Rs.
         1328 2a , D Rs 1438 12a. 10 p   (b) 2   13
5. (a) 200   (b) £19000
6. (a) 3a. 8p. (b) 264 cu ft., Rs 7. 5a. 4p. or 5 miles.

1940.

1 (a) 143   (b) 99.
2 (a) Rs 385 11a. 10½p   (b) Rs. 5100
3 (a) Monday   (b) 2 3.
4. (a) 44 or £19½ (b) 2 miles.
5   (a) Rs 3000.   (b) £3937 10s.
6. 6 32 minutes or $17\frac{5}{11}$ inches.

RAJPUTANA BOARD

1935.

1. (a) 999984=251×83×2⁴×3.
         100149=251×133×3 , G C. M =753.
    (b) 275 times, 003 is the remainder
2. (a) 1 hour 40 min.   (b) 1'0002.
3. (a) Rs 3. 12 a per md.   (b) 21'2 yrs.
4   (a) 12 ft , £1 15s. 7d   (b) 600 soldiers.
5   (a) 5½%.   (b) Rs 2500.

1936.

1 (a) $\frac{5}{3\cdot n}$. (b) No. of girls 42, No. of oranges 9.
        No of Mangoes 11.
2. (a) 65%   (b) £99 13s 9d
3   (a) Rs. 1007 5a 4p   (b) 10 A.M.
4   (a) Total Income Rs. 280, 4$\frac{3}{4}$%. (b) Sum=£2015.
5   (a) 3125.   (b) 1 2.

1937.

1. (a) '0203125.   (b) Rs. 1 6a. 7p.
2. (a) 18 ft   (b) 45 days.
3. (a) £6250 , £6500, £7030. 8s
    (b) cent. per cent. , C passes getting the minimum
        No of marks.
4. (a) Horse Rs 250, carriage Rs 50. (b) 4%, 5%.
5. (a) 301.   (b) Rs. $9\frac{8}{13}$.

1938

1	(a) Rs. 80	(b) 5 years.
2	(a) Rs. 3042 , Rs 2812 8a	(b) £1200, £2800
3	(a) 66⅔%	(b) Rs 187. 8a.
4	(a) 7⅐ sec	(b) 79$\frac{1}{11}$ yds
5	(a) 30, 24, 8 ft	(b) 1 hr 16 min 48 sec.

1939

1 (a) Rs 39   (b) ±2000, 10 p. c.
2 (a) £10200 stock   (b) £7 13s
3 (a) $\frac{20}{3}$ seers of the 1st mixture, $\frac{7}{3}$ seers of the second
   (b) £35   £25
4 (a) Length 24 ft breadth 18 ft height 15 ft
   (b) 3 7
5. (a) 9 miles per hour   (b) 13⅓ days

C P BOARD

1935

1 (a) 6546   (b) Rs. 1955. 8a. 6p
2 2$\frac{9}{37}$ cub ft , Rs 6 3a. 6⅔p
3. Rs. $\frac{1080}{2873}$, Rs. 880½
4. 3 1 5 9$\frac{9}{11}$ days. 6. 19, 35 miles per hour.

1936.

1. Breadth 15 ft. , height 10½ ft
2 They meet at 11-25 A M. , 8$\frac{2}{11}$ miles from A.
3. (a) £1711 15s   (b) Rs. 15   Rs. $\frac{5}{17}$
4. 12%   5. 28 days. 6 A Rs 471 , B Rs 243

1937

1 400 ft   2 (a) Rs 15000   (b) Rs. 1800.
3 56¼%
4 30 seers , Rs 2 per md.
5 88 yards , 22 miles per hour

1938.

1. Length 6 yds Breadth 3 yds Height 4 yds
2 £1200, 6¼ p c , £1439. 7s 1$\frac{5}{13}$d.
3 20 mi. per hour, 30 mi per hour, 12½ mi, 17⅓ mi.
4 (a) ¾ glass, ¼ glass each time   (b) 10, 8

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1935.

1 (a) ·9670. (b) 1012   2. (a) 1. (b) ·03.
3   Rs 400. $14\frac{2}{7}°/_0$ 4.   Rs. 99 $5a.{\sim}4p.$
5.   Rs. 250, Rs. 23 $2as.$ $10\frac{112}{135}p$
6   The remaining work will be finished by A, B, C working together in one day.

1936.

1. (a) 1.   (b) 55 marks.
2. $5 \times 3 \times 3 \times 7 \times 7 \times 11 \times 11 \times 13 \times 13$, 5.   (b) $\frac{3}{4}$.
3   $\frac{702}{875}$ oz   4.   A's share = Rs. 60.   B's share = Rs. 45.
                                 C's share = Rs. 40.
5. $3\frac{1}{3}$ yrs.   6.   Debts = Rs. 600. Assets = Rs. 525

1937.

1   (a) 1   (b) 435 ft., 70, 54
2.   (a) 200 rupees.   (b) £22668. 8s 10d.
3.   320, $6\frac{2}{3}$ per cent.   4.   6 days   5   Rs. 4075.
6.   No change

1938.

1   (a) 2. (b) ·0375.
2.   (a) Rs 2833. 12a. 9p.   (b)   Rs. 369, $15a. \frac{155}{178}p.$
3.   Rs. 9 6a   4.   12 ft.   5.   Rs. 4600
6.   Rs. 525 increase.

1939.

1.   (a) 1. 2. (a) £14, 16s. $9\frac{1}{4}d$. (b) 322. 3. 10 $°/_0$
4   Rs 618. 12a. 5   Rs. 9375. 6. Rs. 7200, Rs. 4800.

1940

1. (a) Rs. 1200. (b) ·264   2 (a) Rs. 283 (b) 625
3   Rs. 13333, 5a. 4p.   4. £27. 13s. 4d.
5   20, 12 ft.   6.   4% loss.

THE END.

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 8 12	17 21 25	29 33 37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 11	15 19 23	26 30 34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 10	14 17 21	24 28 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 6 10	13 16 19	23 26 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9	12 15 18	21 24 27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 8	11 14 17	20 23 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8	11 13 16	18 21 24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2 5 7	10 12 15	17 20 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7	9 12 14	16 19 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7	9 11 13	16 18 20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	13 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5021	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1 2 3	4 5 6	7 8 9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 3	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7753	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 5 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8083	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 1	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1 2 3	4 5 6	7 8 9
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1 1	1 1 1	2 2 2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 2 1	1 1 2	2 2 2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1 1	1 1 2	2 2 2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1091	0 0 1 1	1 1 2	2 2 2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 0 1 1	1 1 2	2 2 2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1 1	1 2 2	2 2 3
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1 1	1 2 2	2 2 2
07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1 1	1 2 2	2 2 3
08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1 1	1 2 2	2 2 3
09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1 1	1 2 2	2 2 3
10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1 1	1 2 2	2 2 3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1 1	2 2 2	2 2 3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1 1	2 2 2	2 2 3
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1 1	2 2 2	2 3 3
14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1 1	2 2 2	2 3 3
15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1 1	2 2 2	3 3 3
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1 1	2 2 2	3 3 3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1 1	2 2 2	3 3 3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1 1	2 2 2	3 3 3
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1 1	2 2 3	3 3 3
20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1 1	2 2 3	3 3 3
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1 2	2 2 3	3 3 3
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1 2	2 2 3	3 3 3
23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1 2	2 2 3	3 3 4
24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1 2	2 2 3	3 3 4
25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1 2	2 2 3	3 3 4
26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1 2	2 3 3	3 3 4
27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1 2	2 3 3	3 4 4
28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1 2	2 3 3	4 4 4
29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1 2	2 3 3	4 4 4
30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1 2	2 3 3	4 4 4
31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1 2	2 3 3	4 4 4
32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1 2	2 3 3	4 4 4
33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1 2	2 3 3	4 4 4
34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2 2	3 3 4	4 5 —
35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2 2	3 3 4	4 5 —
36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2 2	3 3 4	4 5 —
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2 2	3 3 4	4 5 —
38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2 2	3 3 4	4 5 5
39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2 2	3 3 4	5 5 —
40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2 2	3 4 4	5 5 —
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2 2	3 4 4	5 5 —
42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2 3	3 4 4	5 5 6
43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2 3	3 4 —	5 5 6
44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2 3	3 4 4	5 5 6
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2 3	3 4 5	5 6 —
46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2 3	3 4 5	5 6 —
47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2 3	3 4 5	5 6 —
48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2 3	4 4 5	6 6 —
49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2 3	4 4 5	6 6 —

ANTI-LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1 2 3	4 5 6	7 8 9
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	6 6 7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 5 7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 1 5	6 6 7
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 7 7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 8
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 6	6 7 8
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1 2 3	4 5 6	7 8 9
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
64	4365	4375	4395	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 9 10
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6903	2 3 5	6 8 9	11 13 14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2 4 5	7 9 11	13 14 16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2 4 6	7 9 11	13 15 17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20